

Solution of ECE 300 Test 8 S09

1. A voltage $v(t)$ is graphed below. It can be described by a sum of a constant and some scaled and shifted unit step functions of the form

$$v(t) = A + Bu(t-b) + Cu(t-c) + Du(t-d)$$

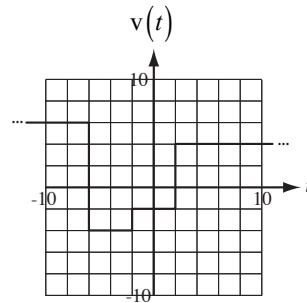
Find the numerical values of the constants.

$$v(t) = 6 - 10u(t+6) + 2u(t+2) + 6u(t-2)$$

$$A = 6, \quad B = -10, \quad C = 2, \quad D = 6$$

$$b = -6, \quad c = -2, \quad d = 2$$

$$v(t) = 6, \quad t < -10 \quad \text{and} \quad v(t) = 4, \quad t > 10$$



2. (a) What is the numerical value of $v_c(2)$?

$$v_c(t) = Ae^{-t/\tau} + v_f, \quad t > 0$$

$$\tau = RC = 1 \text{ s}$$

$$v_f = 3 \text{ and } v_c(0^+) = 0 \Rightarrow A = -3$$

$$v_c(t) = 3(1 - e^{-t}), \quad 0 < t \leq 2$$

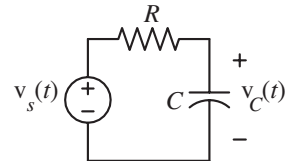
$$v_c(2) = 3(1 - e^{-2}) = 2.59 \text{ V}$$

- (b) What is the maximum current that occurs in this circuit over all time and when does it occur?

Before $t = 0$ the current is zero because the source has not turned on yet. At $t = 0^+$ the capacitor is uncharged and the voltage across the resistor is 3 V so the current is $3 \mu\text{A}$. From that point on in time, the current decreases.

$$v_s(t) = 3u(t) \text{ V}$$

$$R = 1\text{M}\Omega, \quad C = 1\mu\text{F}$$



3. (a) Find the numerical time constant for inductor current in the circuit below.

$$\tau = L / R_{eq} = 1\text{mH}/20\text{k}\Omega = 50 \text{ ns}$$

- (b) Now knowing that the time constant is much less than 1 ms, find the numerical value of the inductor current $i_L(t)$ at the following times.

$$i_L(-3^+) = \text{_____ mA}$$

$$i_L(-2^+) = \text{_____ mA}$$

$$i_L(0^+) = \text{_____ mA}$$

$$i_L(\infty) = \text{_____ mA}$$

At $t = -3^+$, the current source has a value of 3 mA and the voltage source has a value of 40 V and all currents and voltages are constant. By superposition, the current through the inductor is 3 mA plus 2 mA or 5 mA.

At $t = -2^+$ the voltage source has just changed to 90 V and the current source is still 3 mA but the inductor current must still be 5 mA because it cannot change instantaneously.

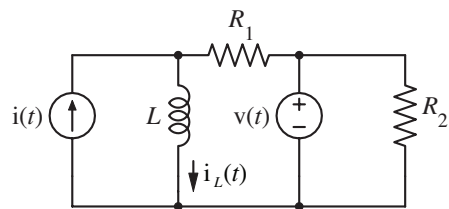
At $t = 0^+$ the current source has just gone to 0 mA and the voltage source is still 90 V. At $t = 0^-$ the transient from the last switching is long since over so the voltages and currents are again all constant. So the inductor current due to the 90V alone and is 4.5 mA and due to the current source alone is 3 mA for a total of 7.5 mA. The inductor current cannot change instantaneously so at $t = 0^+$ it is still 7.5 mA.

At $t \rightarrow \infty$, the current source is 0 mA and the voltage source is 90 V all voltages and currents are constant and the inductor current, which is due to the 90 V source alone is 4.5 mA.

$$i(t) = 3u(-t) \text{ mA}$$

$$v(t) = 40 + 50u(t+2) \text{ V}$$

$$L = 1 \text{ mH} , R_1 = 20 \text{ k}\Omega , R_2 = 50 \text{ k}\Omega$$



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1. A voltage $v(t)$ is graphed below. It can be described by a sum of a constant and some scaled and shifted unit step functions of the form

$$v(t) = A + Bu(t-b) + Cu(t-c) + Du(t-d)$$

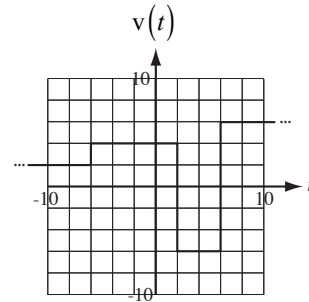
Find the numerical values of the constants.

$$v(t) = 2 + 2u(t+6) - 10u(t-1) + 12u(t-6)$$

$$A = 2, B = 2, C = -10, D = 12$$

$$b = -6, c = 2, d = 6$$

$$v(t) = 2, t < -10 \text{ and } v(t) = 6, t > 10$$



2. (a) What is the numerical value of $v_C(2)$?

$$v_C(t) = Ae^{-t/\tau} + v_f, t > 0$$

$$\tau = RC = 1 \text{ s}$$

$$v_f = 5 \text{ and } v_C(0^+) = 0 \Rightarrow A = -5$$

$$v_C(t) = 5(1 - e^{-t}), 0 < t \leq 2$$

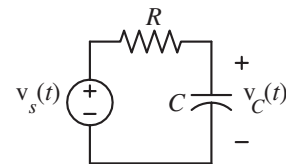
$$v_C(2) = 5(1 - e^{-2}) = 4.32 \text{ V}$$

- (b) What is the maximum current that occurs in this circuit over all time and when does it occur?

Before $t = 0$ the current is zero because the source has not turned on yet. At $t = 0^+$ the capacitor is uncharged and the voltage across the resistor is 5 V so the current is $5 \mu\text{A}$. From that point on in time, the current decreases.

$$v_s(t) = 5u(t) \text{ V}$$

$$R = 1\text{M}\Omega, C = 1\mu\text{F}$$



3. (a) Find the numerical time constant for inductor current in the circuit below.

$$\tau = L / R_{eq} = 2\text{mH}/20\text{k}\Omega = 100 \text{ ns}$$

- (b) Now knowing that the time constant is much less than 1 ms, find the numerical value of the inductor current $i_L(t)$ at the following times.

$$i_L(-3^+) = \text{_____ mA}$$

$$i_L(-2^+) = \text{_____ mA}$$

$$i_L(0^+) = \text{_____ mA}$$

$$i_L(\infty) = \text{_____ mA}$$

At $t = -3^+$, the current source has a value of 2 mA and the voltage source has a value of 50 V and all currents and voltages are constant. By superposition, the current through the inductor is 2 mA plus 2.5 mA or 4.5 mA.

At $t = -2^+$ the voltage source has just changed to 110 V and the current source is still 2 mA but the inductor current must still be 4.5 mA because it cannot change instantaneously.

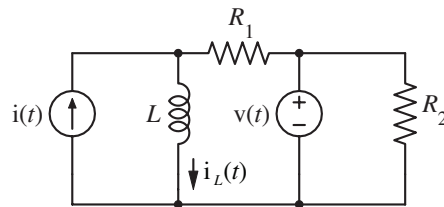
At $t = 0^+$ the current source has just gone to 0 mA and the voltage source is still 110 V. At $t = 0^-$ the transient from the last switching is long since over so the voltages and currents are again all constant. So the inductor current due to the 110V alone and is 5.5 mA and due to the current source alone is 2 mA for a total of 7.5 mA. The inductor current cannot change instantaneously so at $t = 0^+$ it is still 7.5 mA.

At $t \rightarrow \infty$, the current source is 0 mA and the voltage source is 110 V all voltages and currents are constant and the inductor current, which is due to the 110 V source alone is 5.5 mA.

$$i(t) = 2u(-t) \text{ mA}$$

$$v(t) = 50 + 60u(t+2) \text{ V}$$

$$L = 2 \text{ mH} , R_1 = 20 \text{ k}\Omega , R_2 = 50 \text{ k}\Omega$$



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1. A voltage $v(t)$ is graphed below. It can be described by a sum of a constant and some scaled and shifted unit step functions of the form

$$v(t) = A + Bu(t-b) + Cu(t-c) + Du(t-d)$$

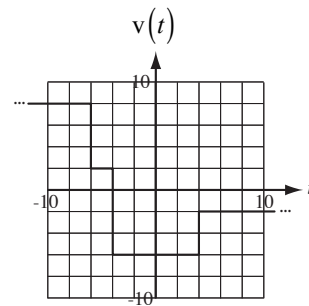
Find the numerical values of the constants.

$$v(t) = 8 - 6u(t+6) - 8u(t+4) + 4u(t-4)$$

$$A = 8, B = -6, C = -8, D = 4$$

$$b = -6, c = -4, d = 4$$

$$v(t) = 8, t < -10 \text{ and } v(t) = -2, t > 10$$



2. (a) What is the numerical value of $v_C(2)$?

$$v_C(t) = Ae^{-t/\tau} + v_f, t > 0$$

$$\tau = RC = 1 \text{ s}$$

$$v_f = 10 \text{ and } v_C(0^+) = 0 \Rightarrow A = -3$$

$$v_C(t) = 10(1 - e^{-t}), 0 < t \leq 2$$

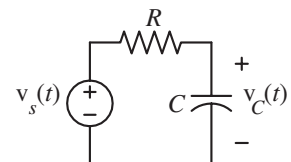
$$v_C(2) = 10(1 - e^{-2}) = 8.63 \text{ V}$$

- (b) What is the maximum current that occurs in this circuit over all time and when does it occur?

Before $t = 0$ the current is zero because the source has not turned on yet. At $t = 0^+$ the capacitor is uncharged and the voltage across the resistor is 10 V so the current is $10 \mu\text{A}$. From that point on in time, the current decreases.

$$v_s(t) = 10u(t) \text{ V}$$

$$R = 1\text{M}\Omega, C = 1\mu\text{F}$$



3. (a) Find the numerical time constant for inductor current in the circuit below.

$$\tau = L / R_{eq} = 3\text{mH}/20\text{k}\Omega = 150 \text{ ns}$$

- (b) Now knowing that the time constant is much less than 1 ms, find the numerical value of the inductor current $i_L(t)$ at the following times.

$$i_L(-3^+) = \text{_____ mA}$$

$$i_L(-2^+) = \text{_____ mA}$$

$$i_L(0^+) = \text{_____ mA}$$

$$i_L(\infty) = \text{_____ mA}$$

At $t = -3^+$, the current source has a value of 5 mA and the voltage source has a value of 30 V and all currents and voltages are constant. By superposition, the current through the inductor is 5 mA plus 1.5 mA or 6.5 mA.

At $t = -2^+$ the voltage source has just changed to 50 V and the current source is still 5 mA but the inductor current must still be 6.5 mA because it cannot change instantaneously.

At $t = 0^+$ the current source has just gone to 0 mA and the voltage source is still 50 V. At $t = 0^-$ the transient from the last switching is long since over so the voltages and currents are again all constant. So the inductor current due to the 50V alone and is 2.5 mA and due to the current source alone is 5 mA for a total of 7.5 mA. The inductor current cannot change instantaneously so at $t = 0^+$ it is still 7.5 mA.

At $t \rightarrow \infty$, the current source is 0 mA and the voltage source is 50 V all voltages and currents are constant and the inductor current, which is due to the 50 V source alone is 2.5 mA.

$$i(t) = 5u(-t) \text{ mA}$$

$$v(t) = 30 + 20u(t+2) \text{ V}$$

$$L = 3 \text{ mH} , R_1 = 20 \text{ k}\Omega , R_2 = 50 \text{ k}\Omega$$

