Solution of ECE 300 Test 8 S10

1. A voltage is described by $v(t) = 5\,u(t) - 6\,u(t-2) + 3u(t+2) - 4\,u(-t)$. Write in the numerical voltages at the times indicated in the table below. (Be sure to notice the "$-t$" in the last term.)

t	-4	-1	1	3
$v(t)$	________	________	________	________

$$v(t) = 5\,u(-4) - 6\,u(-4-2) + 3u(-4+2) - 4\,u(-(-4)) = 5\underbrace{u(-4)}_{=0} - 6\underbrace{u(-6)}_{=0} + 3\underbrace{u(-2)}_{=0} - 4\underbrace{u(4)}_{=1} = -4$$

$$v(t) = 5\,u(-1) - 6\,u(-1-2) + 3u(-1+2) - 4\,u(-(-1)) = 5\underbrace{u(-1)}_{=0} - 6\underbrace{u(-3)}_{=0} + 3\underbrace{u(1)}_{=1} - 4\underbrace{u(1)}_{=1} = -1$$

$$v(t) = 5\,u(1) - 6\,u(1-2) + 3u(1+2) - 4\,u(-1) = 5\underbrace{u(1)}_{=1} - 6\underbrace{u(-1)}_{=0} + 3\underbrace{u(3)}_{=1} - 4\underbrace{u(-1)}_{=0} = 8$$

$$v(t) = 5\,u(3) - 6\,u(3-2) + 3u(3+2) - 4\,u(-3) = 5\underbrace{u(3)}_{=1} - 6\underbrace{u(1)}_{=1} + 3\underbrace{u(5)}_{=1} - 4\underbrace{u(-3)}_{=0} = 2$$

2. With reference to the circuit below write in the numerical values of the voltages and currents in the table below.

What is the numerical time constant of this circuit?

Before time $t = 0^-$ the 20 mA current source is active and the voltage source voltage is zero. The capacitor is an open circuit so $i_C(0^-)$ is zero, $v_2(0^-)$ is, by Ohm's law, zero and all the current must flow through R_1. Therefore, $i_x(0^-)$ is 20 mA and, by Ohm's law, $v_I(0^-)$ is 1.6 V. The voltage across the capacitor $v_C(0^-)$ is, by KVL, 1.6 V.

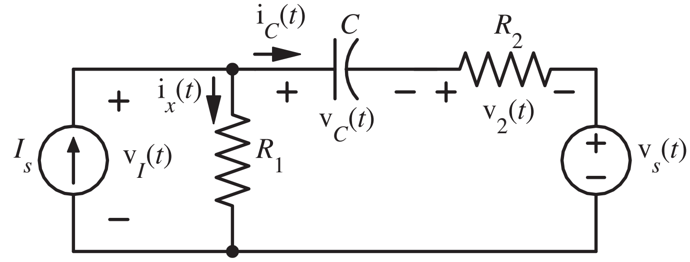
At time $t = 0^+$ we can find the voltages and currents by considering each source one at a time and adding the results. We already have the results for the current source alone because, if the current source were the only source the voltages and currents would not change at time $t = 0$. For the voltage source acting alone, the capacitor is initially uncharged so $v_C(0^+)$ is zero. The current source is set to zero so any current that flows must flow through the voltage source, both resistors and the capacitor. So the current (flowing counterclockwise) times the sum of the resistances must equal the applied voltage. Therefore, that counterclockwise current is $2\text{V}/150\Omega = 13.33\text{ mA}$ and it follows that $i_x(0^+) = 13.33\text{ mA}$ and $i_C(0^+) = -13.33\text{ mA}$. Then, by Ohm's law, $v_I(0^+) = 13.33\text{ mA} \times 80\ \Omega = 1.067\text{ V}$ and $v_2(0^+) = -13.33\text{ mA} \times 70\ \Omega = -0.933\text{ V}$. Then KVL on that loop is satisfied. Now, adding the two results for the two sources acting alone, $v_C(0^+) = 1.6\text{ V} + 0\text{ V} = 1.6\text{ V}$, $i_C(0^+) = 0\text{ A} + (-13.33\text{ mA}) = -13.33\text{ mA}$, $i_x(0^+) = 20\text{ mA} + 13.33\text{ mA} = 33.33\text{ mA}$, $v_2(0^+) = 0\text{ V} + (-0.933\text{ V}) = -0.933\text{ V}$ and $v_I(0^+) = 1.6\text{ V} + 1.067\text{ V} = 2.667\text{ V}$.

At time $t \to \infty$, all voltages and currents are again constant, therefore the capacitor is again an open circuit, $i_C(\infty)$ and $v_2(\infty)$ are zero and the current source current must again all flow through R_1. Therefore, $i_x(\infty) = 20\text{ mA}$ and $v_I(\infty) = 1.6\text{ V}$. Then, by KVL around the right-hand mesh, $v_C(\infty) = -0.4\text{ V}$.

$i_x(0^-) = 20\text{ mA}$	$i_C(0^-) = 0\text{A}$	$v_I(0^-) = 1.6\text{ V}$	$v_C(0^-) = 1.6\text{ V}$	$v_2(0^-) = 0\text{ V}$
$i_x(0^+) = 33.33\text{ mA}$	$i_C(0^+) = -13.33\text{ mA}$	$v_I(0^+) = 2.667\text{ V}$	$v_C(0^+) = 1.6\text{ V}$	$v_2(0^+) = -0.933\text{V}$
$i_x(\infty) = 20\text{ mA}$	$i_C(\infty) = 0\text{ A}$	$v_I(\infty) = 1.6\text{ V}$	$v_C(\infty) = -0.4\text{ V}$	$v_2(\infty) = 0\text{ V}$

The time constant is $\tau = R_{eq}C = 150\ \Omega \times 10\mu\text{F} = 0.00150\text{ s or } 1.5\text{ ms}$.

$I_s = 20\text{mA}$, $v_s(t) = 2\,u(t)$, $R_1 = 80\Omega$, $R_2 = 70\Omega$, $C = 10\mu\text{F}$



$i_C(t)$
C
R_2
$i_x(t)$
+
+
− +
−
$v_C(t)$
$v_2(t)$
I_s
$v_I(t)$
R_1
−
$v_s(t)$

Solution of ECE 300 Test 8 S10

1. A voltage is described by $v(t) = 3u(t) - 8u(t-2) + 2u(t+2) - 7u(-t)$. Write in the numerical voltages at the times indicated in the table below. (Be sure to notice the "$-t$" in the last term.)

t	-4	-1	1	3
$v(t)$	________	________	________	________

$$v(t) = 3u(-4) - 8u(-4-2) + 2u(-4+2) - 7u(-(-4)) = 3\underbrace{u(-4)}_{=0} - 8\underbrace{u(-6)}_{=0} + 2\underbrace{u(-2)}_{=0} - 7\underbrace{u(4)}_{=1} = -7$$

$$v(t) = 3u(-1) - 8u(-1-2) + 2u(-1+2) - 7u(-(-1)) = 3\underbrace{u(-1)}_{=0} - 8\underbrace{u(-3)}_{=0} + 2\underbrace{u(1)}_{=1} - 7\underbrace{u(1)}_{=1} = -5$$

$$v(t) = 3u(1) - 8u(1-2) + 2u(1+2) - 7u(-1) = 3\underbrace{u(1)}_{=1} - 8\underbrace{u(-1)}_{=0} + 2\underbrace{u(3)}_{=1} - 7\underbrace{u(-1)}_{=0} = 5$$

$$v(t) = 3u(3) - 8u(3-2) + 2u(3+2) - 7u(-3) = 3\underbrace{u(3)}_{=1} - 8\underbrace{u(1)}_{=1} + 2\underbrace{u(5)}_{=1} - 7\underbrace{u(-3)}_{=0} = -3$$

2. With reference to the circuit below write in the numerical values of the voltages and currents in the table below.

What is the numerical time constant of this circuit?

Before time $t = 0^-$ the 10 mA current source is active and the voltage source voltage is zero. The capacitor is an open circuit so $i_C(0^-)$ is zero, $v_2(0^-)$ is, by Ohm's law, zero and all the current must flow through R_1. Therefore, $i_x(0^-)$ is 10 mA and, by Ohm's law, $v_I(0^-)$ is 0.8 V. The voltage across the capacitor $v_C(0^-)$ is, by KVL, 0.8 V.

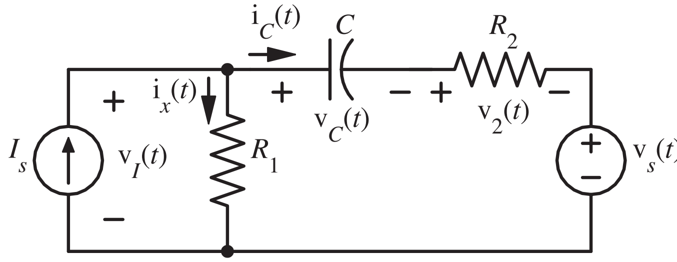
At time $t = 0^+$ we can find the voltages and currents by considering each source one at a time and adding the results. We already have the results for the current source alone because, if the current source were the only source the voltages and currents would not change at time $t = 0$. For the voltage source acting alone, the capacitor is initially uncharged so $v_C(0^+)$ is zero. The current source is set to zero so any current that flows must flow through the voltage source, both resistors and the capacitor. So the current (flowing counterclockwise) times the sum of the resistances must equal the applied voltage. Therefore, that counterclockwise current is $3\text{V}/150\Omega = 20\text{ mA}$ and it follows that $i_x(0^+) = 20\text{ mA}$ and $i_C(0^+) = -20\text{ mA}$. Then, by Ohm's law, $v_I(0^+) = 20\text{ mA} \times 80\ \Omega = 1.6\text{ V}$ and $v_2(0^+) = -20\text{ mA} \times 70\ \Omega = -1.4\text{ V}$. Then KVL on that loop is satisfied. Now, adding the two results for the two sources acting alone, $v_C(0^+) = 0.8\text{ V} + 0\text{ V} = 0.8\text{ V}$, $i_C(0^+) = 0\text{ A} + (-20\text{ mA}) = -20\text{ mA}$, $i_x(0^+) = 10\text{ mA} + 20\text{ mA} = 30\text{ mA}$, $v_2(0^+) = 0\text{ V} + (-1.4\text{ V}) = -1.4\text{ V}$ and $v_I(0^+) = 0.8\text{ V} + 1.6\text{ V} = 2.4\text{ V}$.

At time $t \to \infty$, all voltages and currents are again constant, therefore the capacitor is again an open circuit, $i_C(\infty)$ and $v_2(\infty)$ are zero and the current source current must again all flow through R_1. Therefore, $i_x(\infty) = 10\text{ mA}$ and $v_I(\infty) = 0.8\text{ V}$. Then, by KVL around the right-hand mesh, $v_C(\infty) = -2.2\text{ V}$.

$i_x(0^-) = 10\text{ mA}$	$i_C(0^-) = 0\text{A}$	$v_I(0^-) = 0.8\text{ V}$	$v_C(0^-) = 0.8\text{ V}$	$v_2(0^-) = 0\text{ V}$
$i_x(0^+) = 30\text{ mA}$	$i_C(0^+) = -20\text{ mA}$	$v_I(0^+) = 2.4\text{ V}$	$v_C(0^+) = 0.8\text{ V}$	$v_2(0^+) = -1.4\text{V}$
$i_x(\infty) = 10\text{ mA}$	$i_C(\infty) = 0\text{ A}$	$v_I(\infty) = 0.8\text{ V}$	$v_C(\infty) = -2.2\text{ V}$	$v_2(\infty) = 0\text{ V}$

The time constant is $\tau = R_{eq}C = 150\ \Omega \times 10\mu\text{F} = 0.00150\text{ s or } 1.5\text{ ms}$.

$I_s = 10\text{mA}$, $v_s(t) = 3u(t)$, $R_1 = 80\Omega$, $R_2 = 70\Omega$, $C = 10\mu\text{F}$



Solution of ECE 300 Test 8 S10

1. A voltage is described by $\mathrm{v}(t) = -3\,\mathrm{u}(t) + 4\,\mathrm{u}(t-2) + 5\,\mathrm{u}(t+2) - 8\,\mathrm{u}(-t)$. Write in the numerical voltages at the times indicated in the table below. (Be sure to notice the "$-t$" in the last term.)

t	-4	-1	1	3
$\mathrm{v}(t)$	__________	__________	__________	__________

$$\mathrm{v}(t) = -3\,\mathrm{u}(-4) + 4\,\mathrm{u}(-4-2) + 5\,\mathrm{u}(-4+2) - 8\,\mathrm{u}(-(-4)) = -3\underbrace{\mathrm{u}(-4)}_{=0} + 4\underbrace{\mathrm{u}(-6)}_{=0} + 5\underbrace{\mathrm{u}(-2)}_{=0} - 8\underbrace{\mathrm{u}(4)}_{=1} = -8$$

$$\mathrm{v}(t) = -3\,\mathrm{u}(-1) + 4\,\mathrm{u}(-1-2) + 5\,\mathrm{u}(-1+2) - 8\,\mathrm{u}(-(-1)) = -3\underbrace{\mathrm{u}(-1)}_{=0} + 4\underbrace{\mathrm{u}(-3)}_{=0} + 5\underbrace{\mathrm{u}(1)}_{=1} - 8\underbrace{\mathrm{u}(1)}_{=1} = -3$$

$$\mathrm{v}(t) = -3\,\mathrm{u}(1) + 4\,\mathrm{u}(1-2) + 5\,\mathrm{u}(1+2) - 8\,\mathrm{u}(-1) = -3\underbrace{\mathrm{u}(1)}_{=1} + 4\underbrace{\mathrm{u}(-1)}_{=0} + 5\underbrace{\mathrm{u}(3)}_{=1} - 8\underbrace{\mathrm{u}(-1)}_{=0} = 2$$

$$\mathrm{v}(t) = -3\,\mathrm{u}(3) + 4\,\mathrm{u}(3-2) + 5\,\mathrm{u}(3+2) - 8\,\mathrm{u}(-3) = -3\underbrace{\mathrm{u}(3)}_{=1} + 4\underbrace{\mathrm{u}(1)}_{=1} + 5\underbrace{\mathrm{u}(5)}_{=1} - 8\underbrace{\mathrm{u}(-3)}_{=0} = 6$$

2. With reference to the circuit below write in the numerical values of the voltages and currents in the table below.

   What is the numerical time constant of this circuit?

Before time $t = 0^-$ the 30 mA current source is active and the voltage source voltage is zero. The capacitor is an open circuit so $\mathrm{i}_C(0^-)$ is zero, $\mathrm{v}_2(0^-)$ is, by Ohm's law, zero and all the current must flow through R_1. Therefore, $\mathrm{i}_x(0^-)$ is 30 mA and, by Ohm's law, $\mathrm{v}_I(0^-)$ is 2.4 V. The voltage across the capacitor $\mathrm{v}_C(0^-)$ is, by KVL, 2.4 V.

At time $t = 0^+$ we can find the voltages and currents by considering each source one at a time and adding the results. We already have the results for the current source alone because, if the current source were the only source the voltages and currents would not change at time $t = 0$. For the voltage source acting alone, the capacitor is initially uncharged so $\mathrm{v}_C(0^+)$ is zero. The current source is set to zero so any current that flows must flow through the voltage source, both resistors and the capacitor. So the current (flowing counterclockwise) times the sum of the resistances must equal the applied voltage. Therefore, that counterclockwise current is $4\text{V}/150\Omega = 26.67\text{ mA}$ and it follows that $\mathrm{i}_x(0^+) = 26.67\text{ mA}$ and $\mathrm{i}_C(0^+) = -26.67\text{ mA}$. Then, by Ohm's law, $\mathrm{v}_I(0^+) = 26.67\text{ mA} \times 80\ \Omega = 2.133\text{ V}$ and $\mathrm{v}_2(0^+) = -26.67\text{ mA} \times 70\ \Omega = -1.867\text{ V}$. Then KVL on that loop is satisfied. Now, adding the two results for the two sources acting alone, $\mathrm{v}_C(0^+) = 2.4\text{ V} + 0\text{ V} = 2.4\text{ V}$, $\mathrm{i}_C(0^+) = 0\text{ A} + (-26.67\text{ mA}) = -26.67\text{ mA}$, $\mathrm{i}_x(0^+) = 30\text{ mA} + 26.67\text{ mA} = 56.67\text{ mA}$, $\mathrm{v}_2(0^+) = 0\text{ V} + (-1.867\text{ V}) = -1.867\text{ V}$ and $\mathrm{v}_I(0^+) = 2.4\text{ V} + 2.133\text{ V} = 4.533\text{ V}$.

At time $t \to \infty$, all voltages and currents are again constant, therefore the capacitor is again an open circuit, $\mathrm{i}_C(\infty)$ and $\mathrm{v}_2(\infty)$ are zero and the current source current must again all flow through R_1. Therefore, $\mathrm{i}_x(\infty) = 30\text{ mA}$ and $\mathrm{v}_I(\infty) = 2.4\text{ V}$. Then, by KVL around the right-hand mesh, $\mathrm{v}_C(\infty) = -1.6\text{ V}$.

$\mathrm{i}_x(0^-) = 30\text{ mA}$	$\mathrm{i}_C(0^-) = 0\text{A}$	$\mathrm{v}_I(0^-) = 2.4\text{ V}$	$\mathrm{v}_C(0^-) = 2.4\text{ V}$	$\mathrm{v}_2(0^-) = 0\text{ V}$
$\mathrm{i}_x(0^+) = 56.67\text{ mA}$	$\mathrm{i}_C(0^+) = -26.67\text{ mA}$	$\mathrm{v}_I(0^+) = 4.533\text{ V}$	$\mathrm{v}_C(0^+) = 2.4\text{ V}$	$\mathrm{v}_2(0^+) = -1.867\text{V}$
$\mathrm{i}_x(\infty) = 30\text{ mA}$	$\mathrm{i}_C(\infty) = 0\text{ A}$	$\mathrm{v}_I(\infty) = 2.4\text{ V}$	$\mathrm{v}_C(\infty) = -1.6\text{ V}$	$\mathrm{v}_2(\infty) = 0\text{ V}$

The time constant is $\tau = R_{eq}C = 150\ \Omega \times 10\mu\text{F} = 0.00150\text{ s or }1.5\text{ ms}$.

$I_s = 30\text{mA}$, $v_s(t) = 4\,u(t)$, $R_1 = 80\Omega$, $R_2 = 70\Omega$, $C = 10\mu F$