

Solution of EECS 300 Test 8 F08

1. (a) With reference to the circuit below, fill in the blanks with numbers.

$$\begin{aligned}
 i_5(0^-) &= \text{_____ A}, & i_5(0^+) &= \text{_____ A}, & i_5(\infty) &= \text{_____ A} \\
 v_5(0^-) &= \text{_____ V}, & v_5(0^+) &= \text{_____ V}, & v_5(\infty) &= \text{_____ V} \\
 i_c(0^-) &= \text{_____ A}, & i_c(0^+) &= \text{_____ A}, & i_c(\infty) &= \text{_____ A} \\
 v_c(0^-) &= \text{_____ V}, & v_c(0^+) &= \text{_____ V}, & v_c(\infty) &= \text{_____ V} \\
 i_v(0^-) &= \text{_____ A}, & i_v(0^+) &= \text{_____ A}, & i_v(\infty) &= \text{_____ A} \\
 v_l(0^-) &= \text{_____ V}, & v_l(0^+) &= \text{_____ V}, & v_l(\infty) &= \text{_____ V} \\
 v_{10}(0^-) &= \text{_____ V}, & v_{10}(0^+) &= \text{_____ V}, & v_{10}(\infty) &= \text{_____ V} \\
 i_L(0^-) &= \text{_____ A}, & i_L(0^+) &= \text{_____ A}, & i_L(\infty) &= \text{_____ A} \\
 v_L(0^-) &= \text{_____ V}, & v_L(0^+) &= \text{_____ V}, & v_L(\infty) &= \text{_____ V}
 \end{aligned}$$

(b) Find these numerical values.

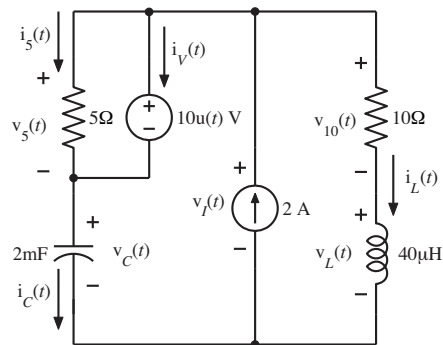
$$\begin{aligned}
 \left. \frac{d}{dt} v_c(t) \right|_{t=0^+} &= \text{_____ V/s} & \left. \frac{d}{dt} i_c(t) \right|_{t=0^+} &= \text{_____ A/s} \\
 \left. \frac{d}{dt} v_L(t) \right|_{t=0^+} &= \text{_____ V/s} & \left. \frac{d}{dt} i_L(t) \right|_{t=0^+} &= \text{_____ A/s}
 \end{aligned}$$

(c) Is the circuit parallel or series *RLC*? Parallel Series

(d) Find the numerical values of α and ω_0 . $\alpha = \text{_____}$, $\omega_0 = \text{_____}$

(e) Is the circuit overdamped, underdamped or critically damped?

Overdamped Underdamped Critically Damped



At $t = 0^-$, all currents and voltages are constant and the capacitor is an open circuit, the inductor is a short circuit and the independent voltage source is a short circuit. Therefore the 2A current must flow through the 10Ω resistor and the inductor. Then the voltage across the 10Ω resistor is 20V which is also the voltage across the 2A current source and across the capacitor. The voltage across the inductor is zero. There is no current through the 5Ω resistor or the capacitor or the voltage source.

At $t = 0^+$, the capacitor voltage of 20V and the inductor current of 2A are unchanged. Therefore the voltage across the 10Ω resistor is also unchanged at 20V. The voltage across the 5Ω resistor jumps discontinuously to 10V because the voltage source turns on at $t = 0$ and the current through the 5Ω resistor jumps discontinuously to 2A. Summing currents at the top node yields

$$i_5 + i_V + i_L = 2 \Rightarrow i_V = 2 - i_5 - i_L = 2 - 2 - 2 = -2$$

and the current through the voltage source must be $-2A$. Summing currents at the node above the capacitor

$$i_5 + i_V = i_C \Rightarrow i_C = 2 + (-2) = 0$$

and the capacitor current must be zero. The voltage across the current source is the sum of the voltages across the voltage source and across the capacitor which is $30V$. That is also the voltage across the series combination of the 10Ω resistor and the inductor. Therefore the voltage across the inductor must be $10V$.

At $t = 0^+$, the derivative of the capacitor voltage is the capacitor current divided by the capacitance which is zero because the capacitor current is zero. The derivative of the inductor current is the inductor voltage divided by the inductance which is $250,000A/s$. From the derivative of the KCL equation at the bottom node

$$i_C + i_L = 2 \Rightarrow i'_C = -i'_L$$

the derivative of capacitor current must be the negative of the derivative of the inductor current. Therefore the derivative of the capacitor current is $-250,000A/s$. From the derivative of the KVL equation around the outer loop

$$-v_C - 10 + v_{10} + v_L = 0 \Rightarrow v'_L = v'_C - v'_{10}$$

the derivative of the inductor voltage must be the derivative of the capacitor voltage minus the derivative of the 10Ω resistor voltage or

$$v'_L = 0 - i'_L \times 10 = -2,500,000 \text{ V/s} .$$

At $t \rightarrow \infty$, all the voltages and currents are again constant and the capacitor is an open circuit and the inductor is a short circuit. The current through the 5Ω resistor is still $2A$ because the voltage source sets a voltage of $10V$ across it. The current through the voltage source must be $-2A$ because the capacitor current is zero. Therefore all the current from the voltage source flows through the 5Ω resistor and the $2A$ from the current source must flow through the 10Ω resistor and the inductor. So the voltage across the 10Ω resistor is $20V$ and the voltage across the inductor is zero. Therefore the voltage across the current source is $20V$. Summing voltages around the outer loop and solving for the capacitor voltage yields $10V$.

Replacing the voltage source with a short circuit and the current source with an open circuit, the circuit is a series RLC circuit with $R = 10\Omega$.

$$\alpha = \frac{R}{2L} = \frac{10\Omega}{2 \times 40\mu\text{H}} = 125,000 / \text{s} , \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\text{mF} \times 40\mu\text{H}}} = 3536 \text{ rad/s} .$$

The circuit is overdamped.

2. Find the numerical value of $i_c(t)$ at time $t = 1\text{ms}$ in the circuit below.

Initial capacitor voltage and inductor current at time $t = 0^+$ are both zero.
Parallel RLC .

$$\alpha = \frac{1}{2RC} = 500, \quad \omega_0 = \frac{1}{\sqrt{LC}} = 2236.1, \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 2179.4$$

Underdamped.

$$i_c(t) = e^{-\alpha t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)) + i_c(\infty)$$

$$i_c(\infty) = 0$$

At $t = 0^+$, the inductor current is zero and the capacitor voltage is zero. Since the resistor is in parallel with the capacitor, the voltage across it is zero and its current must be zero. Also the inductor voltage is 20V positive on the right side. The voltage source cannot immediately force a current through the inductor so, by KCL at the left node, the initial capacitor current is zero. Therefore

$$i_c(0^+) = 0 = B_1 + \underbrace{i_c(\infty)}_{=0} \Rightarrow B_1 = 0$$

By KCL at the left node, the derivative of the capacitor current is

$$\left. \frac{d}{dt} i_c(t) \right|_{t=0^+} = - \left. \frac{d}{dt} i_L(t) \right|_{t=0^+} - \left. \frac{d}{dt} i_R(t) \right|_{t=0^+}$$

where i_R and i_L both point to the right. The derivative of the inductor current is

$$\left. \frac{d}{dt} i_L(t) \right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-20}{0.1H} = -200 \text{ A/s}$$

The derivative of the resistor current is the derivative of the resistor voltage (which is the same as the derivative of the capacitor voltage, since they are in parallel) and the derivative of the capacitor voltage is the capacitor current divided by the capacitance. But the capacitor current is zero at $t = 0^+$. So the derivative of the resistor current is also zero and

$$\left. \frac{d}{dt} i_c(t) \right|_{t=0^+} = -(-200 \text{ A/s}) = 200 \text{ A/s} = -\alpha B_1 + \omega_d B_2$$

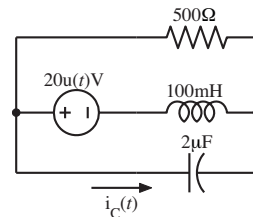
$B_1 = 0$, so

$$B_2 = \frac{200 \text{ A/s}}{2179.4 / \text{s}} = 0.0918 \text{ A}$$

and

$$i_c(t) = 91.8 e^{-500t} \sin(2179.4t) \text{ mA}$$

$$i_c(1 \text{ ms}) = 91.8 e^{-0.500} \sin(2.1794) \text{ mA} = 45.68 \text{ mA}$$



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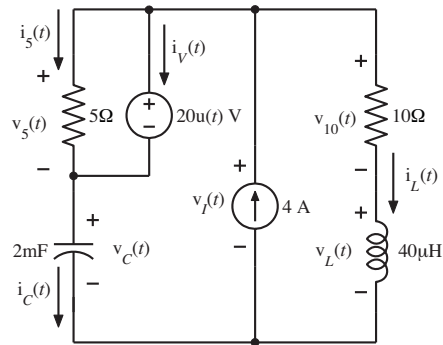
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Overdamped Underdamped Critically Damped



At $t = 0^-$, all currents and voltages are constant and the capacitor is an open circuit, the inductor is a short circuit and the independent voltage source is a short circuit. Therefore the 4A current must flow through the 10Ω resistor and the inductor. Then the voltage across the 10Ω resistor is 40V which is also the voltage across the 4A current source and across the capacitor. The voltage across the inductor is zero. There is no current through the 5Ω resistor or the capacitor or the voltage source.

At $t = 0^+$, the capacitor voltage of 40V and the inductor current of 4A are unchanged. Therefore the voltage across the 10Ω resistor is also unchanged at 40V. The voltage across the 5Ω resistor jumps discontinuously to 20V because the voltage source turns on at $t = 0$ and the current through the 5Ω resistor jumps discontinuously to 4A. Summing currents at the top node yields

$$i_5 + i_V + i_L = 4 \Rightarrow i_V = 4 - i_5 - i_L = 4 - 4 - 4 = -4$$

and the current through the voltage source must be -4A . Summing currents at the node above the capacitor

$$i_5 + i_V = i_C \Rightarrow i_C = 4 + (-4) = 0$$

and the capacitor current must be zero. The voltage across the current source is the sum of the voltages across the voltage source and across the capacitor which is 60V . That is also the voltage across the series combination of the 10Ω resistor and the inductor. Therefore the voltage across the inductor must be 20V .

At $t = 0^+$, the derivative of the capacitor voltage is the capacitor current divided by the capacitance which is zero because the capacitor current is zero. The derivative of the inductor current is the inductor voltage divided by the inductance which is $500,000\text{A/s}$. From the derivative of the KCL equation at the bottom node

$$i_C + i_L = 4 \Rightarrow i'_C = -i'_L$$

the derivative of capacitor current must be the negative of the derivative of the inductor current. Therefore the derivative of the capacitor current is $-500,000\text{A/s}$. From the derivative of the KVL equation around the outer loop

$$-v_C - 20 + v_{10} + v_L = 0 \Rightarrow v'_L = v'_C - v'_{10}$$

the derivative of the inductor voltage must be the derivative of the capacitor voltage minus the derivative of the 10Ω resistor voltage or

$$v'_L = 0 - i'_L \times 10 = -5,000,000 \text{ V/s} .$$

At $t \rightarrow \infty$, all the voltages and currents are again constant and the capacitor is an open circuit and the inductor is a short circuit. The current through the 5Ω resistor is still 4A because the voltage source sets a voltage of 20V across it. The current through the voltage source must be -4A because the capacitor current is zero. Therefore all the current from the voltage source flows through the 5Ω resistor and the 4A from the current source must flow through the 10Ω resistor and the inductor. So the voltage across the 10Ω resistor is 40V and the voltage across the inductor is zero. Therefore the voltage across the current source is 40V . Summing voltages around the outer loop and solving for the capacitor voltage yields 20V .

Replacing the voltage source with a short circuit and the current source with an open circuit, the circuit is a series RLC circuit with $R = 10\Omega$.

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$$i_c(t) = e^{-\alpha t} (B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)) + i_c(\infty)$$

$$i_c(\infty) = 0$$

At $t = 0^+$, the inductor current is zero and the capacitor voltage is zero. Since the resistor is in parallel with the capacitor, the voltage across it is zero and its current must be zero. Also the inductor voltage is 40V positive on the right side. The voltage source cannot immediately force a current through the inductor so, by KCL at the left node, the initial capacitor current is zero. Therefore

$$i_c(0^+) = 0 = B_1 + \underbrace{i_c(\infty)}_{=0} \Rightarrow B_1 = 0$$

By KCL at the left node, the derivative of the capacitor current is

$$\left. \frac{d}{dt} i_c(t) \right|_{t=0^+} = - \left. \frac{d}{dt} i_L(t) \right|_{t=0^+} - \left. \frac{d}{dt} i_R(t) \right|_{t=0^+}$$

where i_R and i_L both point to the right. The derivative of the inductor current is

$$\left. \frac{d}{dt} i_L(t) \right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{-40}{0.1\text{H}} = -400 \text{ A/s}$$

The derivative of the resistor current is the derivative of the resistor voltage (which is the same as the derivative of the capacitor voltage, since they are in parallel) and the derivative of the capacitor voltage is the capacitor current divided by the capacitance. But the capacitor current is zero at $t = 0^+$. So the derivative of the resistor current is also zero and

$$\left. \frac{d}{dt} i_c(t) \right|_{t=0^+} = -(-400 \text{ A/s}) = 400 \text{ A/s} = -\alpha B_1 + \omega_d B_2$$

$B_1 = 0$, so

$$B_2 = \frac{400 \text{ A/s}}{2179.4 / \text{s}} = 0.1836 \text{ A}$$

and

$$i_c(t) = 183.6 e^{-500t} \sin(2179.4t) \text{ mA}$$

$$i_c(1 \text{ ms}) = 183.6 e^{-0.500} \sin(2.1794) \text{ mA} = 91.36 \text{ mA}$$

