Solution of ECE 300 Test 8 S09

1. The current through the inductor in a series RLC circuit is $i_L(t) = 32e^{-4t}\sin(12t)$, $t > 0$. The capacitance is $C = 80$ mF . Find the numerical values of ω_0 , R and L.

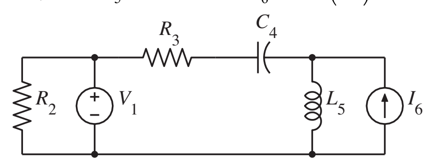
Underdamped, $\alpha = 4$, $\omega_d = 12 \Rightarrow \omega_0^2 = \omega_d^2 + \alpha^2 = 160 \Rightarrow \omega_0 = \sqrt{160} = 12.649$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = 78.125 \text{ mH}$$

$$\alpha = \frac{R}{2L} \Rightarrow R = 2\alpha L = 0.625\ \Omega$$

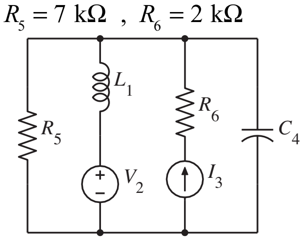
2. In each circuit below find the numerical values of α and ω_0 .

$V_1 = 20$ V , $R_2 = 10$ kΩ , $R_3 = 20$ kΩ
$C_4 = 5\ \mu$F , $L_5 = 50$ mH , $I_6 = 2\,\mathrm{u}(-t)$ mA



Series RLC. $\alpha = \frac{R}{2L} = \frac{20{,}000\ \Omega}{0.1\ \text{H}} = 200{,}000$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\text{mH} \times 5\mu\text{F}}} = 2000$

$L_1 = 20$ mH , $V_2 = 40\,\mathrm{u}(t)$ V , $I_3 = 100\ \mu$A
$C_4 = 200\ \mu$F , $R_5 = 7$ kΩ , $R_6 = 2$ kΩ



Parallel RLC.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 7000\Omega \times 200 \times 10^{-6}\text{F}} = 0.357 \ , \ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\text{mH} \times 200\mu\text{F}}} = 500$$

3. For the circuit below, find these numerical values.

$$\mathrm{i}(0^-) = 0 \;,\; \mathrm{i}(0^+) = 2\text{ mA} \;,\; \mathrm{i}(\infty) = 0$$
$$\mathrm{v}(0^-) = 0 \;,\; \mathrm{v}(0^+) = -40\text{ V} \;,\; \mathrm{v}(\infty) = 0$$

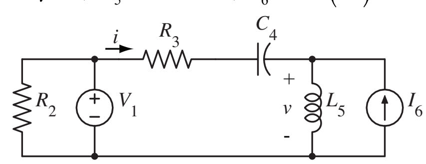
$$\left.\frac{d\,\mathrm{i}(t)}{dt}\right|_{t=0^+} = -400\text{ A/s} \;,\; \left.\frac{d\,\mathrm{v}(t)}{dt}\right|_{t=0^+} = 15.9996\times10^6\text{ V/s}$$

At time $t=0^-$, there is no current through R_3 because it is in series with a capacitor. At that same time, the voltage across the capacitor (positive polarity on the left) is 20V due to the voltage source alone and 0V due to the current source alone for a total of 20V. The current through the inductor (pointing downward) is 0A due to the voltage source alone and 2mA due to the current source alone for a total of 2mA. At $t=0^+$ these values are unchanged. At $t=0^+$ the current source has just turned off and the inductor current is unchanged so that 2mA current has to come from the capacitor (pointing to the right). The capacitor voltage is still 20V and the voltage across R_3 is $20\text{ k}\Omega\times 2\text{ mA} = 40\text{ V}$ (positive polarity on the left). So the voltage across the inductor must be $\mathrm{v}(0^+) = -40\text{ V}$. The rate of change of inductor current at $t=0^+$ is $-40\text{ V} / 50\text{ mH} = -800\text{ A/s}$. Since that current must also flow through R_3 its rate of change is the same. Summing voltages around the middle mesh we get $-20 + iR_3 + v_c + v = 0$ where v_c is positive on the left. Differentiating, $\frac{d\,\mathrm{i}(t)}{dt}R_3 + \frac{d\,\mathrm{v}_c(t)}{dt} + \frac{d\,\mathrm{v}(t)}{dt} = 0$. Solving for $\frac{d\,\mathrm{v}(t)}{dt}$ at time $t=0^+$,

$$\frac{d\,\mathrm{v}(t)}{dt} = -(-800\text{ A/s})20{,}000 - \frac{2\text{ mA}}{5\;\mu\text{F}} = 16\times10^6\text{ V/s} - 400\text{ V/s} = 15.9996\times10^6\text{ V/s}.$$

$V_1 = 20\text{ V}$, $R_2 = 10\text{ k}\Omega$, $R_3 = 20\text{ k}\Omega$

$C_4 = 5\;\mu\text{F}$, $L_5 = 50\text{ mH}$, $I_6 = 2\,\mathrm{u}(-t)$ mA



Solution of ECE 300 Test 8 S09

1. The current through the inductor in a series RLC circuit is $\mathrm{i}_L(t) = 32e^{-4t}\sin(16t)$, $t > 0$. The capacitance is $C = 80\ \mathrm{mF}$. Find the numerical values of ω_0 , R and L.

$$\text{Underdamped, } \alpha = 4 \text{ , } \omega_d = 16 \Rightarrow \omega_0^2 = \omega_d^2 + \alpha^2 = 272 \Rightarrow \omega_0 = \sqrt{272} = 16.492$$

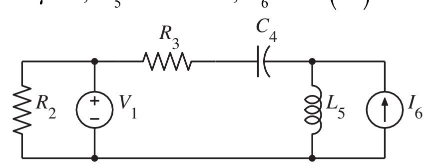
$$\omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = 45.9\ \mathrm{mH}$$

$$\alpha = \frac{R}{2L} \Rightarrow R = 2\alpha L = 0.3676\ \Omega$$

2. In each circuit below find the numerical values of α and ω_0 .

$V_1 = 20\ \mathrm{V}$, $R_2 = 10\ \mathrm{k\Omega}$, $R_3 = 15\ \mathrm{k\Omega}$

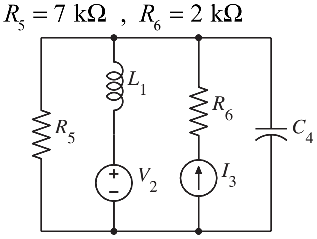
$C_4 = 10\ \mu\mathrm{F}$, $L_5 = 50\ \mathrm{mH}$, $I_6 = 2\mathrm{u}(-t)\ \mathrm{mA}$



$$\text{Series } RLC. \quad \alpha = \frac{R}{2L} = \frac{15{,}000\ \Omega}{0.1\ \mathrm{H}} = 150{,}000 \text{ , } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\mathrm{mH} \times 10\mu\mathrm{F}}} = 1414.2$$

$L_1 = 20\ \mathrm{mH}$, $V_2 = 40\mathrm{u}(t)\ \mathrm{V}$, $I_3 = 100\ \mu\mathrm{A}$

$C_4 = 50\ \mu\mathrm{F}$, $R_5 = 7\ \mathrm{k\Omega}$, $R_6 = 2\ \mathrm{k\Omega}$



Parallel RLC.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 7000\Omega \times 50 \times 10^{-6}\mathrm{F}} = 1.429 \text{ , } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\mathrm{mH} \times 50\mu\mathrm{F}}} = 1000$$

3. For the circuit below, find these numerical values.

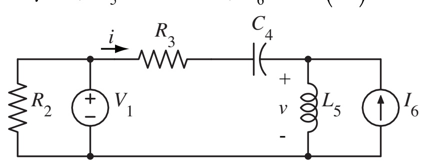
$$\mathrm{i}\left(0^-\right)=0 \;\;, \mathrm{i}\left(0^+\right)=2\text{ mA} \;\;, \mathrm{i}\left(\infty\right)=0$$
$$\mathrm{v}\left(0^-\right)=0 \;\;, \mathrm{v}\left(0^+\right)=-20\text{ V} \;\;, \mathrm{v}\left(\infty\right)=0$$

$$\left.\frac{d\,\mathrm{i}(t)}{dt}\right|_{t=0^+}=-400\text{ A/s} \;,\; \left.\frac{d\,\mathrm{v}(t)}{dt}\right|_{t=0^+}=3.9996\times10^6\text{ V/s}$$

At time $t=0^-$, there is no current through R_3 because it is in series with a capacitor. At that same time, the voltage across the capacitor (positive polarity on the left) is 10V due to the voltage source alone and 0V due to the current source alone for a total of 10V. The current through the inductor (pointing downward) is 0A due to the voltage source alone and 2mA due to the current source alone for a total of 2mA. At $t=0^+$ these values are unchanged. At $t=0^+$ the current source has just turned off and the inductor current is unchanged so that 2mA current has to come from the capacitor (pointing to the right). The capacitor voltage is still 10V and the voltage across R_3 is $10\text{ k}\Omega\times2\text{ mA}=20\text{ V}$ (positive polarity on the left). So the voltage across the inductor must be $\mathrm{v}\left(0^+\right)=-20\text{ V}$. The rate of change of inductor current at $t=0^+$ is $-20\text{ V}/50\text{ mH}=-400\text{ A/s}$. Since that current must also flow through R_3 its rate of change is the same. Summing voltages around the middle mesh we get $-10+iR_3+v_c+v=0$ where v_c is positive on the left. Differentiating, $\frac{d\,\mathrm{i}(t)}{dt}R_3+\frac{d\,\mathrm{v}_c(t)}{dt}+\frac{d\,\mathrm{v}(t)}{dt}=0$. Solving for $\frac{d\,\mathrm{v}(t)}{dt}$ at time $t=0^+$,

$$\frac{d\,\mathrm{v}(t)}{dt}=-\left(-400\text{ A/s}\right)10{,}000-\frac{2\text{ mA}}{5\ \mu\text{F}}=4\times10^6\text{ V/s }-400\text{ V/s}=3.9996\times10^6\text{ V/s}.$$

$V_1=10\text{ V}\;,\;R_2=10\text{ k}\Omega\;,\;R_3=10\text{ k}\Omega$
$C_4=5\ \mu\text{F}\;,\;L_5=50\text{ mH}\;,\;I_6=2\,\mathrm{u}(-t)\text{ mA}$



Solution of ECE 300 Test 8 S09

1. The current through the inductor in a series RLC circuit is $\mathrm{i}_L(t) = 32e^{-4t}\sin(8t)$, $t > 0$. The capacitance is $C = 80$ mF . Find the numerical values of ω_0 , R and L.

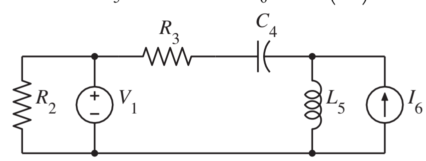
$$\text{Underdamped, } \alpha = 4 \text{ , } \omega_d = 8 \ \Rightarrow \omega_0^2 = \omega_d^2 + \alpha^2 = 80 \Rightarrow \omega_0 = \sqrt{80} = 8.944$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = 156.25 \text{ mH}$$

$$\alpha = \frac{R}{2L} \Rightarrow R = 2\alpha L = 1.25\ \Omega$$

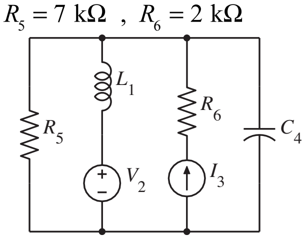
2. In each circuit below find the numerical values of α and ω_0 .

$V_1 = 25$ V , $R_2 = 10$ kΩ , $R_3 = 15$ kΩ
$C_4 = 15\ \mu$F , $L_5 = 50$ mH , $I_6 = 2\mathrm{u}(-t)$ mA



Series RLC. $\alpha = \dfrac{R}{2L} = \dfrac{15{,}000\ \Omega}{0.1\ \text{H}} = 150{,}000$, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{50\text{mH} \times 15\mu\text{F}}} = 1154.7$

$L_1 = 20$ mH , $V_2 = 40\mathrm{u}(t)$ V , $I_3 = 100\ \mu$A
$C_4 = 100\ \mu$F , $R_5 = 7$ kΩ , $R_6 = 2$ kΩ



Parallel RLC.

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 7000\Omega \times 100 \times 10^{-6}\text{F}} = 0.714 \text{ , } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\text{mH} \times 100\mu\text{F}}} = 707.11$$

3. For the circuit below, find these numerical values.

$$\mathrm{i}(0^-) = 0 \ , \ \mathrm{i}(0^+) = 2 \text{ mA} \ , \ \mathrm{i}(\infty) = 0$$
$$\mathrm{v}(0^-) = 0 \ , \ \mathrm{v}(0^+) = -30 \text{ V} \ , \ \mathrm{v}(\infty) = 0$$

$$\left.\frac{d\,\mathrm{i}(t)}{dt}\right|_{t=0^+} = -600 \text{ A/s} \ , \ \left.\frac{d\,\mathrm{v}(t)}{dt}\right|_{t=0^+} = 8.9996\times 10^6 \text{ V/s}$$

At time $t = 0^-$, there is no current through R_3 because it is in series with a capacitor. At that same time, the voltage across the capacitor (positive polarity on the left) is 25V due to the voltage source alone and 0V due to the current source alone for a total of 25V. The current through the inductor (pointing downward) is 0A due to the voltage source alone and 2mA due to the current source alone for a total of 2mA. At $t = 0^+$ these values are unchanged. At $t = 0^+$ the current source has just turned off and the inductor current is unchanged so that 2mA current has to come from the capacitor (pointing to the right). The capacitor voltage is still 25V and the voltage across R_3 is $15 \text{ k}\Omega \times 2 \text{ mA} = 30 \text{ V}$ (positive polarity on the left). So the voltage across the inductor must be $\mathrm{v}(0^+) = -30 \text{ V}$. The rate of change of inductor current at $t = 0^+$ is $-30 \text{ V} / 50 \text{ mH} = -600 \text{ A/s}$. Since that current must also flow through R_3 its rate of change is the same. Summing voltages around the middle mesh we get $-25 + iR_3 + v_c + v = 0$ where v_c is positive on the left. Differentiating, $\frac{d\,\mathrm{i}(t)}{dt}R_3 + \frac{d\,\mathrm{v}_c(t)}{dt} + \frac{d\,\mathrm{v}(t)}{dt} = 0$. Solving for $\frac{d\,\mathrm{v}(t)}{dt}$ at time $t = 0^+$,

$$\frac{d\,\mathrm{v}(t)}{dt} = -(-600 \text{ A/s})15{,}000 - \frac{2 \text{ mA}}{5\ \mu\text{F}} = 9\times 10^6 \text{ V/s} - 400 \text{ V/s} = 8.9996\times 10^6 \text{ V/s}.$$

$$\mathrm{i}(0^-) = 0 \ , \ \mathrm{i}(0^+) = 2 \text{ mA} \ , \ \mathrm{i}(\infty) = 0$$
$$\mathrm{v}(0^-) = 0 \ , \ \mathrm{v}(0^+) = -30 \text{ V} \ , \ \mathrm{v}(\infty) = 0$$

$$\left.\frac{d\,\mathrm{i}(t)}{dt}\right|_{t=0^+} = -600 \text{ A/s} \ , \ \left.\frac{d\,\mathrm{v}(t)}{dt}\right|_{t=0^+} = 8.9996\times 10^6 \text{ V/s}$$

$V_1 = 25 \text{ V} \ , \ R_2 = 10 \text{ k}\Omega \ , \ R_3 = 15 \text{ k}\Omega$

$C_4 = 5\ \mu\text{F} \ , \ L_5 = 50 \text{ mH} \ , \ I_6 = 2\,\mathrm{u}(-t) \text{ mA}$