Solution of ECE 300 Test 9 S10

1. The voltage across the capacitor in a parallel *RLC* circuit for the time range $t > 0$ is $\mathrm{v}_C(t) = 3e^{-400t} + 7e^{-900t}$ volts. If the capacitance of the capacitor is $50\ \mu\text{F}$ find the numerical values of

   (a) The initial capacitor voltage $\mathrm{v}_C(0^+)$

   (b) The initial derivative of the capacitor voltage $\frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+}$

   (c) The damping factor α

   (d) The natural radian frequency ω_0

   (e) The resistance *R*.

   (f) The inductance *L*.

$$\mathrm{v}_C(0^+) = 3 + 7 = 10\ \text{V}$$

$$\frac{d}{dt}(\mathrm{v}_C(t)) = -1200e^{-400t} - 6300e^{-900t} \Rightarrow \frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+} = -7500\ \text{V/s}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -400 / s$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -900 / s$$

$$s_1 + s_2 = -2\alpha = -1300 \Rightarrow \alpha = 650 / s$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2} = 500 \Rightarrow \alpha^2 - \omega_0^2 = 250^2 \Rightarrow \omega_0^2 = 650^2 - 250^2 = 360{,}000 \Rightarrow \omega_0 = 600\ \text{/s}$$

OR

$$-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -400 \Rightarrow \alpha^2 - \omega_0^2 = (-400 + \alpha)^2$$

$$\omega_0^2 = \alpha^2 - (-400 + \alpha)^2 = 422{,}500 - 62{,}500 = 360{,}000 \Rightarrow \omega_0 = 600 / s$$

$$\alpha = \frac{1}{2RC} \Rightarrow R = \frac{1}{2\alpha C} = \frac{1}{2 \times 650 / s \times 50 \times 10^{-6}\,\text{F}} = 15.38\ \Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{360{,}000 / s^2 \times 50 \times 10^{-6}\,\text{F}} = 55.56\ \text{mH}$$

2. With reference to the series *RLC* circuit below, the initial current $i(0^+)$ is 50 mA and the initial capacitor voltage $v_C(0^+)$ is 12 V. Is this circuit overdamped, critically damped or underdamped?

$$\alpha = \frac{R}{2L} = \frac{300\Omega}{2 \times 80\text{mH}} = 1875 / s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 645.49 / s \qquad \text{Therefore, overdamped.}$$

Find these numerical values.

(a) $v_R(0^+) =$ ____________ V (b) $v_L(0^+) =$ ____________ V

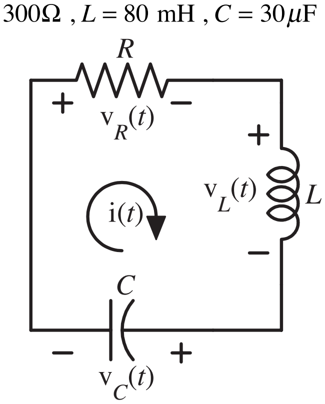
(c) $\frac{d}{dt}(i(t))_{t=0^+} =$ ____________ A/s (d) $\frac{d}{dt}(v_R(t))_{t=0^+} =$ ____________ V/s

(e) $\frac{d}{dt}(v_C(t))_{t=0^+} =$ ____________ V/s (f) $\frac{d}{dt}(v_L(t))_{t=0^+} =$ ____________ V/s

(Be sure that KVL is satisfied and that the derivative of KVL is also satisfied at $t = 0^+$.)

The initial resistor voltage is the initial current times the resistance or 15 volts. By KVL, the initial inductor voltage must be -27 V. The initial rate of change of the inductor current is the initial voltage divided by the inductance or -337.5 A/s. The initial rate of change of the resistor voltage is the initial rate of change of the current, times the resistance, or -101,250 V/s. The initial rate of change of the capacitor voltage is the initial current divided by the capacitance or 1666.7 V/s. From the derivative of KVL, the initial rate of change of inductor voltage is the negative of the sum of the initial rates of change of the resistor voltage and the capacitor voltage or 99583 V/s.

$R = 300\Omega$, $L = 80$ mH , $C = 30\mu$F



Solution of ECE 300 Test 9 S10

1. The voltage across the capacitor in a parallel *RLC* circuit for the time range $t > 0$ is $\mathrm{v}_C(t) = 2e^{-800t} + 7e^{-900t}$ volts. If the capacitance of the capacitor is $50\ \mu\mathrm{F}$ find the numerical values of

   (a) The initial capacitor voltage $\mathrm{v}_C(0^+)$

   (b) The initial derivative of the capacitor voltage $\frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+}$

   (c) The damping factor α

   (d) The natural radian frequency ω_0

   (e) The resistance R.

   (f) The inductance L.

$$\mathrm{v}_C(0^+) = 2 + 7 = 9\ \mathrm{V}$$

$$\frac{d}{dt}(\mathrm{v}_C(t)) = -1600e^{-800t} - 6300e^{-900t} \Rightarrow \frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+} = -7900\ \mathrm{V/s}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -800 / s$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -900 / s$$

$$s_1 + s_2 = -2\alpha = -1700 \Rightarrow \alpha = 850 / s$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2} = 100 \Rightarrow \alpha^2 - \omega_0^2 = 50^2 \Rightarrow \omega_0^2 = 850^2 - 50^2 = 720{,}000 \Rightarrow \omega_0 = 848.53\ /\mathrm{s}$$

OR

$$-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -800 \Rightarrow \alpha^2 - \omega_0^2 = (-800 + \alpha)^2$$

$$\omega_0^2 = \alpha^2 - (-800 + \alpha)^2 = 722{,}500 - 2500 = 720{,}000 \Rightarrow \omega_0 = 848.53 / s$$

$$\alpha = \frac{1}{2RC} \Rightarrow R = \frac{1}{2\alpha C} = \frac{1}{2 \times 850 / s \times 50 \times 10^{-6}\mathrm{F}} = 11.76\ \Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{720{,}000 / s^2 \times 50 \times 10^{-6}\mathrm{F}} = 27.8\ \mathrm{mH}$$

2. With reference to the series *RLC* circuit below, the initial current $i(0^+)$ is 100 mA and the initial capacitor voltage $v_C(0^+)$ is 9 V. Is this circuit overdamped, critically damped or underdamped?

$$\alpha = \frac{R}{2L} = \frac{250\Omega}{2 \times 80\text{mH}} = 1562.5 / s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 500 / s \quad \text{Therefore, overdamped.}$$

Find these numerical values.

(a) $v_R(0^+) =$ ____________ V

(b) $v_L(0^+) =$ ____________ V

(c) $\frac{d}{dt}(i(t))_{t=0^+} =$ ____________ A/s

(d) $\frac{d}{dt}(v_R(t))_{t=0^+} =$ ____________ V/s

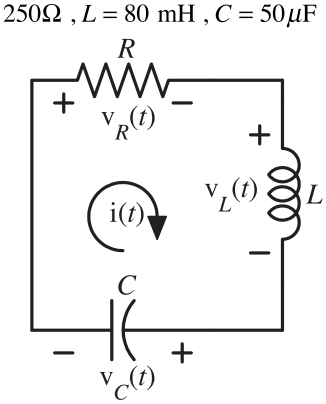
(e) $\frac{d}{dt}(v_C(t))_{t=0^+} =$ ____________ V/s

(f) $\frac{d}{dt}(v_L(t))_{t=0^+} =$ ____________ V/s

(Be sure that KVL is satisfied and that the derivative of KVL is also satisfied at $t = 0^+$.)

The initial resistor voltage is the initial current times the resistance or 25 volt. By KVL, the initial inductor voltage must be -34 V. The initial rate of change of the inductor current is the initial voltage divided by the inductance or -425 A/s. The initial rate of change of the resistor voltage is the initial rate of change of the current, times the resistance, or -106,025 V/s. The initial rate of change of the capacitor voltage is the initial current divided by the capacitance or 2000 V/s. From the derivative of KVL, the initial rate of change of inductor voltage is the negative of the sum of the initial rates of change of the resistor voltage and the capacitor voltage or 104,250 V/s.

$R = 250\Omega$, $L = 80$ mH , $C = 50\mu$F



Solution of ECE 300 Test 9 S10

1. The voltage across the capacitor in a parallel *RLC* circuit for the time range $t > 0$ is $\mathrm{v}_C(t) = 3e^{-400t} - 7e^{-600t}$ volts. If the capacitance of the capacitor is $50\ \mu\text{F}$ find the numerical values of

   (a) The initial capacitor voltage $\mathrm{v}_C(0^+)$

   (b) The initial derivative of the capacitor voltage $\frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+}$

   (c) The damping factor α

   (d) The natural radian frequency ω_0

   (e) The resistance *R*.

   (f) The inductance *L*.

$$\mathrm{v}_C(0^+) = 3 - 7 = -4\ \text{V}$$

$$\frac{d}{dt}(\mathrm{v}_C(t)) = -1200e^{-400t} + 4200e^{-900t} \Rightarrow \frac{d}{dt}(\mathrm{v}_C(t))_{t=0^+} = 3000\ \text{V/s}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -400\ /\ s$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -600\ /\ s$$

$$s_1 + s_2 = -2\alpha = -1000 \Rightarrow \alpha = 500\ /\ s$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2} = 200 \Rightarrow \alpha^2 - \omega_0^2 = 100^2 \Rightarrow \omega_0^2 = 500^2 - 100^2 = 240{,}000 \Rightarrow \omega_0 = 489.9\ /\text{s}$$

OR

$$-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -400 \Rightarrow \alpha^2 - \omega_0^2 = (-400 + \alpha)^2$$

$$\omega_0^2 = \alpha^2 - (-400 + \alpha)^2 = 250{,}000 - 10{,}000 = 240{,}000 \Rightarrow \omega_0 = 489.9\ /\ s$$

$$\alpha = \frac{1}{2RC} \Rightarrow R = \frac{1}{2\alpha C} = \frac{1}{2 \times 500\ /\ s \times 50 \times 10^{-6}\,\text{F}} = 20\ \Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{240{,}000\ /\ s^2 \times 50 \times 10^{-6}\,\text{F}} = 83.3\ \text{mH}$$

2. With reference to the series *RLC* circuit below, the initial current $i(0^+)$ is 200 mA and the initial capacitor voltage $v_C(0^+)$ is 10 V. Is this circuit overdamped, critically damped or underdamped?

$$\alpha = \frac{R}{2L} = \frac{50\Omega}{2 \times 80\text{mH}} = 312.5 / s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 790.57 / s \qquad \text{Therefore, underdamped.}$$

Find these numerical values.

(a) $v_R(0^+) =$ ____________ V

(b) $v_L(0^+) =$ ____________ V

(c) $\frac{d}{dt}(i(t))_{t=0^+} =$ ____________ A/s

(d) $\frac{d}{dt}(v_R(t))_{t=0^+} =$ ____________ V/s

(e) $\frac{d}{dt}(v_C(t))_{t=0^+} =$ ____________ V/s

(f) $\frac{d}{dt}(v_L(t))_{t=0^+} =$ ____________ V/s

(Be sure that KVL is satisfied and that the derivative of KVL is also satisfied at $t = 0^+$.)

The initial resistor voltage is the initial current times the resistance or 10 volts. By KVL, the initial inductor voltage must be -20 V. The initial rate of change of the inductor current is the initial voltage divided by the inductance or -250 A/s. The initial rate of change of the resistor voltage is the initial rate of change of the current, times the resistance, or -12,500 V/s. The initial rate of change of the capacitor voltage is the initial current divided by the capacitance or 10,000 V/s. From the derivative of KVL, the initial rate of change of inductor voltage is the negative of the sum of the initial rates of change of the resistor voltage and the capacitor voltage or -2500 V/s.

$R = 50\Omega$, $L = 80$ mH , $C = 20\mu$F