Solution of EECS 300 Test 12 F08

1. Referring to the circuit below

(a) Find the numerical values of the three mutual inductances M_{12} , M_{23} , M_{13}.

M_{12} = _____ , M_{23} = _____ , M_{13} = _____

(b) Fill in the blanks in the matrix equation for the mesh currents with magnitudes in ohms or volts and phases in degrees. (All magnitudes must be greater than or equal to zero.)

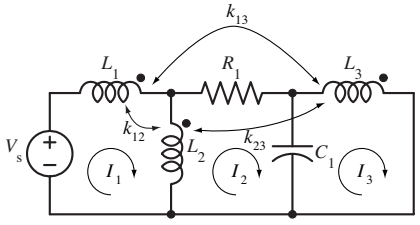
$$\begin{bmatrix} \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega \\ \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega \\ \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega & \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\Omega \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\text{V} \\ \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\text{V} \\ \_\_\_\_\_\angle\_\_\_\_\_^\circ\,\text{V} \end{bmatrix}$$

(a)
$$M_{12} = k_{12}\sqrt{L_1 L_2} = 0.8\sqrt{200\text{ mH}\times 400\text{ mH}} = 0.8\times 282.8\text{ mH} = 226.27\text{ mH}$$
$$M_{23} = k_{23}\sqrt{L_2 L_3} = 0.9\sqrt{400\text{ mH}\times 50\text{ mH}} = 0.9\times 141.4\text{ mH} = 127.3\text{ mH}$$
$$M_{13} = k_{13}\sqrt{L_1 L_3} = 0.4\sqrt{200\text{ mH}\times 50\text{ mH}} = 0.4\times 100\text{ mH} = 40\text{ mH}$$

(b)
$$-120 + j\omega L_1 I_1 - j\omega M_{12}\left(I_1 - I_2\right) + j\omega M_{13} I_3 + j\omega L_2\left(I_1 - I_2\right) - j\omega M_{12} I_1 - j\omega M_{23} I_3 = 0$$
$$j\omega L_2\left(I_2 - I_1\right) + j\omega M_{12} I_1 + j\omega M_{23} I_3 + R_1 I_2 + \left(1/\ j\omega C_1\right)\left(I_2 - I_3\right) = 0$$
$$\left(1/\ j\omega C_1\right)\left(I_3 - I_2\right) + j\omega L_3 I_3 + j\omega M_{13} I_1 - j\omega M_{23}\left(I_1 - I_2\right) = 0$$

$$\begin{bmatrix} j\omega\left(L_1 - M_{12} + L_2 - M_{12}\right) & j\omega\left(M_{12} - L_2\right) & j\omega\left(M_{13} - M_{23}\right) \\ j\omega\left(-L_2 + M_{12}\right) & j\omega L_2 + R_1 + 1/\ j\omega C_1 & j\omega M_{23} - 1/\ j\omega C_1 \\ j\omega\left(M_{13} - M_{23}\right) & -1/\ j\omega C_1 + j\omega M_{23} & 1/\ j\omega C_1 + j\omega L_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 55.5924\angle 90^\circ & 65.4962\angle -90^\circ & 32.9121\angle -90^\circ \\ 65.4962\angle -90^\circ & 101.6381\angle 10.3^\circ & 180.6181\angle 90^\circ \\ 32.9121\angle -90^\circ & 180.6181\angle 90^\circ & 113.776\angle -90^\circ \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$



$\omega = 377$, $V_s = 120\angle 0^\circ$

$L_1 = 200$ mH , $L_2 = 400$ mH

$R_1 = 100\ \Omega$, $C_1 = 20\ \mu$F

$L_3 = 50$ mH , $k_{12} = 0.8$

$k_{23} = 0.9$, $k_{13} = 0.4$

2. Find the numerical values of I_1 , V_2 and V_3 in the circuit below.

I_1 = _____ A , V_2 = _____ V , V_3 = _____ V

The input resistance of the top transformer is $\left(4/3\right)^2 \times 10 = 17.78\ \Omega$.

The input resistance of the bottom transformer is $\left(1/4\right)^2 \times 100 = 6.25\ \Omega$.

So the input current of 10 A splits between the $5\ \Omega$ and $17.78 + 6.25 = 24.03\ \Omega$ resistances.

$$I_1 = \frac{24.03}{5 + 24.03} \times 10 = 8.28\ \text{A}$$

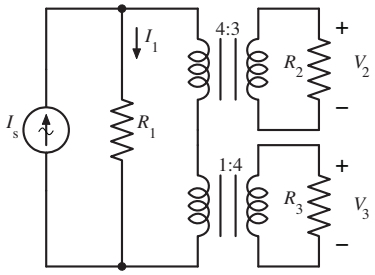
Therefore the current through the primary of both transformers is $10 - 8.28 = 1.72\ \text{A}$.

The current in the secondary of the top transformer is $1.72 \times \left(4/3\right) = 2.296\ \text{A}$.

The current in the secondary of the bottom transformer is $1.72 \times \left(1/4\right) = 0.43\ \text{A}$.

$V_2 = 2.296\ \text{A} \times 10\ \Omega = 22.96\ \text{V}$

$V_3 = 0.43\ \text{A} \times 100\ \Omega = 43\ \text{V}$



$I_s = 10\angle 0°\ \text{A}$, $R_1 = 5\ \Omega$

$R_2 = 10\ \Omega$, $R_3 = 100\ \Omega$

$I_s = 10\angle 0°\ \text{A}$, $R_1 = 20\ \Omega$

$R_2 = 15\ \Omega$, $R_3 = 80\ \Omega$