Solution of ECE 300 Test 13 S09

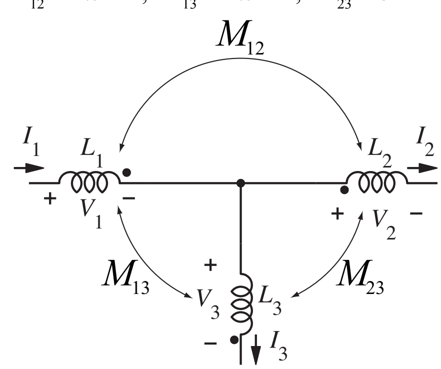
1. In the partial circuit below, the voltages can be written in terms of the inductances, mutual inductances and the currents. Fill in the coefficient matrix below with numerical magnitudes and angles in degrees.

$$V_1 = j40I_1 - j30I_2 + j50I_3$$
$$V_2 = j80I_2 - j30I_1 - j60I_3$$
$$V_3 = j100I_3 + j50I_1 - j60I_2$$

$$\begin{bmatrix} 40\angle 90° & 30\angle -90° & 50\angle 90° \\ 30\angle -90° & 80\angle 90° & 60\angle -90° \\ 50\angle 90° & 60\angle -90° & 100\angle 90° \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

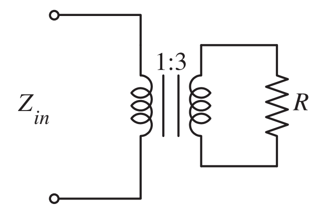
$L_1 = 2\text{H}$, $L_2 = 4\text{H}$, $L_3 = 5\text{H}$, $\omega = 20$

$M_{12} = 1.5\text{H}$, $M_{13} = 2.5\text{H}$, $M_{23} = 3\text{H}$



2. In the circuit below, if $R = 15$ and $\omega = 377$ and the transformer is ideal, what is the numerical value of Z_{in} ?

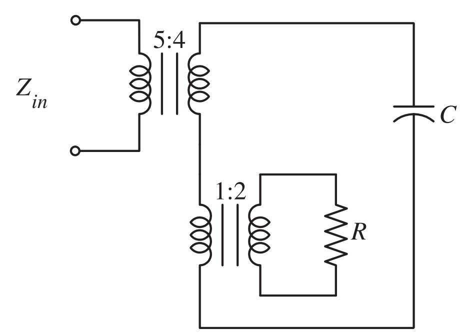
Input resistance on the primary (the left side) of the transformer is $Z_{in2} = (1/3)^2 \times 15\Omega = 1.667\Omega$.



3. In the circuit below, the transformers are both ideal, $R = 20\Omega$, $C = 20\mu\text{F}$ and $\omega = 377$. Find the numerical value of Z_{in}.

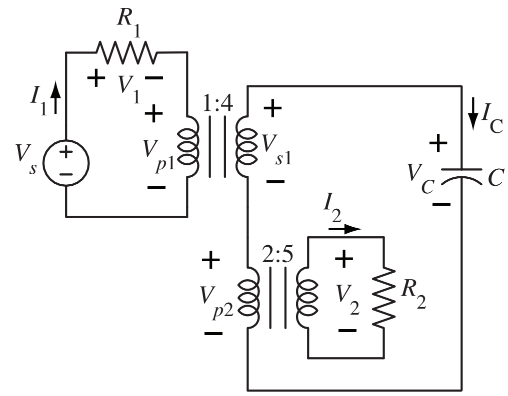
Input resistance on the primary (the left side) of the lower transformer is $Z_{in2} = (1/2)^2 \times 20\Omega = 5\Omega$. Therefore the load on the secondary (the right side) of the upper transformer is $Z_{L1} = \dfrac{1}{j377 \times 20 \times 10^{-6}} + 5 = 5 - j132.63 = 132.72\angle -87.84°\ \Omega$. Then the input impedance of the upper transformer is

$$Z_{in1} = (5/4)^2 \left(132.72\angle -87.84°\ \Omega\right) = 7.8125 - j207.23 = 207.38\angle -87.84°$$



4. In the circuit below, the transformers are both ideal.

$V_s = 120\angle 0°$ rms , $R_1 = 10\Omega$, $R_2 = 50\Omega$, $C = 80\mu\text{F}$, $\omega = 377$



Find the following numerical values: (Be sure KVL is satisfied for all closed loops in the circuit.)

The input impedance of the lower transformer on the primary side (the left side) = ________ ∠ ______ Ω
The load impedance on the secondary of the upper transformer (the right side) = ________ ∠ ______ Ω
The input impedance of the upper transformer on the primary side (the left side) = ________ ∠ ______ Ω
The current I_1 flowing out of the voltage source = ________ ∠ ______ A rms
The voltage V_1 across the resistor R_1 = ________ ∠ ______ Vrms
The voltage V_{p1} across the primary of the upper transformer = ________ ∠ ______ Vrms
The voltage V_{s1} across the secondary of the upper transformer = ________ ∠ ______ Vrms
The current I_C = ________ ∠ ______ Arms
The voltage V_C across the capacitor = ________ ∠ ______ Vrms
The current I_2 = ________ ∠ ______ Arms
The voltage V_2 across the secondary of the lower transformer = ________ ∠ ______ Vrms
The voltage V_{p2} across the primary of the lower transformer = ________ ∠ ______ Vrms

Input resistance on the primary (the left side) of the lower transformer is $Z_{in2} = \left(2/5\right)^2 \times 50\Omega = 8\Omega$. Therefore the load on the secondary (the right side) of the upper transformer is $Z_{L1} = \dfrac{1}{j377 \times 80 \times 10^{-6}} + 8 = 8 - j33.157 = 34.11\angle -76.43°\ \Omega$. Then the input impedance of the upper transformer is

$$Z_{in1} = \left(1/4\right)^2 \left(34.11\angle -76.43°\ \Omega\right) = 0.5 - j2.072 = 2.1317\angle -76.44°$$

The current from the voltage source is $I_1 = \dfrac{V_s}{R_1 + Z_{in1}} = \dfrac{120\angle 0°\ \text{Vrms}}{10 + 0.5 - j2.072\ \Omega} = 11 + j2.17 = 11.212\angle 11.16°$. The voltage across R_1 is $V_1 = I_1 R_1 = 11.212\angle 11.16° \times 10\Omega = 112.12\angle 11.16°$. The voltage across the primary of the upper transformer is $V_{p1} = 120\angle 0° - 112.12\angle 11.16° = 23.9\angle -65.27°$. The voltage across the secondary of the upper transformer is $V_{s1} = V_{p1} \times 4 = 95.6\angle -65.27°$. The current through the capacitor is the secondary current of the upper transformer, $I_C = 11.212\angle 11.16° / 4 = 2.803\angle 11.16°$ and this is also the primary current of the lower transformer. The voltage V_C across the capacitor is $V_C = 2.803\angle 11.16° \times \dfrac{1}{j377 \times 80 \times 10^{-6}} = 92.94\angle -78.84°$. The current I_2 is the secondary current of the lower transformer and is

$I_2 = -I_C \times (2/5) = -2.803\angle 11.16° \times 2/5 = 1.1212\angle -168.84°$. The voltage V_2 across the secondary of the lower transformer is $V_2 = 1.1212\angle -168.84° \times 50 = 56.06\angle -168.84°$. The voltage across the primary of the lower transformer is $V_{p2} = 56.06\angle -168.84° \times (2/5) = 22.424\angle -168.84°$

Solution of ECE 300 Test 13 S09

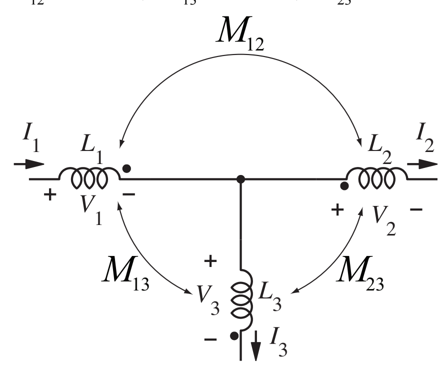
1. In the partial circuit below, the voltages can be written in terms of the inductances, mutual inductances and the currents. Fill in the coefficient matrix below with numerical magnitudes and angles in degrees.

$$V_1 = j60I_1 - j45I_2 + j75I_3$$
$$V_2 = j120I_2 - j45I_1 - j90I_3$$
$$V_3 = j150I_3 + j75I_1 - j90I_2$$

$$\begin{bmatrix} 60\angle 90° & 45\angle -90° & 75\angle 90° \\ 45\angle -90° & 120\angle 90° & 90\angle -90° \\ 75\angle 90° & 90\angle -90° & 150\angle 90° \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

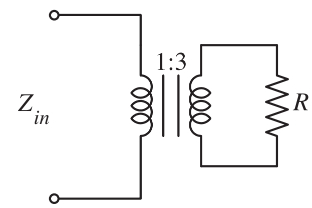
$L_1 = 2\text{H}$, $L_2 = 4\text{H}$, $L_3 = 5\text{H}$, $\omega = 30$

$M_{12} = 1.5\text{H}$, $M_{13} = 2.5\text{H}$, $M_{23} = 3\text{H}$



2. In the circuit below, if $R = 25$ and $\omega = 377$ and the transformer is ideal, what is the numerical value of Z_{in} ?

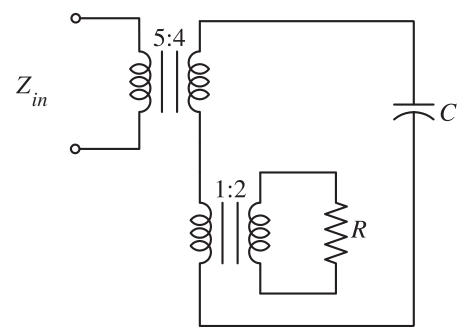
Input resistance on the primary (the left side) of the transformer is $Z_{in2} = \left(1/3\right)^2 \times 25\Omega = 2.778\Omega$.



3. In the circuit below, the transformers are both ideal, $R = 30\Omega$, $C = 20\mu\text{F}$ and $\omega = 377$. Find the numerical value of Z_{in} .

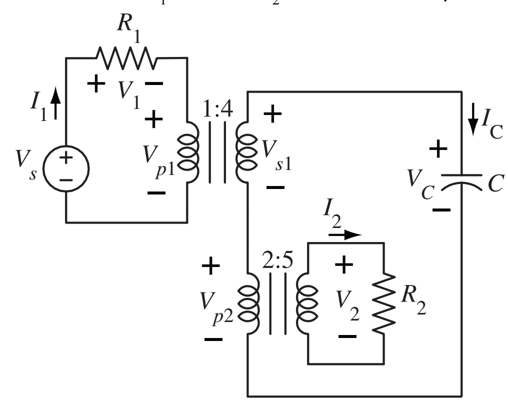
Input resistance on the primary (the left side) of the lower transformer is $Z_{in2} = \left(1/2\right)^2 \times 30\Omega = 7.5\Omega$. Therefore the load on the secondary (the right side) of the upper transformer is $Z_{L1} = \dfrac{1}{j377 \times 20 \times 10^{-6}} + 7.5 = 7.5 - j132.63 = 132.84\angle -86.76°\ \Omega$. Then the input impedance of the upper transformer is

$$Z_{in1} = \left(5/4\right)^2 \left(132.84\angle -86.76°\ \Omega\right) = 11.72 - j207.23 = 207.56\angle -86.76°$$



4. In the circuit below, the transformers are both ideal.

$V_s = 120\angle 0°$ rms , $R_1 = 10\Omega$, $R_2 = 30\Omega$, $C = 80\mu$F , $\omega = 377$



Find the following numerical values: (Be sure KVL is satisfied for all closed loops in the circuit.)

The input impedance of the lower transformer on the primary side (the left side) = ________∠______Ω
The load impedance on the secondary of the upper transformer (the right side) = ________∠______ Ω
The input impedance of the upper transformer on the primary side (the left side) = ________∠______ Ω
The current I_1 flowing out of the voltage source = ________∠______ A rms
The voltage V_1 across the resistor R_1 = ________∠______ Vrms
The voltage V_{p1} across the primary of the upper transformer = ________∠______ Vrms
The voltage V_{s1} across the secondary of the upper transformer = ________∠______ Vrms
The current I_C = ________∠______ Arms
The voltage V_C across the capacitor = ________∠______ Vrms
The current I_2 =________∠______ Arms
The voltage V_2 across the secondary of the lower transformer = ________∠______ Vrms
The voltage V_{p2} across the primary of the lower transformer = ________∠______ Vrms

Input resistance on the primary (the left side) of the lower transformer is $Z_{in2} = (2/5)^2 \times 30\Omega = 4.8\Omega$. Therefore the load on the secondary (the right side) of the upper transformer is $Z_{L1} = \frac{1}{j377 \times 80 \times 10^{-6}} + 4.8 = 4.8 - j33.5 = 33.5\angle -81.76°\ \Omega$. Then the input impedance of the upper transformer is

$$Z_{in1} = (1/4)^2 \left(33.5\angle -81.76°\ \Omega\right) = 0.3 - j2.07 = 2.09\angle -81.76°$$

The current from the voltage source is $I_1 = \frac{V_s}{R_1 + Z_{in1}} = \frac{120\angle 0°\text{ Vrms}}{10 + 0.3 - j2.07\ \Omega} = 11.197 + j2.253 = 11.422\angle 11.38°$. The voltage across R_1 is $V_1 = I_1 R_1 = 11.422\angle 11.38° \times 10\Omega = 114.216\angle 11.38°$. The voltage across the primary of the upper transformer is $V_{p1} = 120\angle 0° - 114.216\angle 11.38° = 23.92\angle -70.39°$. The voltage across the secondary of the upper transformer is $V_{s1} = V_{p1} \times 4 = 95.66\angle -70.39°$. The current through the capacitor is the secondary current of the upper transformer, $I_C = 11.42\angle 11.38° / 4 = 2.855\angle 11.38°$ and this is also the primary current of the lower transformer. The voltage V_C across the capacitor is $V_C = 2.855\angle 11.38° \times \frac{1}{j377 \times 80 \times 10^{-6}} = 94.68\angle -78.62°$. The current I_2 is the secondary current of the lower transformer and is $I_2 = -I_C \times (2/5) = -2.855\angle 11.38° \times 2/5 = 1.142\angle -168.62°$. The voltage V_2 across the secondary of the lower transformer is $V_2 = 1.142\angle -168.62° \times 30 = 34.26\angle -168.62°$. The voltage across the primary of the lower transformer is $V_{p2} = 34.26\angle -168.62° \times (2/5) = 13.71\angle -168.62°$