Solution of EECS 300 Test 11 F08

1. With reference to the circuit below, let

$$I = 20\angle 0° \text{ operating at } \omega = 1000,\ R = 50\ \Omega\ ,\ C = 30\ \mu\text{F}\ ,\ L = 80\ \text{mH}$$

and find the following numerical values.

(a) $|V_C|$ = ________ V , $\angle V_C$ = ________ °

(b) $|V_R|$ = ________ V , $\angle V_R$ = ________ °

(c) $|V_L|$ = ________ V , $\angle V_L$ = ________ °

(d) $|I_C|$ = ________ A , $\angle I_C$ = ________ °

(e) $|I_L|$ = ________ A , $\angle I_L$ = ________ °

(f) The average power supplied by the current source P_s. P_s = ________ W

(g) The average power absorbed by the resistor P_R. P_R = ________ W

(h) The complex power absorbed by the current source $\mathbf{S}_s$. $\mathbf{S}_s$ = ______ ∠ ______ ° VA

(i) The complex power absorbed by the capacitor $\mathbf{S}_C$. $\mathbf{S}_C$ = ______ ∠ ______ ° VA

(j) The complex power absorbed by the resistor $\mathbf{S}_R$. $\mathbf{S}_R$ = ______ ∠ ______ ° VA

(k) The complex power absorbed by the inductor $\mathbf{S}_L$. $\mathbf{S}_L$ = ______ ∠ ______ ° VA

The impedance driven by the current source is

$$Z = (1/\ j\omega C)\,\|\,(R + j\omega L) = \frac{(1/\ j0.03)(50 + j80)}{1/\ j0.03 + 50 + j80} = \frac{-j33.33(50 + j80)}{-j33.33 + 50 + j80}$$

$$Z = \frac{-j1666.7 + 2666.7}{50 + j46.67} = 45.98\angle -75.033°\ \Omega$$

The voltage across the current source (and the capacitor) is

$$V_s = IZ = 20\angle 0° \times 45.98\angle -75.03° = 919.6\angle -75.03°\ \text{V}$$

The current through the capacitor is

$$I_C = V_s \times j\omega C = 919.6\angle -75.03° \times j0.03 = 27.58\angle 14.96°\ \text{A}$$

The current through the resistor and inductor is

$$I_L = I - I_C = 20\angle 0° - 27.58\angle 14.96° = 9.747\angle -133.03°\ \text{A}$$

The voltage across the resistor is $V_R = I_L R = 487.37\angle -133.03°\ \text{V}$.
The voltage across the inductor is $V_L = I_L \times j\omega L = 779.79\angle -43.03°\ \text{V}$.

The average power supplied by the current source is $P_s = (|V_s||I|/2)\cos(\angle V_s - \angle I) = 2375.3\ \text{W}$
The average power absorbed by the resistor is $P_R = |I_R|^2 R/2 = 2375.3\ \text{W}$

The complex power absorbed by the source is

$$\mathbf{S}_s = \mathbf{V}_{eff}\mathbf{I}^*_{eff} = -\frac{919.6}{\sqrt{2}}\angle -75.03° \times \frac{20}{\sqrt{2}}\angle 0° = 9196\angle 104.96°\ \text{VA}$$

The complex power absorbed by the capacitor is

$$\mathbf{S}_C = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{919.6}{\sqrt{2}}\angle -75.03° \times \frac{27.58}{\sqrt{2}}\angle -14.96° = 12,684\angle -90° \text{ VA}$$

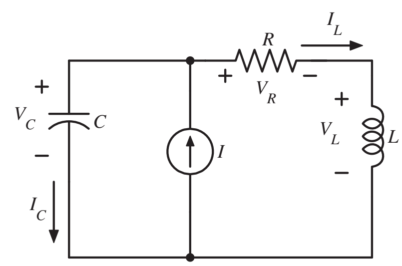
The complex power absorbed by the resistor is

$$\mathbf{S}_R = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{487.37}{\sqrt{2}}\angle -1333.03° \times \frac{9.747}{\sqrt{2}}\angle 133.03° = 2375.3\angle 0° \text{ VA}$$

The complex power absorbed by the inductor is

$$\mathbf{S}_L = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{779.79}{\sqrt{2}}\angle -43.03° \times \frac{9.747}{\sqrt{2}}\angle 133.03° = 3800.5\angle 90° \text{ VA}$$

The absorbed complex powers add to zero as they should.



2. With reference to the circuit below, let

$$V_s = 120\angle 0° \text{ rms operating at } \omega = 377,\ R = 20\ \text{k}\Omega\ ,\ C = 90\ \text{nF}$$

Find the numerical average power delivered to

(a) the resistor

(b) the dependent current source

(c) Find the numerical instantaneous power delivered by the dependent voltage source at time $t = 10$ ms.

$$I_R = V_s / R = \frac{120\angle 0° \text{ Vrms}}{20{,}000\ \Omega} = 6\angle 0° \text{ mArms} \Rightarrow P_R = I_{eff}^2 R = \left(6\times 10^{-3}\text{ A}\right)^2 20{,}000\ \Omega = 0.72\text{ W}$$

$$V_C \times j\omega C + 0.5 I_R = 0$$

$$V_C = \frac{-0.5V_s}{j\omega RC} = -\frac{-60\angle 0° \text{ Vrms}}{j\left(377\text{ rad/s}\right)\left(20{,}000\ \Omega\right)\left(90\times 10^{-9}\text{ F}\right)} = j88.417\text{ Vrms}$$

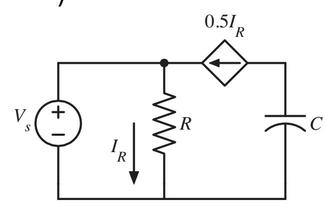
$$V_{DS} = V_s - V_C = 120\angle 0° \text{ Vrms} - \left(j88.417\text{ Vrms}\right) = 149.06\angle 143.62° \text{ Vrms (positive on left)}$$

$$I_{DS} = 0.5 I_R = 3\angle 0° \text{ mArms}$$

$$P_{DS} = -\left|V_{DS}\right|\left|I_{DS}\right|\cos\left(143.62°\right) = -149.06\times 0.003\times\left(-0.805\right) = 0.36\text{ W}$$

$$\text{p}_{DS}\left(t\right) = 149.06\sqrt{2}\cos\left(377t + 143.62°\right)0.003\sqrt{2}\cos\left(377t\right)\text{ W}$$

$$\text{p}_{DS}\left(10\text{ ms}\right) = 149.06\sqrt{2}\cos\left(377\times 10\text{ ms} + 143.62°\right)0.003\sqrt{2}\cos\left(377\times 10\text{ ms}\right) = -0.7235\text{ W}$$



Solution of EECS 300 Test 11 F08

1. With reference to the circuit below, let

$$I = 8\angle 0° \text{ operating at } \omega = 1000,\ R = 50\ \Omega\ ,\ C = 30\ \mu\text{F}\ ,\ L = 80\ \text{mH}$$

and find the following numerical values.

(a) $|V_C|$ = ________ V , $\angle V_C$ = ________ °

(b) $|V_R|$ = ________ V , $\angle V_R$ = ________ °

(c) $|V_L|$ = ________ V , $\angle V_L$ = ________ °

(d) $|I_C|$ = ________ A , $\angle I_C$ = ________ °

(e) $|I_L|$ = ________ A , $\angle I_L$ = ________ °

(f) The average power supplied by the current source P_s. P_s = ________ W

(g) The average power absorbed by the resistor P_R. P_R = ________ W

(h) The complex power absorbed by the current source $\mathbf{S}_s$. $\mathbf{S}_s$ = ______ $\angle$ ______ ° VA

(i) The complex power absorbed by the capacitor $\mathbf{S}_C$. $\mathbf{S}_C$ = ______ $\angle$ ______ ° VA

(j) The complex power absorbed by the resistor $\mathbf{S}_R$. $\mathbf{S}_R$ = ______ $\angle$ ______ ° VA

(k) The complex power absorbed by the inductor $\mathbf{S}_L$. $\mathbf{S}_L$ = ______ $\angle$ ______ ° VA

The impedance driven by the current source is

$$Z = \left(1/\ j\omega C\right) \| \left(R + j\omega L\right) = \frac{\left(1/\ j0.03\right)\left(50 + j80\right)}{1/\ j0.03 + 50 + j80} = \frac{-j33.33\left(50 + j80\right)}{-j33.33 + 50 + j80}$$

$$Z = \frac{-j1666.7 + 2666.7}{50 + j46.67} = 45.98\angle -75.033°\ \Omega$$

The voltage across the current source (and the capacitor) is

$$V_s = IZ = 8\angle 0° \times 45.98\angle -75.03° = 367.8\angle -75.03°\ \text{V}$$

The current through the capacitor is

$$I_C = V_s \times j\omega C = 367.8\angle -75.03° \times j0.03 = 11.032\angle 14.96°\ \text{A}$$

The current through the resistor and inductor is

$$I_L = I - I_C = 8\angle 0° - 11.032\angle 14.96° = 3.899\angle -133.03°\ \text{A}$$

The voltage across the resistor is $V_R = I_L R = 194.95\angle -133.03°\ \text{V}$.

The voltage across the inductor is $V_L = I_L \times j\omega L = 311.92\angle -43.03°\ \text{V}$.

The average power supplied by the current source is $P_s = \left(|V_s||I|/2\right)\cos\left(\angle V_s - \angle I\right) = 380\ \text{W}$

The average power absorbed by the resistor is $P_R = |I_R|^2 R/2 = 380\ \text{W}$

The complex power absorbed by the source is

$$\mathbf{S}_s = \mathbf{V}_{eff}\mathbf{I}^*_{eff} = -\frac{367.8}{\sqrt{2}}\angle -75.03° \times \frac{8}{\sqrt{2}}\angle 0° = 1471.4\angle 104.96°\ \text{VA}$$

The complex power absorbed by the capacitor is

$$\mathbf{S}_C = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{367.8}{\sqrt{2}}\angle -75.03° \times \frac{11.03}{\sqrt{2}}\angle -14.96° = 2029.6\angle -90° \text{ VA}$$

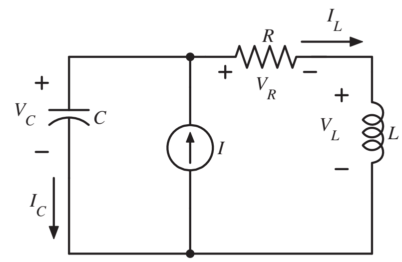
The complex power absorbed by the resistor is

$$\mathbf{S}_R = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{194.95}{\sqrt{2}}\angle -1333.03° \times \frac{3.899}{\sqrt{2}}\angle 133.03° = 380\angle 0° \text{ VA}$$

The complex power absorbed by the inductor is

$$\mathbf{S}_L = \mathbf{V}_{eff}\mathbf{I}_{eff}^* = \frac{311.9}{\sqrt{2}}\angle -43.03° \times \frac{3.899}{\sqrt{2}}\angle 133.03° = 608.1\angle 90° \text{ VA}$$

The absorbed complex powers add to zero as they should.



2. With reference to the circuit below, let

$$V_s = 90\angle 0° \text{ rms operating at } \omega = 377,\ R = 20\ \text{k}\Omega\ ,\ C = 90\ \text{nF}$$

Find the numerical average power delivered to

(a) the resistor

(b) the dependent current source

(c) Find the numerical instantaneous power delivered by the dependent voltage source at time $t = 10$ ms.

$$I_R = V_s / R = \frac{90\angle 0°\ \text{Vrms}}{20{,}000\ \Omega} = 4.5\angle 0°\ \text{mArms} \Rightarrow P_R = I_{eff}^2 R = \left(4.5\times 10^{-3}\ \text{A}\right)^2 20{,}000\ \Omega = 0.405\ \text{W}$$

$$V_C \times j\omega C + 0.5 I_R = 0$$

$$V_C = \frac{-0.5 V_s}{j\omega RC} = -\frac{-45\angle 0°\ \text{Vrms}}{j(377\ \text{rad/s})(20{,}000\ \Omega)(90\times 10^{-9}\ \text{F})} = j66.31\ \text{Vrms}$$

$$V_{DS} = V_s - V_C = 90\angle 0°\ \text{Vrms} - (j66.31\ \text{Vrms}) = 111.8\angle 143.62°\ \text{Vrms (positive on left)}$$

$$I_{DS} = 0.5 I_R = 2.25\angle 0°\ \text{mArms}$$

$$P_{DS} = -|V_{DS}||I_{DS}|\cos(143.62°) = -111.8\times 0.00225\times(-0.805) = 0.2025\ \text{W}$$

$$p_{DS}(t) = 111.8\sqrt{2}\cos(377t + 143.62°)0.00225\sqrt{2}\cos(377t)\ \text{W}$$

$$p_{DS}(10\ \text{ms}) = 111.8\sqrt{2}\cos(377\times 10\ \text{ms} + 143.62°)0.00225\sqrt{2}\cos(377\times 10\ \text{ms}) = -0.407\ \text{W}$$