# Solution of EECS 300 Test 9 F08

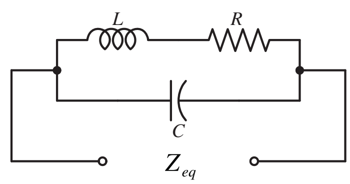
1. Real physical inductors do not behave exactly like ideal inductors. Their real impedance can be modeled by the circuit below. Let $L = 400\text{ mH}$ , $R = 2\ \Omega$ , $C = 300\text{ pF}$. Find the numerical value of the impedance $Z_{eq}$ at $\omega = 90{,}000$.

$$Z_{eq} = \frac{(j\omega L + R)(1/j\omega C)}{j\omega L + R + 1/j\omega C} = \frac{(j90000 \times 0.4 + 2)(1/j90000 \times 300 \times 10^{-12})}{j90000 \times 0.4 + 2 + 1/j90000 \times 300 \times 10^{-12}}$$

$$Z_{eq} = \frac{(j36000 + 2)(-j37037)}{j36000 + 2 - j37037} = \frac{1.3333 \times 10^9 - j74074}{2 - j1037} = \frac{1.3333 \times 10^9 \measuredangle 0°}{1037 \measuredangle -90°} = 1.2854 \times 10^6 \measuredangle 90°$$

A slightly more accurate result (avoiding some roundoff errors) is

$$Z_{eq} = 2551 + j1.286 \times 10^6\ \Omega = 1.286 \times 10^6 \measuredangle 1.569\ \Omega = 1.286 \times 10^6 \measuredangle 89.88°\ \Omega$$




2. Let $i(t) = 120\cos(\omega t)$, $R = 200\ \Omega$, $C = 200\mu\text{F}$ and $L = 2\text{ H}$. If $\omega = 377$ and $v_L(t) = A\cos(\omega t + \phi)$, find the numerical values of $A$ and $\phi$.

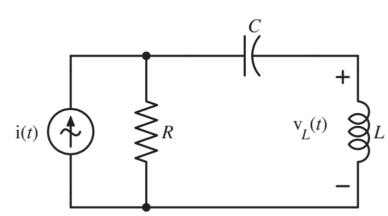
$$Z_{RLC} = \frac{R(j\omega L + 1/j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{200(j377 \times 2 + 1/j377 \times 200 \times 10^{-6})}{200 + j377 \times 2 + 1/j377 \times 200 \times 10^{-6}}$$

$$Z_{RLC} = \frac{200(j377 \times 2 + 1/j377 \times 200 \times 10^{-6})}{200 + j377 \times 2 + 1/j377 \times 200 \times 10^{-6}} = \frac{200(j754 - j13.3)}{200 + j754 - j13.3}$$

$$Z_{RLC} = \frac{200(j754 - j13.3)}{200 + j754 - j13.3} = \frac{200(j740.7)}{200 + j740.7} = 193.1\measuredangle 15.11°$$

$$V_L = IZ_{RLC}\frac{j\omega L}{j\omega L + 1/j\omega C} = 120\measuredangle 0° \times 193.1\measuredangle 15.11° \times \frac{j754}{j740.7}$$

$$V_L = 23{,}589\measuredangle 15.11° \Rightarrow v_L(t) = 23{,}589\cos(377t + 15.11°)$$



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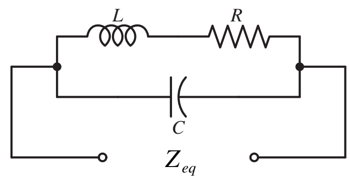
$$Z_{eq} = \frac{(j\omega L + R)(1/\,j\omega C)}{j\omega L + R + 1/\,j\omega C} = \frac{(j100000 \times 0.4 + 2)(1/\,j100000 \times 300 \times 10^{-12})}{j100000 \times 0.4 + 2 + 1/\,j100000 \times 300 \times 10^{-12}}$$

$$Z_{eq} = \frac{(j40000 + 2)(-j33333)}{j40000 + 2 - j33333} = \frac{1.3333 \times 10^9 - j66666}{2 + j6667} = \frac{1.3333 \times 10^9 \measuredangle 0°}{6667 \measuredangle 90°} = 2 \times 10^5 \measuredangle -90°$$

A slightly more accurate result (avoiding some roundoff errors) is

$$Z_{eq} = 50 - j2 \times 10^5\ \Omega = 2 \times 10^5 \measuredangle -1.5705\ \Omega = 2 \times 10^5 \measuredangle -89.99\ \Omega$$




2. Let $\text{i}(t) = 120\cos(\omega t)$, $R = 200\ \Omega$, $C = 200\mu\text{F}$ and $L = 2\ \text{H}$. If $\omega = 200$ and $\text{v}_L(t) = A\cos(\omega t + \phi)$, find the numerical values of $A$ and $\phi$.

$$Z_{RLC} = \frac{R(j\omega L + 1/\,j\omega C)}{R + j\omega L + 1/\,j\omega C} = \frac{200(j200 \times 2 + 1/\,j200 \times 200 \times 10^{-6})}{200 + j200 \times 2 + 1/\,j200 \times 200 \times 10^{-6}}$$

$$Z_{RLC} = \frac{200(j400 - j25)}{200 + j400 - j25} = \frac{200(j375)}{200 + j375} = 176.5\measuredangle 28.07°$$

$$V_L = IZ_{RLC}\frac{j\omega L}{j\omega L + 1/\,j\omega C} = 120\measuredangle 0° \times 176.5\measuredangle 28.07° \times \frac{j400}{j375}$$

$$V_L = 22{,}592\measuredangle 28.07° \Rightarrow \text{v}_L(t) = 22{,}592\cos(377t + 28.07°)$$