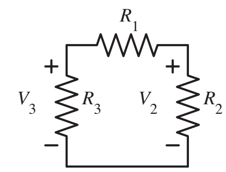
Solution of EECS 300 Test 12 F08

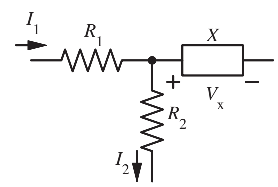
1. In the circuit below $R_2 = 20\Omega$, $R_3 = 10\Omega$, $V_2 = 2\text{V}$, $V_3 = 5\text{V}$ and the power absorbed in resistor R_1 is 15W. What is the numerical value of the resistance of resistor R_1?



$V_1 = V_3 - V_2 \Rightarrow V_1 = 3\text{V} \Rightarrow P_1 = V_1^2 / R_1 = (3\text{V})^2 / R_1 = 15\text{W} \Rightarrow R_1 = 9\text{V}^2 / 15\text{W} = 0.6\Omega$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any non-zero voltages at all. That means that we cannot find the current in resistor R_1 by saying that it is the same as in the other two resistors and we cannot find the power dissipated in it by saying the sum of all the powers must be zero.

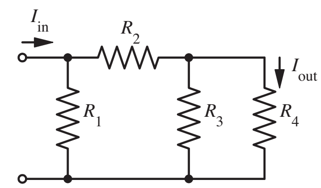
2. In the circuit below $R_1 = 35\Omega$, $R_2 = 200\Omega$, $I_1 = 3\text{A}$, $I_2 = -5\text{A}$ and $V_X = 20\text{V}$. What is the numerical value of the power being <u>delivered by</u> the unknown element X?



$P_X = -V_X I_X$, $I_X = I_1 - I_2 = 8\text{A} \Rightarrow P_X = -(20\text{V})(8\text{A}) = -160\text{W}$ delivered

Note that this circuit is a part of a larger circuit. Otherwise there would not be any currents flowing at all. We cannot find the power delivered by the unknown element by saying the sum of all the powers must be zero.

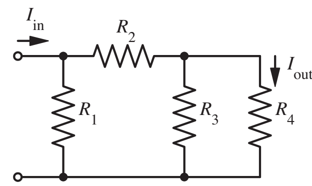
3. In the circuit below $R_1 = 8\Omega$, $R_2 = 9\Omega$, $R_3 = 4\Omega$, $R_4 = 10\Omega$ and $I_{in} = 4\text{A}$. Find the numerical value of I_{out} .



$$I_{out} = I_2 \frac{R_3}{R_3 + R_4} \quad \text{where} \quad I_2 = I_{in} \frac{R_1}{R_1 + R_x} \quad \text{and } R_x = R_2 + R_3 \parallel R_4$$

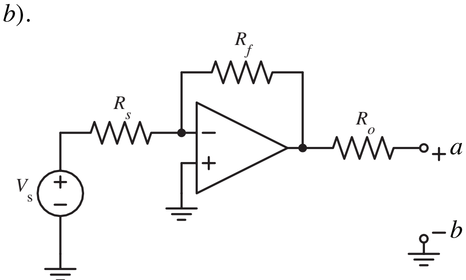
$$R_x = 9\Omega + 4\Omega \parallel 10\Omega = 11.857\Omega \quad , \; I_2 = 4\text{A}\frac{8\Omega}{8\Omega + 11.857\Omega} = 1.612\text{A} \quad , \; I_{out} = 1.612\text{A}\frac{4\Omega}{4\Omega + 10\Omega} = 0.461\text{A}$$

4. In the circuit below $R_1 = 5\Omega$, $R_2 = 20\Omega$, $R_3 = 7\Omega$, $R_4 = 11\Omega$. Find the numerical value of the equivalent input resistance between the two terminals on the left.



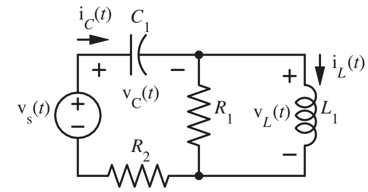
$$R_{in} = R_1 \parallel (R_2 + R_3 \parallel R_4) = 5\Omega \parallel (20\Omega + 4.278\Omega) = 5\Omega \parallel 24.278\Omega = 4.146\Omega$$

5. In the circuit below $R_s = 1\text{k}\Omega$, $R_f = 6\text{k}\Omega$, $R_o = 200\Omega$, $V_s = -5\text{V}$. Find the numerical value of the Thevenin equivalent voltage and resistance at the two terminals marked *a* and *b* and also the Norton equivalent current at those same terminals (flowing from *a* to *b*).



The output resistance of the operational amplifier is zero. Therefore the Thevenin equivalent resistance is simply the 200Ω resistor. The Thevenin equivalent voltage is the voltage at *a* with respect to *b*. No current flows through R_o . Therefore the output voltage of the operational amplifier is the Thevenin equivalent voltage. The voltage gain of the feedback amplifier is $-\frac{R_f}{R_s} = -\frac{6000\Omega}{1000\Omega} = -6$. So the operational amplifier output voltage is $(-6)\times(-5) = 30\text{V}$. The Norton equivalent current is the Thevenin equivalent voltage divided by the Thevenin equivalent resistance or $\frac{30\text{V}}{200\Omega} = 0.15\text{A}$ or 150mA .

6. In the circuit below $R_1 = 50\Omega$, $R_2 = 200\Omega$, $L_1 = 50\mathrm{mH}$, $C_1 = 80\mu\mathrm{F}$ and $\mathrm{v}_s(t) = 4\,\mathrm{u}(t) - 2$. Find the numerical values of $\mathrm{i}_L(0^-)$, $\mathrm{v}_L(0^+)$, $\mathrm{v}_C(0^-)$ and $\mathrm{i}_C(0^+)$.



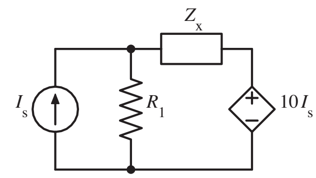
Before time $t = 0$ the capacitor current and the inductor voltage are both zero. No current flows anywhere because the capacitor blocks it. Therefore $\mathrm{i}_L(0^-) = \mathrm{i}_s(0^-) = 0$. Since there is no voltage across anything at that time except the voltage source and the capacitor $\mathrm{v}_C(0^-) = -2\mathrm{V}$.

At time $t = 0^+$ the capacitor voltage and the inductor current are unchanged. So the change in voltage from the source must appear across the other two branches with R_1 and R_2 in them. The new source voltage is 2V. The currents through R_1 and R_2 are the same because $i_L(0^+) = 0$. Let the current through the voltage source (out of the positive terminal) be $\mathrm{i}_s(0^+)$. Then

$$\mathrm{i}_s(0^+)R_1 + \mathrm{i}_s(0^+)R_2 = \mathrm{i}_s(0^+)(R_1 + R_2) = 4\mathrm{V} \Rightarrow \mathrm{i}_s(0^+) = \frac{4\mathrm{V}}{R_1 + R_2} = \frac{4\mathrm{V}}{250\Omega} = 16\mathrm{mA}$$

Since the voltage source and capacitor are in series, their currents are the same and $\mathrm{i}_C(0^+) = 16\mathrm{mA}$.
$\mathrm{v}_L(0^+) = 16\mathrm{mA} \times 50\ \Omega = 0.8\ \mathrm{V}$

7. In the circuit below $R_1 = 200\Omega$, $Z_X = 50 - j100$ and $I_s = 20\angle 0°$ rms. Find the numerical value of the complex power delivered to Z_X.



Let V_s be the voltage across the current source, positive on top. Then

$$\frac{V_s - 10I_s}{Z_X} + \frac{V_s}{R_1} = I_s \Rightarrow \frac{V_s - 200}{50 - j100} + \frac{V_s}{200} = 20 \Rightarrow V_s\left(0.00894\angle 63.43° + 0.005\right) = 20 + 1.79\angle 63.43°$$

$$V_s\left(0.01204\angle 41.62°\right) = 20.862\angle 4.4° \Rightarrow V_s = 1732.7\angle -37.22°$$

$$V_X = 1732.7\angle -37.22° - 200 = 1578.1\angle -41.62°$$

$$I_X = I_s - \frac{V_s}{R_1} = 20 - \frac{1732.7\angle -37.22°}{200} = 20 - 8.66\angle -37.22° = 14.11\angle 21.8°$$

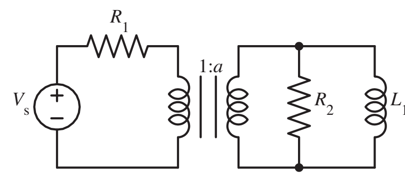
The complex power delivered to Z_X is

$$\mathbf{S}_X = V_X I_X^* = 1578.1\angle -41.62° \text{ V} \times 14.11\angle -21.8° \text{ A} = 22267\angle -63.43° \text{ VA}$$

8. In the circuit below the transformer is ideal and

$$R_1 = 4\Omega \ \ , \ R_2 = 40\Omega \ \ , \ a = 5 \ \ , \ L_1 = 50\,\mathrm{mH} \ \text{ and } V_s = 24\angle 0° \text{ V rms}.$$

The circuit is operating at a radian frequency of 377 radians/second. Find the numerical value of the average power delivered to R_2.



The input impedance to the transformer primary is

$$Z_{in} = \left(1/a^2\right)\left(40\Omega \parallel j(377/\mathrm{s})(0.05\mathrm{H})\right) = \frac{1}{25} \times \frac{40(j18.85)}{40 + j18.85} = 0.682\angle 64.77°\ \Omega = 0.291 + j0.617\ \Omega$$

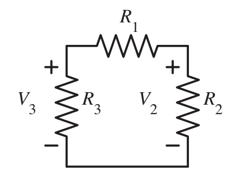
Using voltage division, the voltage across the primary is therefore

$$V_s \frac{Z_{in}}{R_1 + Z_{in}} = 24\angle 0° \text{ V rms} \times \frac{0.291 + j0.617\ \Omega}{4\Omega + (0.291 + j0.617)\Omega} = \ 3.77\angle 56.57° \text{ V rms}$$

The voltage across the secondary is therefore $3.77\angle 56.57° \text{ V rms} \times 5 = 18.85\angle 56.57° \text{ V rms}$
The power delivered to R_2 is then $(18.85 \text{ V rms})^2 / 40\Omega = 8.88\mathrm{W}$

Solution of EECS 300 Test 12 F08

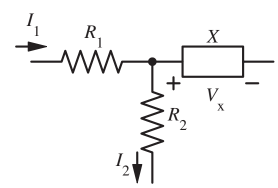
1. In the circuit below $R_2 = 20\Omega$, $R_3 = 10\Omega$, $V_2 = 2\text{V}$, $V_3 = 5\text{V}$ and the power absorbed in resistor R_1 is 10W. What is the numerical value of the resistance of resistor R_1 ?



$V_1 = V_3 - V_2 \Rightarrow V_1 = 3\text{V} \Rightarrow P_1 = V_1^2 / R_1 = (3\text{V})^2 / R_1 = 10\text{W} \Rightarrow R_1 = 9\text{V}^2 / 10\text{W} = 0.9\Omega$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any non-zero voltages at all. That means that we cannot find the current in resistor R_1 by saying that it is the same as in the other two resistors and we cannot find the power dissipated in it by saying the sum of all the powers must be zero.

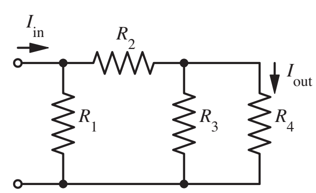
2. In the circuit below $R_1 = 35\Omega$, $R_2 = 200\Omega$, $I_1 = 3\text{A}$, $I_2 = 5\text{A}$ and $V_X = 20\text{V}$. What is the numerical value of the power being <u>delivered by</u> the unknown element X?



$P_X = -V_X I_X$, $I_X = I_1 - I_2 = -2\text{A} \Rightarrow P_X = -(20\text{V})(-2\text{A}) = 40\text{W}$ delivered

Note that this circuit is a part of a larger circuit. Otherwise there would not be any currents flowing at all. We cannot find the power delivered by the unknown element by saying the sum of all the powers must be zero.

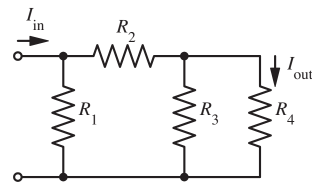
3. In the circuit below $R_1 = 8\Omega$, $R_2 = 9\Omega$, $R_3 = 8\Omega$, $R_4 = 10\Omega$ and $I_{in} = 4\text{A}$. Find the numerical value of I_{out} .



$$I_{out} = I_2 \frac{R_3}{R_3 + R_4} \quad \text{where} \quad I_2 = I_{in} \frac{R_1}{R_1 + R_x} \quad \text{and } R_x = R_2 + R_3 \parallel R_4$$

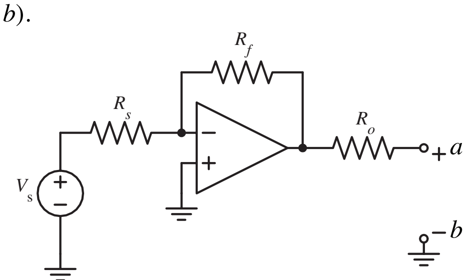
$$R_x = 9\Omega + 8\Omega \parallel 10\Omega = 13.444\Omega \quad , \; I_2 = 4\text{A} \frac{8\Omega}{8\Omega + 13.444\Omega} = 1.49\text{A} \quad , \; I_{out} = 1.49\text{A} \frac{8\Omega}{8\Omega + 10\Omega} = 0.663\text{A}$$

4. In the circuit below $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 7\Omega$, $R_4 = 11\Omega$. Find the numerical value of the equivalent input resistance between the two terminals on the left.



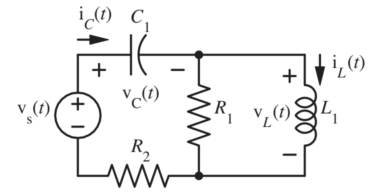
$R_{in} = R_1 \parallel (R_2 + R_3 \parallel R_4) = 5\Omega \parallel (10\Omega + 4.278\Omega) = 5\Omega \parallel 14.278\Omega = 3.7\Omega$

5. In the circuit below $R_s = 1\text{k}\Omega$, $R_f = 9\text{k}\Omega$, $R_o = 200\Omega$, $V_s = -5\text{V}$. Find the numerical value of the Thevenin equivalent voltage and resistance at the two terminals marked *a* and *b* and also the Norton equivalent current at those same terminals (flowing from *a* to *b*).



The output resistance of the operational amplifier is zero. Therefore the Thevenin equivalent resistance is simply the 200Ω resistor. The Thevenin equivalent voltage is the voltage at *a* with respect to *b*. No current flows through R_o. Therefore the output voltage of the operational amplifier is the Thevenin equivalent voltage. The voltage gain of the feedback amplifier is $-\dfrac{R_f}{R_s} = -\dfrac{9000\Omega}{1000\Omega} = -9$. So the operational amplifier output voltage is $(-9)\times(-5) = 45\text{V}$. The Norton equivalent current is the Thevenin equivalent voltage divided by the Thevenin equivalent resistance or $\dfrac{45\text{V}}{200\Omega} = 0.225\text{A}$ or 225mA.

6. In the circuit below $R_1 = 100\Omega$, $R_2 = 200\Omega$, $L_1 = 50\,\mathrm{mH}$, $C_1 = 80\mu\mathrm{F}$ and $\mathrm{v}_s(t) = 4\,\mathrm{u}(t) - 2$. Find the numerical values of $\mathrm{i}_L(0^-)$, $\mathrm{v}_L(0^+)$, $\mathrm{v}_C(0^-)$ and $\mathrm{i}_C(0^+)$.



Before time $t = 0$ the capacitor current and the inductor voltage are both zero. No current flows anywhere because the capacitor blocks it. Therefore $\mathrm{i}_L(0^-) = \mathrm{i}_s(0^-) = 0$. Since there is no voltage across anything at that time except the voltage source and the capacitor $\mathrm{v}_C(0^-) = -2\mathrm{V}$.

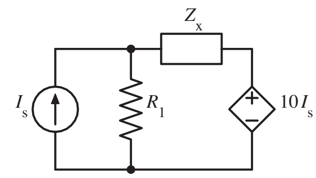
At time $t = 0^+$ the capacitor voltage and the inductor current are unchanged. So the change in voltage from the source must appear across the other two branches with R_1 and R_2 in them. The new source voltage is 2V. The currents through R_1 and R_2 are the same because $i_L(0^+) = 0$. Let the current through the voltage source (out of the positive terminal) be $\mathrm{i}_s(0^+)$. Then

$$\mathrm{i}_s(0^+)R_1 + \mathrm{i}_s(0^+)R_2 = \mathrm{i}_s(0^+)(R_1 + R_2) = 4\mathrm{V} \Rightarrow \mathrm{i}_s(0^+) = \frac{4\mathrm{V}}{R_1 + R_2} = \frac{4\mathrm{V}}{300\Omega} = 13.33\mathrm{mA}$$

Since the voltage source and capacitor are in series, their currents are the same and $\mathrm{i}_C(0^+) = 13.33\mathrm{mA}$.

$\mathrm{v}_L(0^+) = 13.33\ \mathrm{mA} \times 100\ \Omega = 1.333\ \mathrm{V}$

7. In the circuit below $R_1 = 100\Omega$, $Z_X = 50 - j100$ and $I_s = 20\angle 0°$ rms. Find the numerical value of the complex power delivered to Z_X.



Let V_s be the voltage across the current source, positive on top. Then

$$\frac{V_s - 10I_s}{Z_X} + \frac{V_s}{R_1} = I_s \Rightarrow \frac{V_s - 200}{50 - j100} + \frac{V_s}{100} = 20 \Rightarrow V_s\left(0.00894\angle 63.43° + 0.01\right) = 20 + 1.79\angle 63.43°$$

$$V_s\left(0.01612\angle 29.73°\right) = 20.862\angle 4.4° \Rightarrow V_s = 1294.2\angle -25.33°$$

$$V_X = 1294.2\angle -25.33° - 200 = 1116.7\angle -29.72°$$

$$I_X = I_s - \frac{V_s}{R_1} = 20 - \frac{1294.2\angle -25.33°}{100} = 20 - 12.942\angle -25.33° = 9.985\angle 33.69°$$

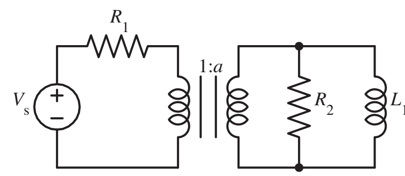
The complex power delivered to Z_X is

$$\mathbf{S}_X = V_X I_X^* = 1116.7\angle -29.72° \text{ V} \times 9.985\angle -33.69° \text{ A} = 11145.4\angle -63.43° \text{ VA}$$

8. In the circuit below the transformer is ideal and

$$R_1 = 4\Omega \ \ , \ R_2 = 40\Omega \ \ , \ a = 4 \ \ , \ L_1 = 50\,\mathrm{mH} \ \text{ and } V_s = 24\angle 0° \text{ V rms}.$$

The circuit is operating at a radian frequency of 377 radians/second. Find the numerical value of the average power delivered to R_2.



The input impedance to the transformer primary is

$$Z_{in} = \left(1/a^2\right)\left(40\Omega \parallel j\left(377/\mathrm{s}\right)\left(0.05\mathrm{H}\right)\right) = \frac{1}{16} \times \frac{40\left(j18.85\right)}{40 + j18.85} = 1.066\angle 64.77° \ \Omega = 0.455 + j0.964 \ \Omega$$

Using voltage division, the voltage across the primary is therefore

$$V_s \frac{Z_{in}}{R_1 + Z_{in}} = 24\angle 0° \text{ V rms} \times \frac{0.455 + j0.964 \ \Omega}{4\Omega + \left(0.455 + j0.964\right)\Omega} = \ 5.613\angle 52.52° \text{ V rms}$$

The voltage across the secondary is therefore $5.613\angle 52.52° \text{ V rms} \times 4 = 22.45\angle 52.52° \text{ V rms}$
The power delivered to R_2 is then $\left(22.45 \text{ V rms}\right)^2 / 40\Omega = 12.6\text{W}$