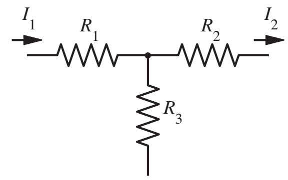
Solution of EECS 300 Test 12 F08

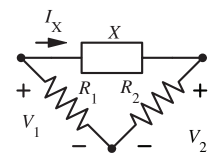
1. In the circuit below $R_1 = 20\Omega$, $R_2 = 10\Omega$, $I_1 = 2\text{A}$, $I_2 = 5\text{A}$ and the power absorbed in resistor R_3 is 15W. What is the numerical value of the resistance of resistor R_3?



$$I_3 + I_1 = I_2 \Rightarrow I_3 = 3\text{A} \Rightarrow P_3 = I_3^2 R_3 = (3\text{A})^2 R_3 = 15\text{W} \Rightarrow R_3 = \frac{15\text{W}}{9\text{A}^2} = 1.67\Omega$$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any currents flowing at all. We cannot find the power dissipated in R_3 by saying the sum of all the powers must be zero.

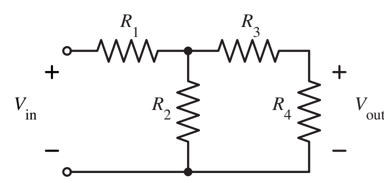
2. In the circuit below $R_1 = 35\Omega$, $R_2 = 200\Omega$, $V_1 = 15\text{V}$, $V_2 = -10\text{V}$ and $I_X = 8\text{A}$. What is the numerical value of the power being <u>delivered by</u> the unknown element *X*?



$$P_X = V_X I_X \ , \ V_X = V_2 - V_1 = -25\text{V} \Rightarrow P_X = (-25\text{V})(8\text{A}) = -200\text{W delivered}$$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any non-zero voltages at all. That means that we cannot find the current in the unknown element by saying that it is the same as in the other two resistors and we cannot find the power delivered by it by saying the sum of all the powers must be zero.

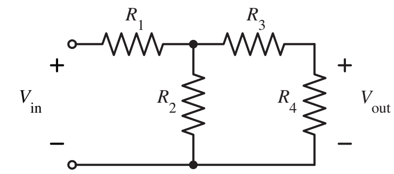
3. In the circuit below $R_1 = 8\Omega$, $R_2 = 9\Omega$, $R_3 = 4\Omega$, $R_4 = 10\Omega$ and $V_{in} = 24\text{V}$. Find the numerical value of V_{out}.



$$V_{out} = V_2 \frac{R_4}{R_3 + R_4} \quad \text{where } V_2 = V_{in} \frac{R_\pi}{R_1 + R_\pi} \text{ and } R_\pi = R_2 \parallel (R_3 + R_4)$$

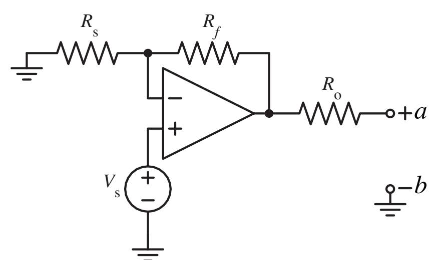
$$R_\pi = \frac{9\Omega \times 14\Omega}{9\Omega + 14\Omega} = 5.478\Omega \ , \ V_2 = 24\text{V} \frac{5.478\Omega}{8\Omega + 5.478\Omega} = 9.76\text{V} \ , \ V_{out} = 9.76\text{V} \frac{10\Omega}{4\Omega + 10\Omega} = 6.97\text{V}$$

4. In the circuit below $R_1 = 5\Omega$, $R_2 = 20\Omega$, $R_3 = 7\Omega$, $R_4 = 11\Omega$. Find the numerical value of the equivalent input resistance between the two terminals on the left.



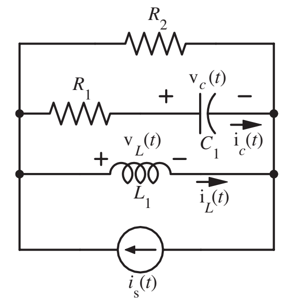
$$R_{in} = R_1 + R_2 \parallel (R_3 + R_4) = 5\Omega + 20\Omega \parallel (18\Omega) = 5\Omega + \frac{20\Omega \times 18\Omega}{38\Omega} = 14.47\Omega$$

5. In the circuit below $R_s = 1\text{k}\Omega$, $R_f = 6\text{k}\Omega$, $R_o = 200\Omega$, $V_s = -5\text{V}$. Find the numerical value of the Thevenin equivalent voltage and resistance at the two terminals marked *a* and *b* and also the Norton equivalent current at those same terminals (flowing from *a* to *b*).



The output resistance of the operational amplifier is zero. Therefore the Thevenin equivalent resistance is simply the 200Ω resistor. The Thevenin equivalent voltage is the voltage at *a* with respect to *b*. No current flows through R_o. Therefore the output voltage of the operational amplifier is the Thevenin equivalent voltage. The voltage gain of the feedback amplifier is $\frac{R_f + R_s}{R_s} = \frac{6000\Omega + 1000\Omega}{1000\Omega} = 7$. So the operational amplifier output voltage is $7 \times (-5) = -35\text{V}$. The Norton equivalent current is the Thevenin equivalent voltage divided by the Thevenin equivalent resistance or $\frac{-35\text{V}}{200\Omega} = -0.175\text{A}$ or -175mA.

6. In the circuit below $R_1 = 50\Omega$, $R_2 = 200\Omega$, $L_1 = 50\,\mathrm{mH}$, $C_1 = 80\mu\mathrm{F}$ and $\mathrm{i}_s(t) = 2\,\mathrm{u}(t) - 4$. Find the numerical values of $\mathrm{i}_L(0^-)$, $\mathrm{v}_L(0^+)$, $\mathrm{v}_C(0^-)$ and $\mathrm{i}_C(0^+)$.



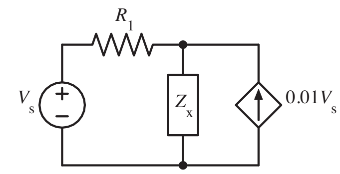
Before time $t = 0$ the capacitor current and the inductor voltage are both zero. All the current flows through the inductor. Therefore $\mathrm{i}_L(0^-) = \mathrm{i}_s(0^-) = -4$. Since there is no current flowing anywhere else at that time (and never has been) all the other currents and voltages are zero. So $\mathrm{v}_C(0^-) = 0$.

At time $t = 0^+$ the capacitor voltage and the inductor current are unchanged. So the change in current from the source must flow through the other two branches with R_1 and R_2 in them. The new source current is -2A. The voltages across R_1 and R_2 are the same because $\mathrm{v}_C(0^+) = 0$. Let the voltage across the current source (positive on the left) be $\mathrm{v}_s(0^+)$. Then

$$\frac{\mathrm{v}_s(0^+)}{R_1} + \frac{\mathrm{v}_s(0^+)}{R_2} = \frac{R_2\,\mathrm{v}_s(0^+) + R_1\,\mathrm{v}_s(0^+)}{R_1R_2} = \mathrm{v}_s(0^+)\frac{R_2 + R_1}{R_1R_2} = 2\mathrm{A} \Rightarrow \mathrm{v}_s(0^+) = \frac{R_1R_2}{R_1 + R_2} \times 2\mathrm{A} = 80\mathrm{V}$$

Since the current source and inductor are in parallel, their voltages are the same and $\mathrm{v}_L(0^+) = 80\mathrm{V}$. Then $\mathrm{i}_C(0^+) = \dfrac{80\mathrm{V}}{50\Omega} = 1.6\mathrm{A}$.

7. In the circuit below $R_1 = 200\Omega$, $Z_X = 50 - j100$ and $V_s = 100\angle 0°$ rms . Find the numerical value of the complex power delivered to Z_X .



Let V_X be the voltage across Z_X , positive on top. Then

$$\frac{V_X - V_s}{R_1} + \frac{V_X}{Z_X} - 0.01V_s = 0 \Rightarrow \frac{V_X - 100\text{V}}{200\Omega} + \frac{V_X}{(50 - j100)\Omega} - 0.01(\text{A/V})(100\text{V}) = 0$$

$$\frac{V_X}{200\Omega} + \frac{V_X}{(50 - j100)\Omega} = 1\text{A} + 0.5\text{A}=1.5\text{A}$$

$$V_X \frac{(250 - j100)\Omega}{(10{,}000 - j20{,}000)\Omega} = V_X \frac{269.26\angle -21.801°}{22{,}360\angle -63.43°} = 1.5\text{A}$$

$$V_X \times 0.012\angle 41.629° = 1.5\text{A} \Rightarrow V_X = 124.56\angle\text{-}41.629° \text{ rms}$$

$$I_X = \frac{V_X}{Z_X} = \frac{124.56\angle\text{-}41.629° \text{ rms}}{111.8\angle -63.43°} = 1.118\angle 21.801° \text{ rms} \quad \text{(flowing downward)}$$

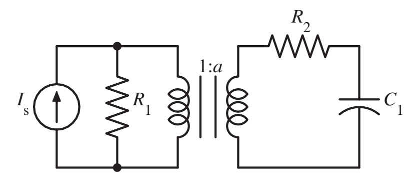
The complex power delivered to Z_X is

$$\mathbf{S}_X = V_X I_X^* = 124.56\angle -41.629° \text{ V} \times 1.118\angle -21.801° \text{ A} = 138.78\angle -63.43° \text{ VA}$$

8. In the circuit below the transformer is ideal and

$$R_1 = 4\Omega \ \ , \ R_2 = 40\Omega \ \ , \ a = 5 \ \ , \ C_1 = 50\mu\text{F} \ \text{and} \ I_s = 6\angle 0° \ \text{A rms}.$$

The circuit is operating at a radian frequency of 377 radians/second. Find the numerical value of the average power delivered to R_2.



The input impedance to the transformer primary is

$$Z_{in} = \left(1/a^2\right)\left(40\Omega + \frac{1}{j\left(377/\text{s}\right)50\times 10^{-6}\text{F}}\right) = \left(1/25\right)\left(40\Omega - j53.05\Omega\right) = 1.6 - j2.122\Omega$$

Using current division, the current in the primary is therefore

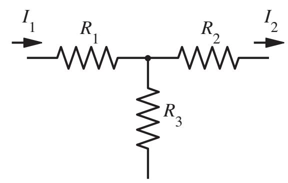
$$I_s \frac{R_1}{R_1 + Z_{in}} = 6\angle 0° \ \text{A rms} \times \frac{4\Omega}{4\Omega + \left(1.6 - j2.122\right)\Omega} = 4.01\angle 20.75° \ \text{A rms}$$

The current in the secondary is therefore $\dfrac{4.01\angle 20.75° \ \text{A rms}}{5} = 0.802\angle 20.75° \ \text{A rms}$

The power delivered to R_2 is then $\left(0.802 \ \text{A rms}\right)^2 \times 40\Omega = 25.73\text{W}$

Solution of EECS 300 Test 12 F08

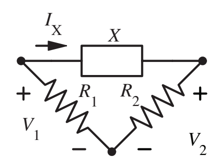
1. In the circuit below $R_1 = 20\Omega$, $R_2 = 10\Omega$, $I_1 = 2\text{A}$, $I_2 = 3\text{A}$ and the power absorbed in resistor R_3 is 15W. What is the numerical value of the resistance of resistor R_3?



$$I_3 + I_1 = I_2 \Rightarrow I_3 = 1\text{A} \Rightarrow P_3 = I_3^2 R_3 = (1\text{A})^2 R_3 = 15\text{W} \Rightarrow R_3 = \frac{15\text{W}}{1\text{A}^2} = 15\Omega$$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any currents flowing at all. We cannot find the power dissipated in R_3 by saying the sum of all the powers must be zero.

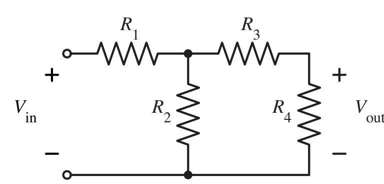
2. In the circuit below $R_1 = 35\Omega$, $R_2 = 200\Omega$, $V_1 = 15\text{V}$, $V_2 = -10\text{V}$ and $I_X = 5\text{A}$. What is the numerical value of the power being <u>delivered by</u> the unknown element *X*?



$$P_X = V_X I_X \ , \ V_X = V_2 - V_1 = -25\text{V} \Rightarrow P_X = (-25\text{V})(5\text{A}) = -125\text{W delivered}$$

Note that this circuit is a part of a larger circuit. Otherwise there would not be any non-zero voltages at all. That means that we cannot find the current in the unknown element by saying that it is the same as in the other two resistors and we cannot find the power delivered by it by saying the sum of all the powers must be zero.

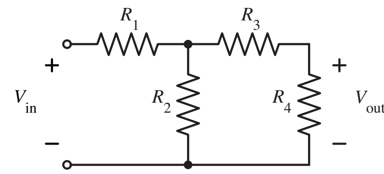
3. In the circuit below $R_1 = 8\Omega$, $R_2 = 12\Omega$, $R_3 = 4\Omega$, $R_4 = 10\Omega$ and $V_{in} = 24\text{V}$. Find the numerical value of V_{out}.



$$V_{out} = V_2 \frac{R_4}{R_3 + R_4} \quad \text{where} \quad V_2 = V_{in} \frac{R_\pi}{R_1 + R_\pi} \quad \text{and } R_\pi = R_2 \parallel (R_3 + R_4)$$

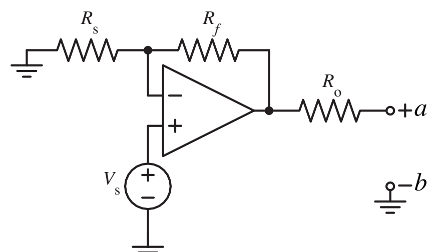
$$R_\pi = \frac{12\Omega \times 14\Omega}{12\Omega + 14\Omega} = 6.462\Omega \ , \ V_2 = 24\text{V}\frac{6.462\Omega}{8\Omega + 6.462\Omega} = 10.72\text{V} \ , \ V_{out} = 10.72\text{V}\frac{10\Omega}{4\Omega + 10\Omega} = 7.66\text{V}$$

4. In the circuit below $R_1 = 5\Omega$, $R_2 = 20\Omega$, $R_3 = 7\Omega$, $R_4 = 8\Omega$. Find the numerical value of the equivalent input resistance between the two terminals on the left.



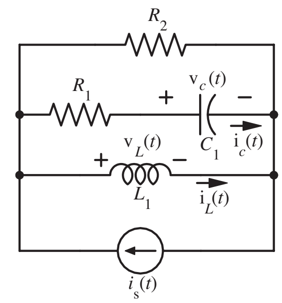
$$R_{in} = R_1 + R_2 \parallel (R_3 + R_4) = 5\Omega + 20\Omega \parallel (15\Omega) = 5\Omega + \frac{20\Omega \times 15\Omega}{35\Omega} = 13.57\Omega$$

5. In the circuit below $R_s = 1\text{k}\Omega$, $R_f = 9\text{k}\Omega$, $R_o = 300\Omega$, $V_s = -5\text{V}$. Find the numerical value of the Thevenin equivalent voltage and resistance at the two terminals marked *a* and *b* and also the Norton equivalent current at those same terminals (flowing from *a* to *b*).



The output resistance of the operational amplifier is zero. Therefore the Thevenin equivalent resistance is simply the 300Ω resistor. The Thevenin equivalent voltage is the voltage at *a* with respect to *b*. No current flows through R_o. Therefore the output voltage of the operational amplifier is the Thevenin equivalent voltage. The voltage gain of the feedback amplifier is $\frac{R_f + R_s}{R_s} = \frac{9000\Omega + 1000\Omega}{1000\Omega} = 10$. So the operational amplifier output voltage is $10 \times (-5) = -50\text{V}$. The Norton equivalent current is the Thevenin equivalent voltage divided by the Thevenin equivalent resistance or $\frac{-50\text{V}}{300\Omega} = -0.1667\text{A}$ or -166.7mA.

6. In the circuit below $R_1 = 50\Omega$, $R_2 = 200\Omega$, $L_1 = 50\,\text{mH}$, $C_1 = 80\mu\text{F}$ and $\text{i}_s(t) = 3\text{u}(t) - 6$. Find the numerical values of $\text{i}_L(0^-)$, $\text{v}_L(0^+)$, $\text{v}_C(0^-)$ and $\text{i}_C(0^+)$.



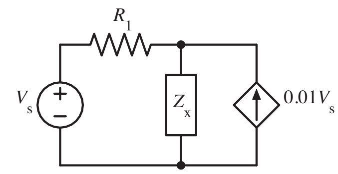
Before time $t = 0$ the capacitor current and the inductor voltage are both zero. All the current flows through the inductor. Therefore $\text{i}_L(0^-) = \text{i}_s(0^-) = -6$. Since there is no current flowing anywhere else at that time (and never has been) all the other currents and voltages are zero. So $\text{v}_C(0^-) = 0$.

At time $t = 0^+$ the capacitor voltage and the inductor current are unchanged. So the change in current from the source must flow through the other two branches with R_1 and R_2 in them. The new source current is -3A. The voltages across R_1 and R_2 are the same because $\text{v}_C(0^+) = 0$. Let the voltage across the current source (positive on the left) be $\text{v}_s(0^+)$. Then

$$\frac{\text{v}_s(0^+)}{R_1} + \frac{\text{v}_s(0^+)}{R_2} = \frac{R_2\,\text{v}_s(0^+) + R_1\,\text{v}_s(0^+)}{R_1 R_2} = \text{v}_s(0^+)\frac{R_2 + R_1}{R_1 R_2} = 3\text{A} \Rightarrow \text{v}_s(0^+) = \frac{R_1 R_2}{R_1 + R_2} \times 3\text{A} = 120\text{V}$$

Since the current source and inductor are in parallel, their voltages are the same and $\text{v}_L(0^+) = 120\text{V}$. Then $\text{i}_C(0^+) = \dfrac{120\text{V}}{50\Omega} = 2.4\text{A}$.

7. In the circuit below $R_1 = 100\Omega$, $Z_X = 50 - j100$ and $V_s = 100\angle 0°$ rms. Find the numerical value of the complex power delivered to Z_X.



Let V_X be the voltage across Z_X, positive on top. Then

$$\frac{V_X - V_s}{R_1} + \frac{V_X}{Z_X} - 0.01V_s = 0 \Rightarrow \frac{V_X - 100\text{V}}{100\Omega} + \frac{V_X}{(50 - j100)\Omega} - 0.01(\text{A/V})(100\text{V}) = 0$$

$$\frac{V_X}{100\Omega} + \frac{V_X}{(50 - j100)\Omega} = 1\text{A} + 1\text{A} = 2\text{A}$$

$$V_X \frac{(150 - j100)\Omega}{(5{,}000 - j10{,}000)\Omega} = V_X \frac{180.28\angle -33.69°}{11{,}180\angle -63.43°} = 2\text{A}$$

$$V_X \times 0.0161\angle 29.74° = 2\text{A} \Rightarrow V_X = 124.03\angle -29.74° \text{ rms}$$

$$I_X = \frac{V_X}{Z_X} = \frac{124.03\angle -29.74° \text{ V rms}}{111.8\angle -63.43° \ \Omega} = 1.11\angle 33.69° \text{ rms} \quad \text{(flowing downward)}$$

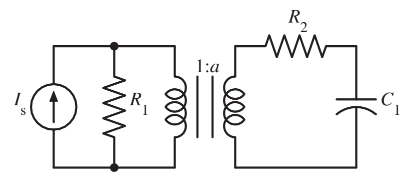
The complex power delivered to Z_X is

$$\mathbf{S}_X = V_X I_X^* = 124.03\angle -29.74° \text{ V} \times 1.11\angle -33.69° \text{ A} = 137.6\angle -63.43° \text{ VA}$$

8. In the circuit below the transformer is ideal and

$$R_1 = 4\Omega \ , \ R_2 = 40\Omega \ , \ a = 4 \ , \ C_1 = 50\mu\text{F} \ \text{and} \ I_s = 6\angle 0° \text{ A rms} .$$

The circuit is operating at a radian frequency of 377 radians/second. Find the numerical value of the average power delivered to R_2.



The input impedance to the transformer primary is

$$Z_{in} = \left(1/a^2\right)\left(40\Omega + \frac{1}{j\left(377/\text{s}\right)50\times10^{-6}\text{F}}\right) = \left(1/16\right)\left(40\Omega - j53.05\Omega\right) = 2.5 - j3.316\Omega$$

Using current division, the current in the primary is therefore

$$I_s \frac{R_1}{R_1 + Z_{in}} = 6\angle 0° \text{ A rms} \times \frac{4\Omega}{4\Omega + \left(2.5 - j3.316\right)\Omega} = 3.288\angle 27.03° \text{ A rms}$$

The current in the secondary is therefore $\frac{3.288\angle 27.03° \text{ A rms}}{4} = 0.822\angle 27.03° \text{ A rms}$

The power delivered to R_2 is then $\left(0.0.822 \text{ A rms}\right)^2 \times 40\Omega = 27.03\text{W}$