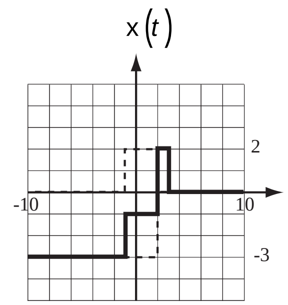
Solution of ECE 315 Test 2 F07

1.

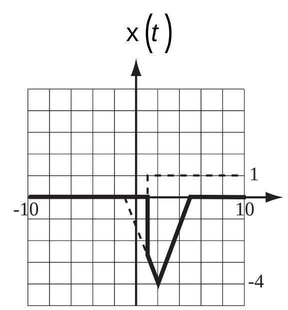
(a) In the space provided below draw a graph of $x(t) = 2\,\mathrm{rect}\left(\frac{t-1}{4}\right) - 3\,u(2-t)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



(b) In the space provided below draw a graph of $x(t) = -4\,\mathrm{tri}\left(\frac{t-2}{3}\right)u(t-1)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

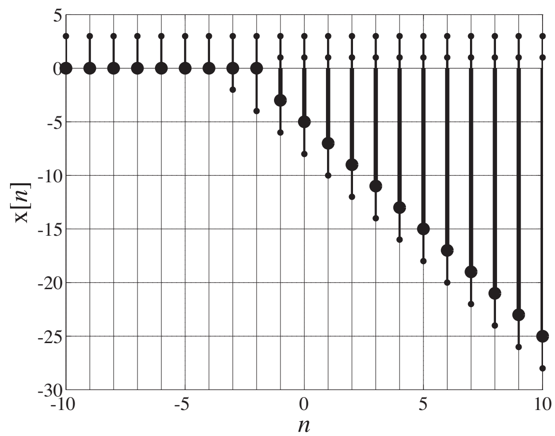
(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



2.

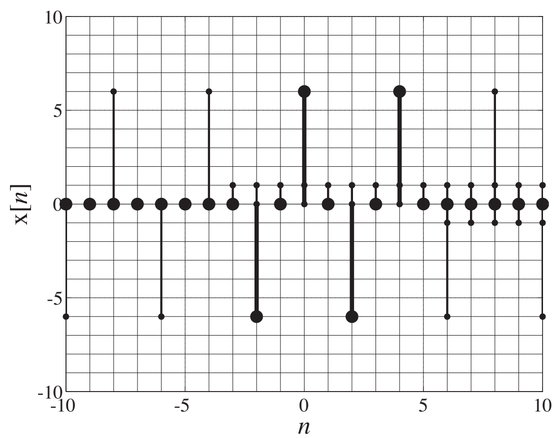
(a) In the space provided below draw a graph of $x[n] = \{3 - \text{ramp}[2(n+4)]\}u[n+1]$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



(b) In the space provided below draw a graph of $x[n] = 6\cos\left(\frac{2\pi n}{4}\right)\{u[n+3] - u[n-6]\}$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



3. For each signal indicate whether or not it is periodic and, if it is periodic, write in its fundamental period. (If you think a signal is periodic it would be a good idea to compute its value at a point in time and also at that point in time plus one fundamental period to be sure you get the same value.)

(a) $x(t) = 4\sin(20\pi t) + 7\sin(24\pi t)$ $\boxed{\text{Periodic}}$ Not Periodic

The two individual fundamental frequencies are 10 Hz and 12 Hz. The GCD is 2 Hz. So $f_0 = 2 \Rightarrow T_0 = 1/2$ and x is periodic.

OR

The two individual fundamental periods are 1/10 and 1/12. The LCM is 1/2. So $T_0 = 1/2$ and x is periodic.

(b) $x(t) = 3\sin(10t) - 9\sin(16\pi t)$ Periodic $\boxed{\text{Not Periodic}}$

The two individual fundamental frequencies are $5/\pi$ Hz and 8 Hz. The GCD is 0. So $f_0 = 0 \Rightarrow T_0 \to \infty$ and x is not periodic.

OR

The two individual fundamental periods are $\pi/5$ and 1/8. The LCM is infinite. So $T_0 \to \infty$ and x is not periodic.

(c) $x[n] = 2\cos\left(\frac{2\pi n}{3}\right) + 8\sin\left(\frac{3\pi n}{7}\right)$ $\boxed{\text{Periodic}}$ Not Periodic

$$x[n] = 2\cos\left(\frac{2\pi n}{3}\right) + 8\sin\left(\frac{2(3)\pi n}{14}\right)$$

The individual periods are 3 and 14. The LCM is 42. So $N_0 = 42$ and x is periodic.

(d) $x[n] = 5\cos\left(\frac{2\pi n}{4}\right) + 4\,\mathrm{sinc}\left(\frac{n}{8}\right)$ Periodic $\boxed{\text{Not Periodic}}$

The sinc function is not periodic, therefore $x[n]$ is not periodic.

4. Find the signal energy of these signals.

(a) $\mathrm{x}(t) = -4\,\mathrm{rect}\left(\frac{t+1}{4}\right) + 3\,\mathrm{rect}\left(\frac{t}{5}\right)$

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left|\mathrm{x}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|-4\,\mathrm{rect}\left(\frac{t+1}{4}\right) + 3\,\mathrm{rect}\left(\frac{t}{5}\right)\right|^2 dt = \int_{-\infty}^{\infty} \left[-4\,\mathrm{rect}\left(\frac{t+1}{4}\right) + 3\,\mathrm{rect}\left(\frac{t}{5}\right)\right]^2 dt$$

$$E_{\mathrm{x}} = 16\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t+1}{4}\right) dt + 9\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t}{5}\right) dt - 24\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t+1}{4}\right)\mathrm{rect}\left(\frac{t}{5}\right) dt$$

$$E_{\mathrm{x}} = 16\int_{-3}^{1} dt + 9\int_{-5/2}^{5/2} dt - 24\int_{-5/2}^{1} dt = 64 + 45 - 84 = 25$$

OR

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left|\mathrm{x}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left[-4\,\mathrm{rect}\left(\frac{t+1}{4}\right) + 3\,\mathrm{rect}\left(\frac{t}{5}\right)\right]^2 dt = \int_{-3}^{-5/2} (-4)^2\, dt + \int_{-5/2}^{1} (-1)^2\, dt + \int_{1}^{5/2} 3^2\, dt$$

$$E_{\mathrm{x}} = 8 + 3.5 + 13.5 = 25$$

(b) $\mathrm{x}[n] = \mathrm{ramp}[4n]\,\mathrm{u}[5-n]$

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|\mathrm{ramp}[4n]\,\mathrm{u}[5-n]\right|^2 = \sum_{n=-\infty}^{5} \left|\mathrm{ramp}[4n]\right|^2$$

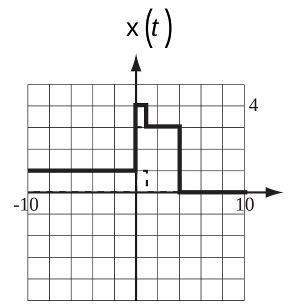
$$E_{\mathrm{x}} = \sum_{n=-\infty}^{5} \left(\mathrm{ramp}[4n]\right)^2 = \sum_{n=0}^{5} (4n)^2 = 16\sum_{n=0}^{5} n^2 = 16(0+1+4+9+16+25) = 16\times 55 = 880$$

Solution of ECE 315 Test 2 F07

1.

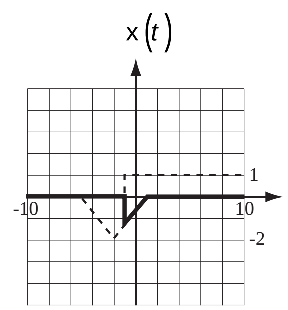
(a) In the space provided below draw a graph of $x(t) = 3\,\text{rect}\left(\frac{t-2}{4}\right) + u(1-t)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



(b) In the space provided below draw a graph of $x(t) = -2\,\text{tri}\left(\frac{t+2}{3}\right)u(t+1)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

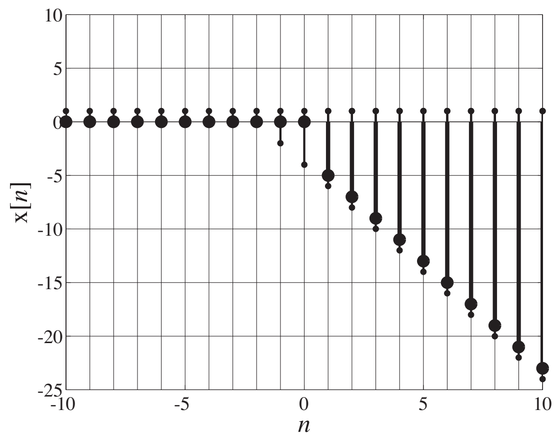
(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



2.

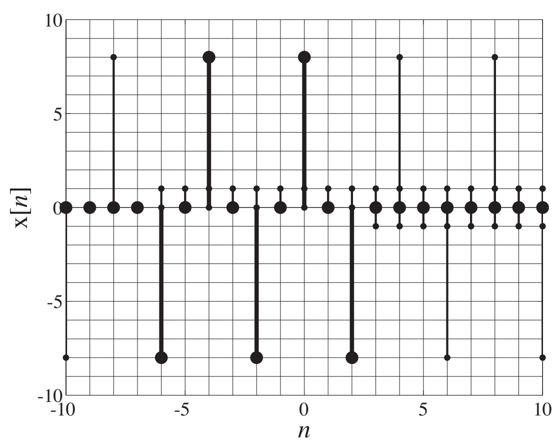
(a) In the space provided below draw a graph of $x[n] = \{1 - \mathrm{ramp}[2(n+2)]\}\mathrm{u}[n-1]$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



(b) In the space provided below draw a graph of $x[n] = 8\cos\left(\frac{2\pi n}{4}\right)\{\mathrm{u}[n+6] - \mathrm{u}[n-3]\}$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



3. For each signal indicate whether or not it is periodic and, if it is periodic, write in its fundamental period. (If you think a signal is periodic it would be a good idea to compute its value at a point in time and also at that point in time plus one fundamental period to be sure you get the same value.)

(a) $x(t) = 4\sin(18\pi t) + 7\sin(30\pi t)$ $\boxed{\text{Periodic}}$ Not Periodic

The two individual fundamental frequencies are 9 Hz and 15 Hz. The GCD is 3 Hz. So $f_0 = 3 \Rightarrow T_0 = 1/3$ and x is periodic.
OR
The two individual fundamental periods are 1/9 and 1/15. The LCM is 1/3. So $T_0 = 1/3$ and x is periodic.

(b) $x(t) = 4\sin(12t) - 5\sin(18\pi t)$ Periodic $\boxed{\text{Not Periodic}}$

The two individual fundamental frequencies are $6/\pi$ Hz and 9 Hz. The GCD is 0. So $f_0 = 0 \Rightarrow T_0 \to \infty$ and x is not periodic.
OR
The two individual fundamental periods are $\pi/6$ and 1/9. The LCM is infinite. So $T_0 \to \infty$ and x is not periodic.

(c) $x[n] = 2\cos\left(\frac{2\pi n}{5}\right) + 8\sin\left(\frac{3\pi n}{5}\right)$ $\boxed{\text{Periodic}}$ Not Periodic

$$x[n] = 2\cos\left(\frac{2\pi n}{5}\right) + 8\sin\left(\frac{2(3)\pi n}{10}\right)$$

The individual periods are 5 and 10. The LCM is 10. So $N_0 = 10$ and x is periodic.

(d) $x[n] = 9\cos\left(\frac{2\pi n}{3}\right) + 2\,\mathrm{sinc}\left(\frac{n}{6}\right)$ Periodic $\boxed{\text{Not Periodic}}$

The sinc function is not periodic, therefore $x[n]$ is not periodic.

4. Find the signal energy of these signals.

(a) $\mathrm{x}(t) = -2\,\mathrm{rect}\left(\frac{t-1}{4}\right) + 6\,\mathrm{rect}\left(\frac{t}{4}\right)$

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2\,dt = \int_{-\infty}^{\infty} \left|-2\,\mathrm{rect}\left(\frac{t-1}{4}\right) + 6\,\mathrm{rect}\left(\frac{t}{4}\right)\right|^2 dt = \int_{-\infty}^{\infty} \left[-2\,\mathrm{rect}\left(\frac{t-1}{4}\right) + 6\,\mathrm{rect}\left(\frac{t}{4}\right)\right]^2 dt$$

$$E_{\mathrm{x}} = 4\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t-1}{4}\right) dt + 36\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t}{4}\right) dt - 24\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t-1}{4}\right)\mathrm{rect}\left(\frac{t}{4}\right) dt$$

$$E_{\mathrm{x}} = 4\int_{-1}^{3} dt + 36\int_{-2}^{2} dt - 24\int_{-1}^{2} dt = 16 + 144 - 72 = 88$$

OR

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2\,dt = \int_{-\infty}^{\infty} \left[-2\,\mathrm{rect}\left(\frac{t-1}{4}\right) + 6\,\mathrm{rect}\left(\frac{t}{4}\right)\right]^2 dt = \int_{2}^{3} (-2)^2\,dt + \int_{-1}^{2} (4)^2\,dt + \int_{-2}^{-1} 6^2\,dt$$

$$E_{\mathrm{x}} = 4 + 48 + 36 = 88$$

(b) $\mathrm{x}[n] = \mathrm{ramp}[3n]\,\mathrm{u}[4-n]$

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} |\mathrm{ramp}[3n]\,\mathrm{u}[4-n]|^2 = \sum_{n=-\infty}^{4} |\mathrm{ramp}[3n]|^2$$

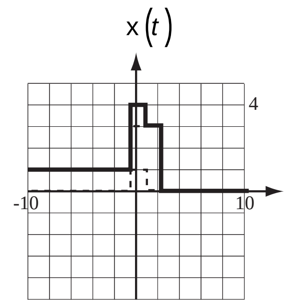
$$E_{\mathrm{x}} = \sum_{n=-\infty}^{4} (\mathrm{ramp}[3n])^2 = \sum_{n=0}^{4} (3n)^2 = 9\sum_{n=0}^{4} n^2 = 9(0+1+4+9+16) = 9 \times 30 = 270$$

Solution of ECE 315 Test 2 F07

1.

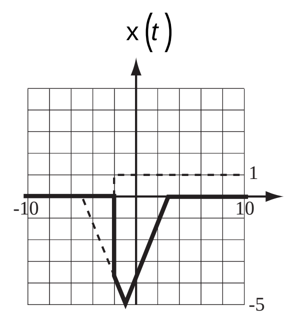
(a) In the space provided below draw a graph of $x(t) = 3\,\text{rect}\left(\frac{t-1}{3}\right) + u(1-t)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



(b) In the space provided below draw a graph of $x(t) = -5\,\text{tri}\left(\frac{t+1}{4}\right)u(t+2)$ over the time range $-10 < t < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

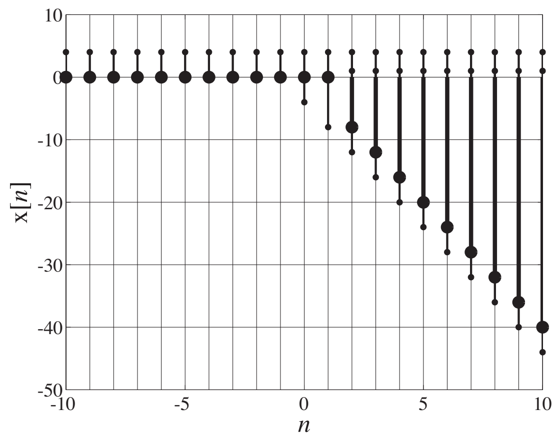
(The heavy solid line is the overall answer. The dashed ones are the individual functions.)



2.

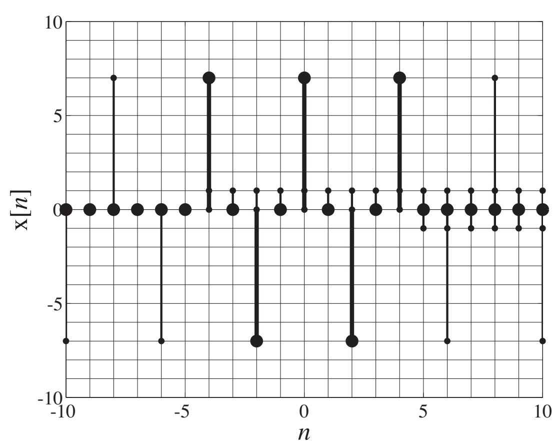
(a) In the space provided below draw a graph of $\mathrm{x}[n] = \{4 - \mathrm{ramp}[4(n+1)]\}\mathrm{u}[n-2]$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



(b) In the space provided below draw a graph of $\mathrm{x}[n] = 7\cos\left(\frac{2\pi n}{4}\right)\{\mathrm{u}[n+4] - \mathrm{u}[n-5]\}$ over the time range $-10 < n < 10$. On the graph put a vertical scale. The graph should be accurate enough to be able to read approximate function values from it.

(The heavy stem plot is the overall answer. The lighter ones are the individual functions.)



3. For each signal indicate whether or not it is periodic and, if it is periodic, write in its fundamental period. (If you think a signal is periodic it would be a good idea to compute its value at a point in time and also at that point in time plus one fundamental period to be sure you get the same value.)

(a) $x(t) = 4\sin(50\pi t) + 7\sin(20\pi t)$ $\boxed{\text{Periodic}}$ Not Periodic

The two individual fundamental frequencies are 25 Hz and 10 Hz. The GCD is 5 Hz. So $f_0 = 5 \Rightarrow T_0 = 1/5$ and x is periodic.
OR
The two individual fundamental periods are 1/25 and 1/10. The LCM is 1/5. So $T_0 = 1/5$ and x is periodic.

(b) $x(t) = 4\sin(16t) - 5\sin(36\pi t)$ Periodic $\boxed{\text{Not Periodic}}$

The two individual fundamental frequencies are $8/\pi$ Hz and 18 Hz. The GCD is 0. So $f_0 = 0 \Rightarrow T_0 \to \infty$ and x is not periodic.
OR
The two individual fundamental periods are $\pi/8$ and 1/18. The LCM is infinite. So $T_0 \to \infty$ and x is not periodic.

(c) $x[n] = 2\cos\left(\frac{2\pi n}{8}\right) + 8\sin\left(\frac{3\pi n}{10}\right)$ $\boxed{\text{Periodic}}$ Not Periodic

$$x[n] = 2\cos\left(\frac{2\pi n}{8}\right) + 8\sin\left(\frac{2(3)\pi n}{20}\right)$$

The individual periods are 8 and 20. The LCM is 40. So $N_0 = 40$ and x is periodic.

(d) $x[n] = 2\cos\left(\frac{2\pi n}{5}\right) - 5\,\mathrm{sinc}\left(\frac{n}{10}\right)$ Periodic $\boxed{\text{Not Periodic}}$

The sinc function is not periodic, therefore $x[n]$ is not periodic.

4. Find the signal energy of these signals.

(a) $\mathrm{x}(t) = -2\,\mathrm{rect}\left(\dfrac{t-3}{4}\right) + 4\,\mathrm{rect}\left(\dfrac{t-2}{2}\right)$

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left|\mathrm{x}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|-2\,\mathrm{rect}\left(\frac{t-3}{4}\right) + 4\,\mathrm{rect}\left(\frac{t-2}{2}\right)\right|^2 dt = \int_{-\infty}^{\infty} \left[-2\,\mathrm{rect}\left(\frac{t-3}{4}\right) + 4\,\mathrm{rect}\left(\frac{t-2}{2}\right)\right]^2 dt$$

$$E_{\mathrm{x}} = 4\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t-3}{4}\right) dt + 16\int_{-\infty}^{\infty} \mathrm{rect}^2\left(\frac{t-2}{2}\right) dt - 16\int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t-3}{4}\right)\mathrm{rect}\left(\frac{t-2}{2}\right) dt$$

$$E_{\mathrm{x}} = 4\int_{1}^{5} dt + 16\int_{1}^{3} dt - 16\int_{1}^{3} dt = 16 + 32 - 32 = 16$$

OR

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} \left|\mathrm{x}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left[-2\,\mathrm{rect}\left(\frac{t-3}{4}\right) + 4\,\mathrm{rect}\left(\frac{t-2}{2}\right)\right]^2 dt = \int_{1}^{3} (2)^2\, dt + \int_{3}^{5} (-2)^2\, dt$$

$$E_{\mathrm{x}} = 8 + 8 = 16$$

(b) $\mathrm{x}[n] = \mathrm{ramp}[5n]\,\mathrm{u}[3-n]$

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|\mathrm{ramp}[5n]\,\mathrm{u}[3-n]\right|^2 = \sum_{n=-\infty}^{3} \left|\mathrm{ramp}[5n]\right|^2$$

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{3} \left(\mathrm{ramp}[5n]\right)^2 = \sum_{n=0}^{3} (5n)^2 = 25\sum_{n=0}^{3} n^2 = 25(0+1+4+9) = 25 \times 14 = 350$$