Solution of ECE 315 Test 2 F09

1. Find the numerical values of the constants in these Laplace transform pairs.

(c)
$$Ae^{at} \mathbf{u}(bt) + Ce^{ct} \mathbf{u}(dt) \longleftrightarrow \frac{s-1}{s(s+2)}, \ \sigma > 0$$

$$\frac{s-1}{s(s+2)} = \frac{-1/2}{s} + \frac{3/2}{s+2}, \ \sigma > 0$$

$$\left[-(1/2) + (3/2)e^{-2t}\right] \mathbf{u}(t) \longleftrightarrow \frac{-1/2}{s} + \frac{3/2}{s+2}, \ \sigma > 0$$

(d)
$$Ae^{at} u(bt) + Ce^{ct} u(dt) \stackrel{\mathscr{L}}{\longleftrightarrow} \frac{s-1}{s(s+2)}$$
, $-2 < \sigma < 0$
 $(1/2)u(-t) + (3/2)e^{-2t} u(t) \stackrel{\mathscr{L}}{\longleftrightarrow} \frac{s-1}{s(s+2)}$, $-2 < \sigma < 0$

2. Find the numerical values of the constants in these Fourier transform pairs.

(a)
$$-2\operatorname{tri}(t-4) \xleftarrow{\mathscr{F}} A\operatorname{sinc}^2(af)e^{bf}$$

 $-2\operatorname{tri}(t-4) \xleftarrow{\mathscr{F}} -2\operatorname{sinc}^2(f)e^{-j8\pi f}$

(b)
$$A\sin(at)\operatorname{sinc}(bt) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-3) - \delta(f+3)\right] * 4\operatorname{rect}(2f)$$

 $-4\sin(6\pi t)\operatorname{sinc}(t/2) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-3) - \delta(f+3)\right] * 4\operatorname{rect}(2f)$
or
 $4\sin(-6\pi t)\operatorname{sinc}(t/2) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-3) - \delta(f+3)\right] * 4\operatorname{rect}(2f)$

(c)
$$A \operatorname{rect}(at) * \operatorname{rect}(bt) \xleftarrow{\mathscr{G}} (14/3) \operatorname{sinc}(f/3) \operatorname{sinc}(2f)$$

 $7 \operatorname{rect}(3t) * \operatorname{rect}(t/2) \xleftarrow{\mathscr{G}} (14/3) \operatorname{sinc}(f/3) \operatorname{sinc}(2f)$

3. If a CTFT in the f form is $24\delta_3(f)$ and in the ω form it is $A\delta_b(c\omega)$, what are the numerical values of A, b and c?

$$[24\delta_3(f)]_{f \to \omega/2\pi} = 24\delta_3(\omega/2\pi) = 24\sum_{k=-\infty}^{\infty} \delta(\omega/2\pi - 3k) = 48\pi\sum_{k=-\infty}^{\infty} \delta(\omega - 6\pi k)$$
$$[24\delta_3(f)]_{f \to \omega/2\pi} = 48\pi\delta_{6\pi}(\omega)$$

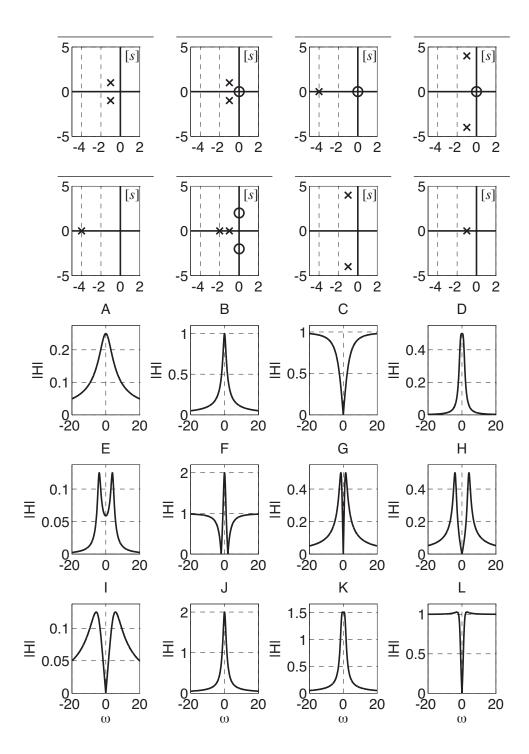
Because of the way the problem was worded, I also accepted the solution

$$24\delta_3(\omega/2\pi)$$

even though that was not what I intended.

4. Below are some pole-zero diagrams and some magnitude frequency responses. Match the frequency responses to the pole-zero diagrams by writing the letter designation of the frequency response in the blank space provided above the pole-zero diagram. If there is no match, just write "None".

D G C H A F E B



Solution of ECE 315 Test 3 F09

1. Find the numerical values of the constants in these Laplace transform pairs.

(a)
$$7e^{3t} u(t) \longleftrightarrow \frac{A}{s-a}$$

What is the ROC? ROC is

$$7e^{3t} u(t) \longleftrightarrow \frac{9}{s-3}$$
, $\sigma > 3$

(b)
$$3\delta(5t-1) \xleftarrow{\mathscr{L}} Ae^{-as}$$

What is the ROC? ROC is
$$3\delta(5t-1) = 3\delta(5(t-1/5)) = (3/5)\delta(t-1/5) \xleftarrow{\mathscr{L}} (3/5)e^{-s/5}$$
, ROC is all s.

(c)
$$Ae^{at} \mathbf{u}(bt) + Ce^{ct} \mathbf{u}(dt) \stackrel{\mathscr{L}}{\longleftrightarrow} \frac{s-2}{s(s+1)}$$
, $\sigma > 0$

$$\frac{s-2}{s(s+1)} = \frac{-2}{s} + \frac{3}{s+1}$$
, $\sigma > 0$

$$\left[-2+3e^{-t}\right]\mathbf{u}(t) \longleftrightarrow -\frac{2}{s} + \frac{3}{s+1} , \sigma > 0$$

(d)
$$Ae^{at} u(bt) + Ce^{ct} u(dt) \longleftrightarrow \frac{s-2}{s(s+1)}$$
, $-1 < \sigma < 0$

$$2\operatorname{u}(-t) + 3e^{-t}\operatorname{u}(t) \longleftrightarrow \frac{s-2}{s(s+1)} , -1 < \sigma < 0$$

2. Find the numerical values of the constants in these Fourier transform pairs.

(a)
$$-4 \operatorname{tri}(t-2) \xleftarrow{\mathcal{F}} A \operatorname{sinc}^2(af) e^{bf}$$

 $-4 \operatorname{tri}(t-2) \xleftarrow{\mathcal{F}} -4 \operatorname{sinc}^2(f) e^{-j4\pi f}$

(b)
$$A\sin(at)\operatorname{sinc}(bt) \xleftarrow{\mathscr{F}} j[\delta(f-5)-\delta(f+5)] * 6\operatorname{rect}(3f)$$

 $-4\sin(10\pi t)\operatorname{sinc}(t/3) \xleftarrow{\mathscr{F}} \xleftarrow{\mathscr{F}} j[\delta(f-5)-\delta(f+5)] * 6\operatorname{rect}(3f)$
or
 $4\sin(-10\pi t)\operatorname{sinc}(t/3) \xleftarrow{\mathscr{F}} \xleftarrow{\mathscr{F}} j[\delta(f-5)-\delta(f+5)] * 6\operatorname{rect}(3f)$

(c)
$$A \operatorname{rect}(at) * \operatorname{rect}(bt) \xleftarrow{\mathscr{G}} (24/5) \operatorname{sinc}(f/5) \operatorname{sinc}(3f)$$

 $8 \operatorname{rect}(5t) * \operatorname{rect}(t/3) \xleftarrow{\mathscr{G}} (24/5) \operatorname{sinc}(f/5) \operatorname{sinc}(3f)$

3. If a CTFT in the f form is $18\delta_5(f)$ and in the ω form it is $A\delta_b(c\omega)$, what are the numerical values of A, b and c?

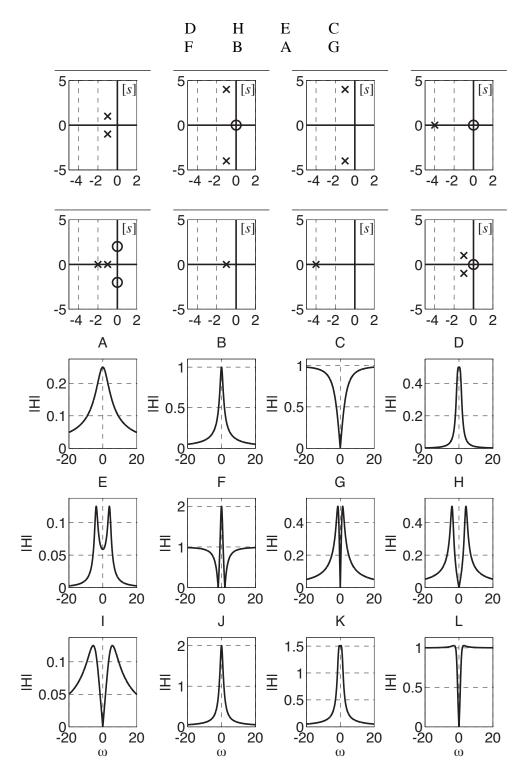
$$[18\delta_{5}(f)]_{f\to\omega/2\pi} = 18\delta_{5}(\omega/2\pi) = 18\sum_{k=-\infty}^{\infty} \delta(\omega/2\pi - 5k) = 36\pi\sum_{k=-\infty}^{\infty} \delta(\omega - 10\pi k)$$
$$[18\delta_{5}(f)]_{f\to\omega/2\pi} = 36\pi\delta_{10\pi}(\omega)$$

Because of the way the problem was worded, I also accepted the solution

$$18\delta_5(\omega/2\pi)$$

even though that was not what I intended.

4. Below are some pole-zero diagrams and some magnitude frequency responses. Match the frequency responses to the pole-zero diagrams by writing the letter designation of the frequency response in the blank space provided above the pole-zero diagram. If there is no match, just write "None".



Solution of ECE 315 Test 2 F09

Find the numerical values of the constants in these Laplace transform pairs. 1.

(a)
$$3e^{5t} u(t) \longleftrightarrow \frac{A}{s-a}$$

What is the ROC? ROC is

$$3e^{5t} \mathbf{u}(t) \longleftrightarrow \frac{3}{s-5}$$
, $\sigma > 5$

(b)
$$4\delta(6t-1) \stackrel{\mathscr{L}}{\longleftrightarrow} Ae^{-as}$$

What is the ROC? ROC is
$$\underline{\hspace{1cm}} 4\delta(6t-1) = 4\delta(6(t-1/6)) = (2/3)\delta(t-1/6) \xleftarrow{\mathscr{L}} (2/3)e^{-s/6}$$
, ROC is all s.

(c)
$$Ae^{at} \mathbf{u}(bt) + Ce^{ct} \mathbf{u}(dt) \longleftrightarrow \frac{s-3}{s(s+5)}, \ \sigma > 0$$

$$\frac{s-3}{s(s+5)} = \frac{-3/5}{s} + \frac{8/5}{s+5} , \ \sigma > 0$$

$$\left[-3/5 + \left(8/5\right)e^{-5t}\right] \mathbf{u}(t) \longleftrightarrow \frac{-3/5}{s} + \frac{8/5}{s+5} , \ \sigma > 0$$

(d)
$$Ae^{at} u(bt) + Ce^{ct} u(dt) \longleftrightarrow \frac{s-3}{s(s+5)}$$
, $-5 < \sigma < 0$

$$(3/5)\mathbf{u}(-t) + (8/5)e^{-5t}\mathbf{u}(t) \longleftrightarrow \frac{s-3}{s(s+5)}, -5 < \sigma < 0$$

2. Find the numerical values of the constants in these Fourier transform pairs.

(a)
$$-7 \operatorname{tri}(t-3) \xleftarrow{\mathcal{F}} A \operatorname{sinc}^2(af) e^{bf}$$

 $-7 \operatorname{tri}(t-3) \xleftarrow{\mathcal{F}} -7 \operatorname{sinc}^2(f) e^{-j6\pi f}$

(b)
$$A\sin(at)\operatorname{sinc}(bt) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-8)-\delta(f+8)\right] * 9\operatorname{rect}(5f)$$

 $-(18/5)\sin(16\pi t)\operatorname{sinc}(t/5) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-8)-\delta(f+8)\right] * 9\operatorname{rect}(5f)$
or $(18/5)\sin(-16\pi t)\operatorname{sinc}(t/5) \stackrel{\mathcal{F}}{\longleftrightarrow} j \left[\delta(f-8)-\delta(f+8)\right] * 9\operatorname{rect}(5f)$

(c)
$$A \operatorname{rect}(at) * \operatorname{rect}(bt) \xleftarrow{\mathscr{F}} 9 \operatorname{sinc}(2f) \operatorname{sinc}(5f)$$

 $(9/10)\operatorname{rect}(t/2) * \operatorname{rect}(t/5) \xleftarrow{\mathscr{F}} 9 \operatorname{sinc}(2f) \operatorname{sinc}(5f)$

3. If a CTFT in the f form is $30\delta_4(f)$ and in the ω form it is $A\delta_b(c\omega)$, what are the numerical values of A, b and c?

$$\left[30\delta_4(f) \right]_{f \to \omega/2\pi} = 30\delta_4(\omega/2\pi) = 30\sum_{k=-\infty}^{\infty} \delta(\omega/2\pi - 4k) = 60\pi\sum_{k=-\infty}^{\infty} \delta(\omega - 8\pi k)$$

$$\left[30\delta_4(f) \right]_{f \to \omega/2\pi} = 60\pi\delta_{8\pi}(\omega)$$

Because of the way the problem was worded, I also accepted the solution

$$30\delta_4(\omega/2\pi)$$

even though that was not what I intended.

4. Below are some pole-zero diagrams and some magnitude frequency responses. Match the frequency responses to the pole-zero diagrams by writing the letter designation of the frequency response in the blank space provided above the pole-zero diagram. If there is no match, just write "None".

H D A F E B G C

