Solution of ECE 315 Test 4 F09

1. Find the numerical values of the constants in these z transform pairs.

(a) $25(-0.5)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{Az}{z-a}$

What is the ROC?

$25(-0.5)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{25z}{z+0.5}$ ROC is $|z| > 0.5$

(b) $5\delta[2n-4] \xleftrightarrow{\mathcal{Z}} Az^a$

What is the ROC?

$5\delta[2n-4] = 5\delta[n-2] \xleftrightarrow{\mathcal{Z}} 5z^{-2}$, ROC is $|z| > 0$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

(c) $A\delta[n+a] + Bb^{n+a} \mathrm{u}[n+a] \xleftrightarrow{\mathcal{Z}} \dfrac{z+1}{z(z+0.8)}$, $|z| > 0.8$

$1.25\delta[n-1] - 0.25(-0.8)^{n-1} \mathrm{u}[n-1] \xleftrightarrow{\mathcal{Z}} \dfrac{1.25}{z} - \dfrac{0.25}{z+0.8}$, $|z| > 0.8$

(d) $(\delta[n+3] - \delta[n-3])5\cos(2\pi n/8) \xleftrightarrow{\mathcal{Z}} A(z^b - z^c)$

What is the ROC?

$(\delta[n+3] - \delta[n-3])5\cos(2\pi n/8) = 5(\delta[n+3]\cos(-6\pi/8) - \delta[n-3]\cos(6\pi/8))$

$(-5\sqrt{2}/2)(\delta[n+3] - \delta[n-3]) \xleftrightarrow{\mathcal{Z}} (5\sqrt{2}/2)(z^{-3} - z^{3})$, ROC is $|z| > 0$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

2. Given the transfer function $\mathrm{H}(z)$ find the numerical magnitude and angle (in radians) of the frequency response $\mathrm{H}(e^{j\Omega})$ at the specified value of Ω.

$$\mathrm{H}(z) = \frac{z}{z-0.7} \quad \text{at } \Omega = \pi/2$$

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-0.7} \Rightarrow \mathrm{H}(e^{j\pi/2}) = \frac{e^{j\pi/2}}{e^{j\pi/2}-0.7} = \frac{j}{j-0.7} = 0.819\angle -0.611 \text{ radians}$$

3. A discrete-time system has a transfer function

$$\mathrm{H}(z) = \frac{z}{(z-0.7+j0.7)(z-0.7-j0.7)}.$$

At what numerical frequencies Ω will the magnitude of its frequency response be a maximum?

Poles at $0.7 \pm j0.7 = 0.98\angle \pm 0.785$. The maximum frequency response maginitude occurs at the closest approach to these poles which is at $\Omega = \pm 0.785$.

4. Given the discrete-time signal $\mathrm{x}[n] = 4 + 12\cos(2\pi n/3)$, let the values of x for $0 \le n < 9$ form the input data vector for a DFT and let the vector returned by the DFT be $\mathrm{X}[k]$ for $0 \le n < 9$. (Be sure to notice the difference between x and X and also the difference between $\le$ and <.)

(a) What is the numerical value of $\mathrm{X}[0]$?

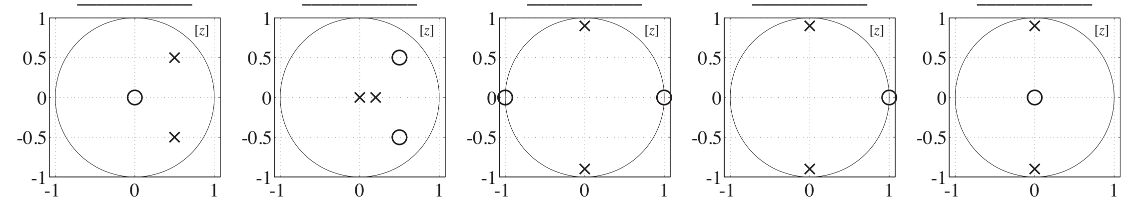
$\mathrm{X}[0]$ is always the sum of the input x values. In this case the sum of the values of $12\cos(2\pi n/3)$ is zero because there are exactly three cycles in 9 points. Therefore the sum of the x values is $4 \times 9 = 36$.

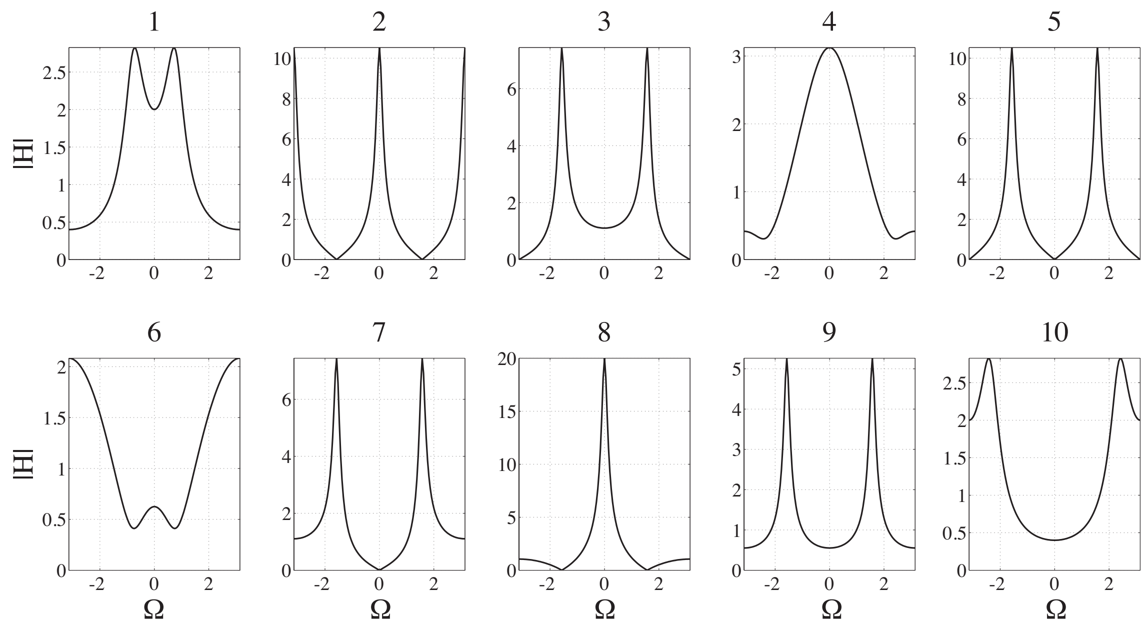
(b) Using the fact that $\mathrm{X}[k]$ is periodic with period 9, at what k values inside the range $0 \le n < 9$ does $\mathrm{X}[k]$ have a value other than zero?

Since this input data vector represents a constant plus three cycles of a sinusoid, the DFT will have non-zero values only at $k = 0$ and $k = \pm 3 + 9q$, q any integer. So the values of k inside $0 \le n < 9$ at which the DFT is non-zero are 0, 3 and 6.

4. Write in the space provided the number designation of the frequency response magnitude corresponding to each pole-zero diagram. (If there is no match, just write "none".) (Each transfer function is of the form $H(z) = A\frac{(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_M)}$ and $A = 1$.)

1	6	5	7	9
______	______	______	______	______





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1. Find the numerical values of the constants in these z transform pairs.

(a) $12(-0.8)^n \mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{Az}{z-a}$

What is the ROC?

$12(-0.8)^n \mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{12z}{z+0.8}$ ROC is $|z| > 0.8$

(b) $3\delta[2n-6] \overset{\mathcal{Z}}{\longleftrightarrow} Az^a$

What is the ROC?

$3\delta[2n-6] = 3\delta[n-3] \overset{\mathcal{Z}}{\longleftrightarrow} 3z^{-3}$, ROC is $|z| > 0$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

(c) $A\delta[n+a] + Bb^{n+a} \mathrm{u}[n+a] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z+0.7}{z(z+0.5)}$, $|z| > 0.5$

$1.4\delta[n-1] - 0.4(-0.5)^{n-1} \mathrm{u}[n-1] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1.4}{z} - \frac{0.4}{z+0.5}$, $|z| > 0.5$

(d) $(\delta[n+4] - \delta[n-4]) 7\cos(2\pi n/10) \overset{\mathcal{Z}}{\longleftrightarrow} A(z^b - z^c)$

What is the ROC?

$(\delta[n+4] - \delta[n-4]) 7\cos(2\pi n/10) = 7(\delta[n+4]\cos(-8\pi/10) - \delta[n-4]\cos(8\pi/10))$

$-5.6631(\delta[n+4] - \delta[n-4]) \overset{\mathcal{Z}}{\longleftrightarrow} -5.6631(z^{-4} - z^4)$, ROC is $|z| > 0$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

2. Given the transfer function $\mathrm{H}(z)$ find the numerical magnitude and angle (in radians) of the frequency response $\mathrm{H}(e^{j\Omega})$ at the specified value of Ω.

$$\mathrm{H}(z) = \frac{z}{z-0.8} \quad \text{at } \Omega = \pi/2$$

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-0.7} \Rightarrow \mathrm{H}(e^{j\pi/2}) = \frac{e^{j\pi/2}}{e^{j\pi/2}-0.8} = \frac{j}{j-0.8} = 0.781\angle -0.675 \text{ radians}$$

3. A discrete-time system has a transfer function

$$\mathrm{H}(z) = \frac{z}{(z+0.7+j0.7)(z+0.7-j0.7)}.$$

At what numerical frequencies Ω will the magnitude of its frequency response be a maximum?

Poles at $-0.7 \pm j0.7 = 0.98\angle \pm 2.356$. The maximum frequency response maginitude occurs at the closest approach to these poles which is at $\Omega = \pm 2.356$.

4. Given the discrete-time signal $\mathrm{x}[n] = 6 + 12\cos(2\pi n/3)$, let the values of x for $0 \le n < 9$ form the input data vector for a DFT and let the vector returned by the DFT be $\mathrm{X}[k]$ for $0 \le n < 9$. (Be sure to notice the difference between x and X and also the difference between $\le$ and <.)

(a) What is the numerical value of $\mathrm{X}[0]$?

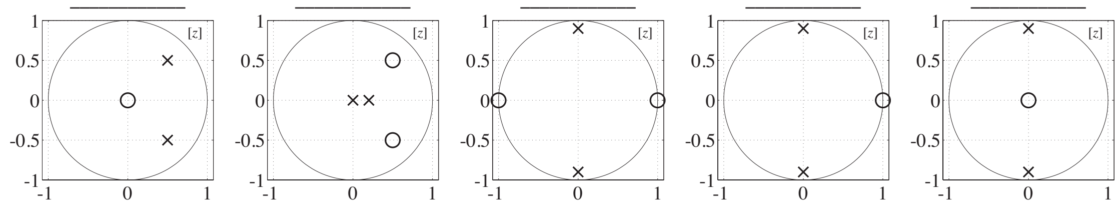
$\mathrm{X}[0]$ is always the sum of the input x values. In this case the sum of the values of $12\cos(2\pi n/3)$ is zero because there are exactly three cycles in 9 points. Therefore the sum of the x values is $6 \times 9 = 54$.

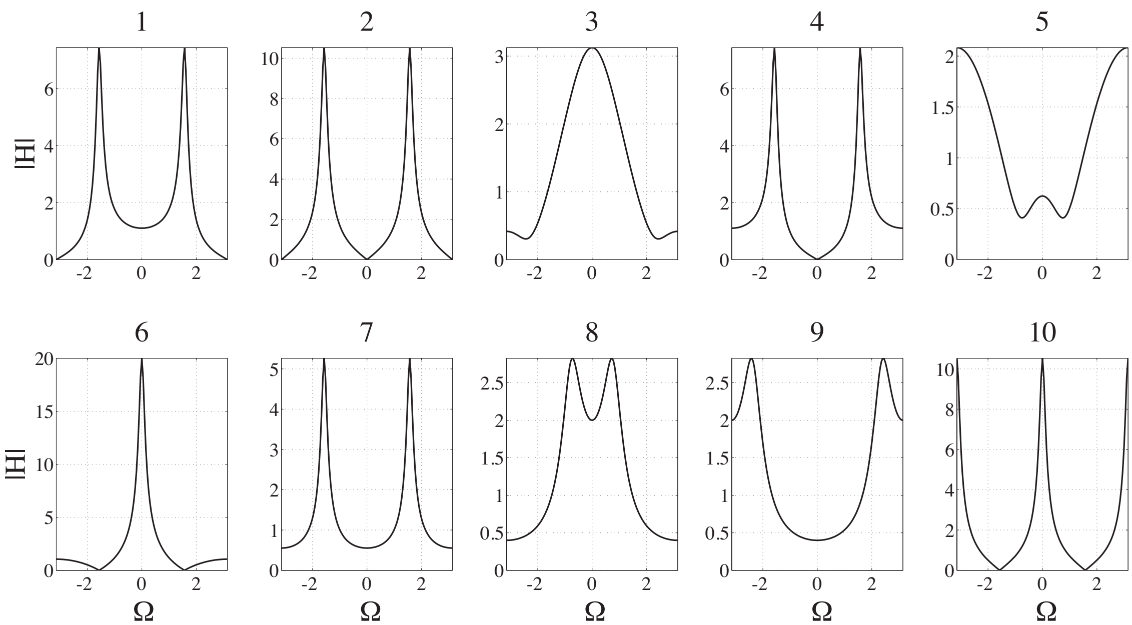
(b) Using the fact that $\mathrm{X}[k]$ is periodic with period 9, at what k values inside the range $0 \le n < 9$ does $\mathrm{X}[k]$ have a value other than zero?

Since this input data vector represents a constant plus three cycles of a sinusoid, the DFT will have non-zero values only at $k = 0$ and $k = \pm 3 + 9q$, q any integer. So the values of k inside $0 \le n < 9$ at which the DFT is non-zero are 0, 3 and 6.

4. Write in the space provided the number designation of the frequency response magnitude corresponding to each pole-zero diagram. (If there is no match, just write "none".) (Each transfer function is of the form $\mathrm{H}(z) = A\dfrac{(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_M)}$ and $A = 1$.)

8 5 2 4 7





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1. Find the numerical values of the constants in these z transform pairs.

(a) $6(-0.3)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{Az}{z-a}$

What is the ROC?

$$6(-0.3)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{6z}{z+0.3} \quad \text{ROC is } |z| > 0.3$$

(b) $15\delta[3n-6] \xleftrightarrow{\mathcal{Z}} Az^a$

What is the ROC?

$$15\delta[3n-6] = 15\delta[n-2] \xleftrightarrow{\mathcal{Z}} 15z^{-2} \text{ , ROC is } |z| > 0$$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

(c) $A\delta[n+a] + Bb^{n+a} \mathrm{u}[n+a] \xleftrightarrow{\mathcal{Z}} \dfrac{z+0.7}{z(z+0.4)} \text{ , } |z| > 0.4$

$$1.75\delta[n-1] - 0.75(-0.4)^{n-1} \mathrm{u}[n-1] \xleftrightarrow{\mathcal{Z}} \frac{1.75}{z} - \frac{0.75}{z+0.4} \text{ , } |z| > 0.4$$

(d) $(\delta[n+2] - \delta[n-2])9\cos(2\pi n/12) \xleftrightarrow{\mathcal{Z}} A(z^b - z^c)$

What is the ROC?

$$(\delta[n+2] - \delta[n-2])9\cos(2\pi n/12) = 9(\delta[n+2]\cos(-4\pi/12) - \delta[n-2]\cos(4\pi/12))$$

$$4.5(\delta[n+2] - \delta[n-2]) \xleftrightarrow{\mathcal{Z}} 4.5(z^2 - z^{-2}) \text{ , ROC is } |z| > 0$$

(I also accepted "all z" as the ROC because one of the tables in the book listed it that way. It is "all z" except for the single point z=0.)

2. Given the transfer function $\mathrm{H}(z)$ find the numerical magnitude and angle (in radians) of the frequency response $\mathrm{H}(e^{j\Omega})$ at the specified value of Ω.

$$\mathrm{H}(z) = \frac{z}{z-0.9} \quad \text{at } \Omega = \pi/2$$

$$\mathrm{H}(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega}-0.9} \Rightarrow \mathrm{H}(e^{j\pi/2}) = \frac{e^{j\pi/2}}{e^{j\pi/2}-0.9} = \frac{j}{j-0.9} = 0.743\angle -0.733 \text{ radians}$$

3. A discrete-time system has a transfer function

$$\mathrm{H}(z) = \frac{z}{(z-0.475+j0.8227)(z-0.475-j0.8227)}.$$

At what numerical frequencies Ω will the magnitude of its frequency response be a maximum?

Poles at $0.4757 \pm j0.8227 = 0.95\angle \pm 1.047$. The maximum frequency response magnitude occurs at the closest approach to these poles which is at $\Omega = \pm 1.047$.

4. Given the discrete-time signal $\mathrm{x}[n] = 3 + 12\cos(2\pi n/3)$, let the values of x for $0 \le n < 9$ form the input data vector for a DFT and let the vector returned by the DFT be $\mathrm{X}[k]$ for $0 \le n < 9$. (Be sure to notice the difference between x and X and also the difference between $\le$ and $<$.)

(a) What is the numerical value of $\mathrm{X}[0]$? $\mathrm{X}[0]$ = __________

$\mathrm{X}[0]$ is always the sum of the input x values. In this case the sum of the values of $12\cos(2\pi n/3)$ is zero because there are exactly three cycles in 9 points. Therefore the sum of the x values is $3 \times 9 = 27$.

(b) Using the fact that $\mathrm{X}[k]$ is periodic with period 9, at what k values inside the range $0 \le n < 9$ does $\mathrm{X}[k]$ have a value other than zero?

Since this input data vector represents a constant plus three cycles of a sinusoid, the DFT will have non-zero values only at $k = 0$ and $k = \pm 3 + 9q$, q any integer . So the values of k inside $0 \le n < 9$ at which the DFT is non-zero are 0, 3 and 6.

4. Write in the space provided the number designation of the frequency response magnitude corresponding to each pole-zero diagram. (If there is no match, just write "none".) (Each transfer function is of the form $\mathrm{H}(z) = A\dfrac{(z-z_1)(z-z_2)\cdots(z-z_M)}{(z-p_1)(z-p_2)\cdots(z-p_M)}$ and $A = 1$.)

8	2	4	1	7