Solution of ECE 315 Test 1 Su10

1. Let $x(t) = -3e^{-t/4}\,u(t-1)$ and let $y(t) = -4\,x(5t)$.

   (a) What is the smallest numerical value of t for which $y(t)$ is not zero?

   $y(t) = -4\,x(5t) = 12e^{-5t/4}\,u(5t-1) \Rightarrow$ Smallest t for which $y(t) \neq 0$ occurs where $5t - 1 = 0$ or $t = 1/5$.

   (b) What is the maximum numerical value of $y(t)$ over all time?

   $$\max(y(t)) = 9.3456$$

   (c) What is the minimum numerical value of $y(t)$ over all time?

   $$\min(y(t)) = 0$$

   (d) What is the numerical value of $y(1)$?

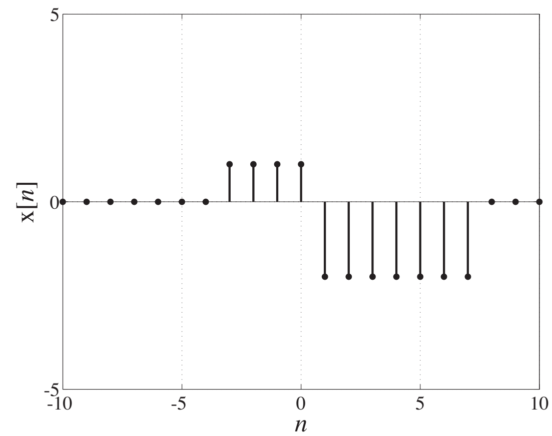
   $$y(t) = 12e^{-5t/4}\,u(5t-1) \Rightarrow y(1) = 12e^{-5/4}\,u(4) = 12e^{-5/4} = 3.4381$$

   (e) What is the largest numerical value of t for which $y(t) > 2$?

   $$y(t) = 2 = 12e^{-5t/4} \Rightarrow e^{-5t/4} = 1/6 \Rightarrow -5t/4 = \ln(1/6) = -1.7918 \Rightarrow t = 4(1.7918)/5 = 1.4334$$

2. Let $\mathrm{x}[n] = \mathrm{u}[n+3] - 3\mathrm{u}[n-1] + 2\mathrm{u}[n-8]$.

(a) Sketch $\mathrm{x}[n]$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical sum of all the values of $\mathrm{x}[n]$?

$$\mathrm{x}[n] = \begin{cases} 0 & , n < -3 \\ 1 & , -2 \le n < 1 \\ -2 & , 1 \le n < 8 \\ 0 & , n \ge 8 \end{cases} \Rightarrow \sum_{n=-\infty}^{\infty} \mathrm{x}[n] = 4(1) + 7(-2) = -10$$

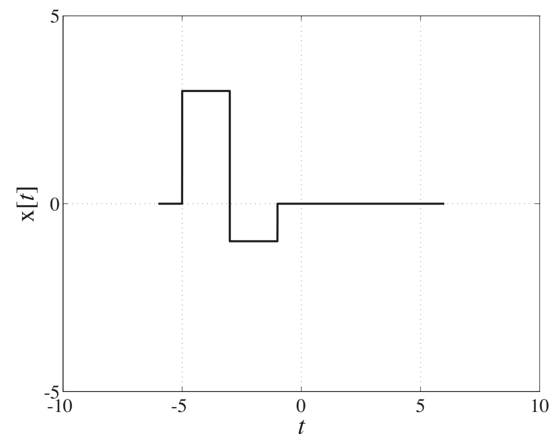
(c) What is the numerical signal energy of $\mathrm{x}[n]$?

$$\left|\mathrm{x}[n]\right|^2 = \begin{cases} 0 & , n < -3 \\ 1 & , -2 \le n < 1 \\ 4 & , 1 \le n < 8 \\ 0 & , n \ge 8 \end{cases} \Rightarrow E_{\mathrm{x}} = \left[4(1) + 7(4)\right] = 3 + 28 = 32$$

3. One period of a periodic continuous-time signal with fundamental period 12 is described by

$$x(t) = 3\,\mathrm{rect}\left(\frac{t+3}{4}\right) - 4\,\mathrm{rect}(t/2+1) \quad , \quad -6 < t < 6 .$$

(a) Sketch this single period of $x(t)$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical average value of $x(t)$?

$$\langle x(t) \rangle = (1/T_0)\int_{T_0} x(t)\,dt = \frac{1}{12}\int_{-6}^{6}\left[3\,\mathrm{rect}\left(\frac{t+3}{4}\right) - 4\,\mathrm{rect}(t/2+1)\right]dt$$

$$\langle x(t) \rangle = \frac{1}{12}\left[3\int_{-5}^{-1} dt - 4\int_{-3}^{-1} dt\right] = \frac{12-8}{12} = 1/3$$

(c) What is the numerical signal power of $x(t)$?

$$P_x = (1/T_0)\int_{T_0} |x(t)|^2\,dt = (1/12)\int_{-6}^{6}\left|3\,\mathrm{rect}\left(\frac{t+3}{4}\right) - 4\,\mathrm{rect}(t/2+1)\right|^2 dt$$

$$P_x = (1/12)\left[9\int_{-5}^{-1} dt + 16\int_{-3}^{-1} dt - 24\int_{-3}^{-1} dt\right] = \frac{36+32-48}{12} = 5/3 \cong 1.667$$

Solution of ECE 315 Test 1 Su10

1. Let $\mathrm{x}(t) = -2e^{-t/5}\,\mathrm{u}(t-2)$ and let $\mathrm{y}(t) = -4\,\mathrm{x}(5t)$.

(a) What is the smallest numerical value of t for which $\mathrm{y}(t)$ is not zero?

$\mathrm{y}(t) = -4\,\mathrm{x}(5t) = 8e^{-5t/5}\,\mathrm{u}(5t-2) \Rightarrow$ Smallest t for which $\mathrm{y}(t) \neq 0$ occurs where $5t - 2 = 0$ or $t = 2/5$.

(b) What is the maximum numerical value of $\mathrm{y}(t)$ over all time?

$$\max(\mathrm{y}(t)) = 5.363$$

(c) What is the minimum numerical value of $\mathrm{y}(t)$ over all time?

$$\min(\mathrm{y}(t)) = 0$$

(d) What is the numerical value of $\mathrm{y}(1)$?

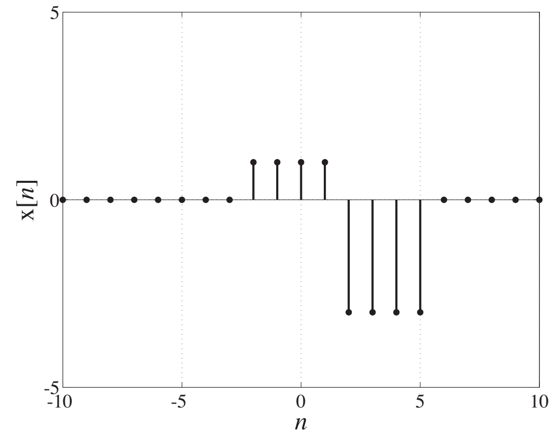
$$\mathrm{y}(t) = 8e^{-5t/5}\,\mathrm{u}(5t-2) \Rightarrow \mathrm{y}(1) = 8e^{-1}\,\mathrm{u}(3) = 8e^{-1} = 2.943$$

(e) What is the largest numerical value of t for which $\mathrm{y}(t) > 2$?

$$\mathrm{y}(t) = 2 = 8e^{-5t/5} \Rightarrow e^{-5t/5} = 2/8 \Rightarrow -t = \ln(2/8) = -1.3863 \Rightarrow t = 1.3863$$

2. Let $\mathrm{x}[n] = \mathrm{u}[n+2] - 4\,\mathrm{u}[n-2] + 3\,\mathrm{u}[n-6]$.

(a) Sketch $\mathrm{x}[n]$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical sum of all the values of $\mathrm{x}[n]$?

$$\mathrm{x}[n] = \begin{Bmatrix} 0 & , \ n < -2 \\ 1 & , \ -2 \le n < 2 \\ -3 & , \ 2 \le n < 6 \\ 0 & , \ n \ge 6 \end{Bmatrix} \Rightarrow \sum_{n=-\infty}^{\infty} \mathrm{x}[n] = 4(1) + 4(-3) = -8$$

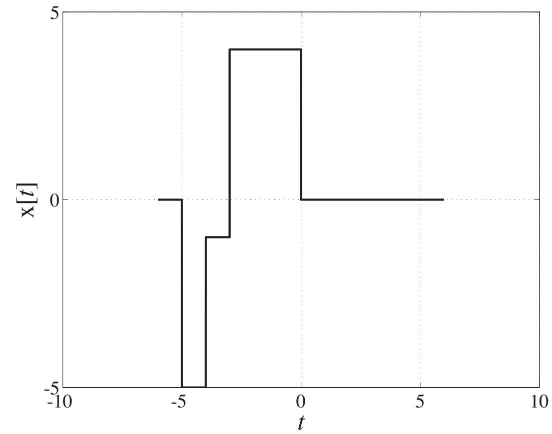
(c) What is the numerical signal energy of $\mathrm{x}[n]$?

$$|\mathrm{x}[n]|^2 = \begin{Bmatrix} 0 & , \ n < -2 \\ 1 & , \ -2 \le n < 2 \\ 9 & , \ 2 \le n < 6 \\ 0 & , \ n \ge 6 \end{Bmatrix} \Rightarrow E_{\mathrm{x}} = [4(1) + 4(9)] = 40$$

3. One period of a periodic continuous-time signal with fundamental period 12 is described by

$$\mathrm{x}(t) = 4\,\mathrm{rect}\left(\frac{t+2}{4}\right) - 5\,\mathrm{rect}(t/2+2) \quad , \quad -6 < t < 6 .$$

(a) Sketch this single period of $\mathrm{x}(t)$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical average value of $\mathrm{x}(t)$?

$$\langle \mathrm{x}(t) \rangle = (1/T_0)\int_{T_0} \mathrm{x}(t)\,dt = \frac{1}{12}\int_{-6}^{6}\left[4\,\mathrm{rect}\left(\frac{t+2}{4}\right) - 5\,\mathrm{rect}(t/2+2)\right]dt$$

$$\langle \mathrm{x}(t) \rangle = \frac{1}{12}\left[4\int_{-4}^{0} dt - 5\int_{-5}^{-3} dt\right] = \frac{16-10}{12} = 1/2$$

(c) What is the numerical signal power of $\mathrm{x}(t)$?

$$P_{\mathrm{x}} = (1/T_0)\int_{T_0} |\mathrm{x}(t)|^2\,dt = (1/12)\int_{-6}^{6}\left|4\,\mathrm{rect}\left(\frac{t+2}{4}\right) - 5\,\mathrm{rect}(t/2+2)\right|^2 dt$$

$$P_{\mathrm{x}} = (1/12)\left[16\int_{-4}^{0} dt + 25\int_{-5}^{-3} dt - 40\int_{-4}^{-3} dt\right] = \frac{64+50-40}{12} = 37/6 \cong 6.1667$$

Solution of ECE 315 Test 1 Su10

1. Let $x(t) = -5e^{-t/3}u(t+1)$ and let $y(t) = -4x(5t)$.

(a) What is the smallest numerical value of t for which $y(t)$ is not zero?

$y(t) = -4x(5t) = 20e^{-5t/3}u(5t+1) \Rightarrow$ Smallest t for which $y(t) \neq 0$ occurs where $5t+1=0$ or $t = -1/5$.

(b) What is the maximum numerical value of $y(t)$ over all time?

$$\max(y(t)) = 27.912$$

(c) What is the minimum numerical value of $y(t)$ over all time?

$$\min(y(t)) = 0$$

(d) What is the numerical value of $y(1)$?

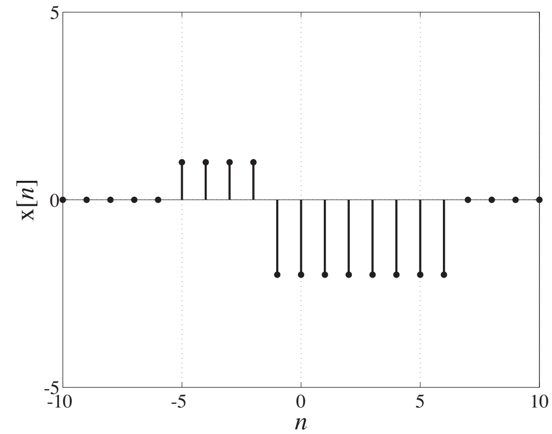
$$y(t) = 20e^{-5/3}u(5+1) \Rightarrow y(1) = 20e^{-5/3}u(4) = 3.778$$

(e) What is the largest numerical value of t for which $y(t) > 2$?

$y(t) = 2 = 20e^{-5t/3} \Rightarrow e^{-5t/3} = 1/10 \Rightarrow -5t/3 = \ln(1/10) = -2.303 \Rightarrow t = 3(2.303)/5 = 1.382$

2. Let $x[n] = u[n+5] - 3u[n+1] + 2u[n-7]$.

(a) Sketch $x[n]$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical sum of all the values of $x[n]$?

$$x[n] = \begin{cases} 0 & , n < -5 \\ 1 & , -5 \le n < -1 \\ -2 & , -1 \le n < 7 \\ 0 & , n \ge 7 \end{cases} \Rightarrow \sum_{n=-\infty}^{\infty} x[n] = 4(1) + 8(-2) = -12$$

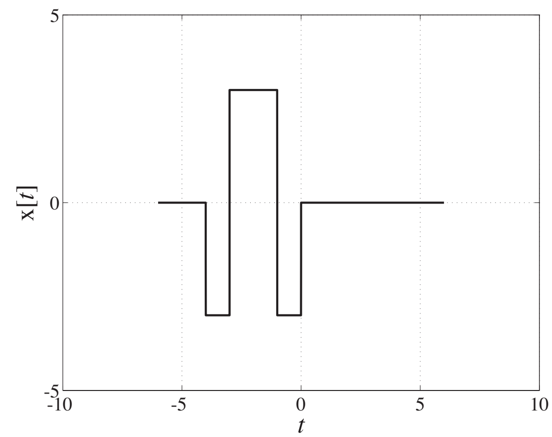
(c) What is the numerical signal energy of $x[n]$?

$$|x[n]|^2 = \begin{cases} 0 & , n < -5 \\ 1 & , -5 \le n < -1 \\ 4 & , -1 \le n < 7 \\ 0 & , n \ge 7 \end{cases} \Rightarrow E_x = [4(1) + 8(4)] = 36$$

3. One period of a periodic continuous-time signal with fundamental period 12 is described by

$$\mathrm{x}(t) = -3\,\mathrm{rect}\left(\frac{t+2}{4}\right) + 6\,\mathrm{rect}(t/2+1) \quad , \quad -6 < t < 6 .$$

(a) Sketch this single period of $\mathrm{x}(t)$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical average value of $\mathrm{x}(t)$?

$$\langle \mathrm{x}(t) \rangle = (1/T_0)\int_{T_0} \mathrm{x}(t)\,dt = \frac{1}{12}\int_{-6}^{6}\left[-3\,\mathrm{rect}\left(\frac{t+2}{4}\right) + 6\,\mathrm{rect}(t/2+1)\right]dt$$

$$\langle \mathrm{x}(t) \rangle = \frac{1}{12}\left[-3\int_{-4}^{0} dt + 6\int_{-3}^{-1} dt\right] = \frac{-12+12}{12} = 0$$

(c) What is the numerical signal power of $\mathrm{x}(t)$?

$$P_{\mathrm{x}} = (1/T_0)\int_{T_0} |\mathrm{x}(t)|^2\,dt = (1/12)\int_{-6}^{6}\left|-3\,\mathrm{rect}\left(\frac{t+2}{4}\right) + 6\,\mathrm{rect}(t/2+1)\right|^2 dt$$

$$P_{\mathrm{x}} = (1/12)\left[9\int_{-4}^{0} dt + 36\int_{-3}^{-1} dt - 36\int_{-3}^{-1} dt\right] = \frac{36+72-72}{12} = 3$$

Solution of ECE 315 Test 1 Su10

1. Let $\mathrm{x}(t) = -5e^{-t/8}\,\mathrm{u}(t+2)$ and let $\mathrm{y}(t) = -4\,\mathrm{x}(5t)$.

   (a) What is the smallest numerical value of t for which $\mathrm{y}(t)$ is not zero?

   $\mathrm{y}(t) = -4\,\mathrm{x}(5t) = 20e^{-5t/8}\,\mathrm{u}(5t+2) \Rightarrow$ Smallest t for which $\mathrm{y}(t) \neq 0$ occurs where $5t+2=0$ or $t = -2/5$.

   (b) What is the maximum numerical value of $\mathrm{y}(t)$ over all time?

$$\max(\mathrm{y}(t)) = 25.68$$

   (c) What is the minimum numerical value of $\mathrm{y}(t)$ over all time?

$$\min(\mathrm{y}(t)) = 0$$

   (d) What is the numerical value of $\mathrm{y}(1)$?

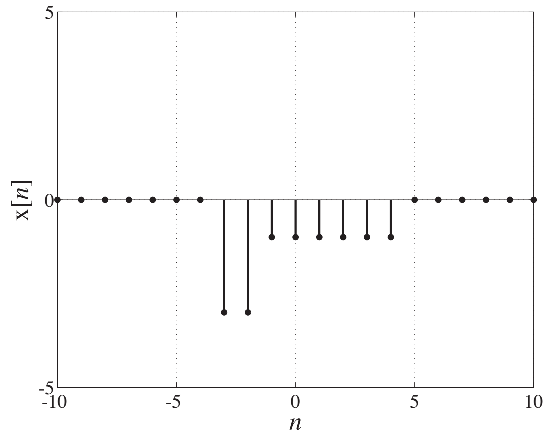
$$\mathrm{y}(1) = 20e^{-5/8}\,\mathrm{u}(7) = 20e^{-5/8} = 10.71$$

   (e) What is the largest numerical value of t for which $\mathrm{y}(t) > 2$?

$$\mathrm{y}(t) = 2 = 20e^{-5t/8} \Rightarrow e^{-5t/8} = 1/10 \Rightarrow -5t/8 = \ln(1/10) = -2.303 \Rightarrow t = 8(2.303)/5 = 3.685$$

2. Let $x[n] = 2u[n+1] - 3u[n+3] + u[n-5]$.

(a) Sketch $x[n]$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical sum of all the values of $x[n]$?

$$x[n] = \begin{cases} 0 & , n < -3 \\ -3 & , -3 \le n < -1 \\ -1 & , -1 \le n < 5 \\ 0 & , n \ge 5 \end{cases} \Rightarrow \sum_{n=-\infty}^{\infty} x[n] = 2(-3) + 6(-1) = -12$$

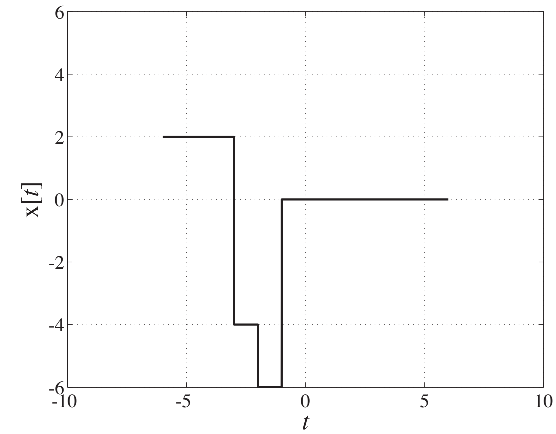
(c) What is the numerical signal energy of $x[n]$?

$$x[n] = \begin{cases} 0 & , n < -3 \\ 9 & , -3 \le n < -1 \\ 1 & , -1 \le n < 5 \\ 0 & , n \ge 5 \end{cases} \Rightarrow \sum_{n=-\infty}^{\infty} x[n] = 2(9) + 6(1) = 24$$

3. One period of a periodic continuous-time signal with fundamental period 12 is described by

$$\mathrm{x}(t) = 2\,\mathrm{rect}\left(\frac{t+5}{6}\right) - 6\,\mathrm{rect}(t/2+1) \quad , \quad -6 < t < 6\,.$$

(a) Sketch this single period of $\mathrm{x}(t)$ in the space provided below. Mark the vertical scale so actual numerical values could be read from it.



(b) What is the numerical average value of $\mathrm{x}(t)$?

$$\langle \mathrm{x}(t)\rangle = (1/T_0)\int_{T_0} \mathrm{x}(t)\,dt = \frac{1}{12}\int_{-6}^{6}\left[2\,\mathrm{rect}\left(\frac{t+5}{6}\right) - 6\,\mathrm{rect}(t/2+1)\right]dt$$

$$\langle \mathrm{x}(t)\rangle = \frac{1}{12}\left[2\int_{-6}^{-2} dt - 6\int_{-3}^{-1} dt\right] = \frac{8-12}{12} = -1/3$$

(c) What is the numerical signal power of $\mathrm{x}(t)$?

$$P_{\mathrm{x}} = (1/T_0)\int_{T_0} |\mathrm{x}(t)|^2\,dt = (1/12)\int_{-6}^{6}\left|2\,\mathrm{rect}\left(\frac{t+5}{6}\right) - 6\,\mathrm{rect}(t/2+1)\right|^2 dt$$

$$P_{\mathrm{x}} = (1/12)\left[4\int_{-6}^{-2} dt + 36\int_{-3}^{-1} dt - 24\int_{-3}^{-2} dt\right] = \frac{16+72-24}{12} = 16/3 \cong 5.333$$