Solution of ECE 315 Test 2 Su10

In all problems below "x" is the excitation, "y" is the response and "h" is the impulse response.

1. A discrete-time system is described by $y[n] - 0.4\,y[n-1] = 2\,x[n] - 3\,x[n-1]$. Fill in the table below with numbers.

n	0	1	2
$h[n]$	______	______	______

$$h[n] - 0.4\,h[n-1] = 2\delta[n] - 3\delta[n-1]$$

$$h[0] - 0.4\,h[-1] = 2\delta[0] - 3\delta[0-1] \Rightarrow h[0] = 2$$

$$h[1] - 0.4\,h[0] = 2\delta[1] - 3\delta[0] \Rightarrow h[1] - 0.8 = -3 \Rightarrow h[1] = -2.2$$

$$h[2] - 0.4\,h[1] = 2\delta[2] - 3\delta[1] \Rightarrow h[2] + 0.88 = 0 \Rightarrow h[2] = -0.88$$

2. A continuous-time system is described by $y'(t) = x(t) - 2\,y(t)$. Is it stable?

   The eigenvalue is -2. Its real part is negative. The system is stable.

3. A discrete-time system is described by $y[n] = x[n] - 2\,y[n-1]$. Is it stable?

   The eigenvalue is -2. Its magnitude is greater than one. The system is unstable.

4. A continuous-time system has an impulse response $\text{rect}(t) * [\delta_8(t-1) - \delta_8(t-5)]u(t)$. Is it stable?

   The impulse response is an alternating sequence of rectangular pulses of equal size. It is not absolutely integrable. It is unstable.

5. A continuous-time system is described by the differential equation $y'(t) + 6\,y(t) = x'(t)$.

   (a) Write the differential equation for the special case of impulse excitation and impulse response.

   $$h'(t) + 6\,h(t) = \delta'(t)$$

   (b) The impulse response is $h(t) = -6e^{-6t}\,u(t) + \delta(t)$. What is the numerical value of the integral from $t = 0^-$ to $t = 0^+$ of $h(t)$?

   The integral of $h(t)$ over this infinitesimal range is 1.

   (c) What is the numerical value of the integral from $t = 0^-$ to $t = 0^+$ of $h'(t)$?

   The integral of $h'(t)$ over this infinitesimal range is -6.

6. If $x(t) = \text{rect}(t/10) * 3\text{rect}((t-1)/8)$ find the following numerical values.

(a) For $t = 1$, the product of $\text{rect}((t-\tau)/10)$ and $3\text{rect}((\tau-1)/8)$ is $3\text{rect}((1-\tau)/10)\text{rect}((\tau-1)/8)$. Since the rect function is an even function we know that $\text{rect}((1-\tau)/10) = \text{rect}((\tau-1)/10)$ and, therefore, that the product is $3\text{rect}((\tau-1)/10)\text{rect}((\tau-1)/8)$. That product is a rectangle of height 3 between -3 and 5 so the area under the product is 24.

(b) For $t = 5$, the product of $\text{rect}((t-\tau)/10)$ and $3\text{rect}((\tau-1)/8)$ is $3\text{rect}((5-\tau)/10)\text{rect}((\tau-1)/8)$. Since the rect function is an even function we know that $\text{rect}((5-\tau)/10) = \text{rect}((\tau-5)/10)$ and, therefore, that the product is $3\text{rect}((\tau-5)/10)\text{rect}((\tau-1)/8)$. That product is a rectangle of height 3 between 0 and 5 so the area under the product is 15.

7. A continuous-time system is described by $y(t) = \int_{-\infty}^{t+t_0} x(\lambda)d\lambda$. What range of values of t_0 guarantees that the system is causal? Range is ______________________________

Range is $t_0 \leq 0$

8. A continuous-time system is described by $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$. What range of values of a guarantees that the system is causal? For causality a must be 1.

If $a > 1$ and $t = 1$ $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 < a < 1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a < -1$ and $t = -1$ $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 > a > -1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a = -1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to time $t = 1$. Not Causal

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In all problems below "x" is the excitation, "y" is the response and "h" is the impulse response.

1. A discrete-time system is described by $y[n] - 0.4\,y[n-1] = 3\,x[n] - 2\,x[n-1]$. Fill in the table below with numbers.

n	0	1	2
$h[n]$	______	______	______

$$h[n] - 0.4\,h[n-1] = 3\delta[n] - 2\delta[n-1]$$

$$h[0] - 0.4\,h[-1] = 3\delta[0] - 2\delta[0-1] \Rightarrow h[0] = 3$$

$$h[1] - 0.4\,h[0] = 3\delta[1] - 2\delta[0] \Rightarrow h[1] - 1.2 = -2 \Rightarrow h[1] = -0.8$$

$$h[2] - 0.4\,h[1] = 3\delta[2] - 2\delta[1] \Rightarrow h[2] + 0.32 = 0 \Rightarrow h[2] = -0.32$$

2. A continuous-time system is described by $y'(t) = x(t) + 0.5\,y(t)$. Is it stable?

   The eigenvalue is 0.5. Its real part is positive. The system is unstable.

3. A discrete-time system is described by $y[n] = x[n] + 0.5\,y[n-1]$. Is it stable?

   The eigenvalue is 0.5. Its magnitude is less than one. The system is stable.

4. A continuous-time system has an impulse response $\mathrm{rect}(t) * [\delta_8(t-1) - \delta_8(t-5)]\,u(t)$. Is it stable?

   The impulse response is an alternating sequence of rectangular pulses of equal size. It is not absolutely integrable. It is unstable.

5. A continuous-time system is described by the differential equation $y'(t) + 4\,y(t) = x'(t)$.

   (a) Write the differential equation for the special case of impulse excitation and impulse response.

   $$h'(t) + 4\,h(t) = \delta'(t)$$

   (b) The impulse response is $h(t) = -4e^{-4t}\,u(t) + \delta(t)$. What is the numerical value of the integral from $t = 0^-$ to $t = 0^+$ of $h(t)$?

   The integral of $h(t)$ over this infinitesimal range is 1.

   (c) What is the numerical value of the integral from $t = 0^-$ to $t = 0^+$ of $h'(t)$?

   The integral of $h'(t)$ over this infinitesimal range is -4.

6. If $x(t) = \text{rect}(t/12) * 3\text{rect}((t-1)/10)$ find the following numerical values.

(a) For $t = 1$, the product of $\text{rect}((t-\tau)/12)$ and $3\text{rect}((\tau-1)/10)$ is $3\text{rect}((1-\tau)/12)\text{rect}((\tau-1)/10)$. Since the rect function is an even function we know that $\text{rect}((1-\tau)/12) = \text{rect}((\tau-1)/12)$ and, therefore, that the product is $3\text{rect}((\tau-1)/12)\text{rect}((\tau-1)/10)$. That product is a rectangle of height 3 between -4 and 6 so the area under the product is 30.

(b) For $t = 5$, the product of $\text{rect}((t-\tau)/12)$ and $3\text{rect}((\tau-1)/10)$ is $3\text{rect}((5-\tau)/12)\text{rect}((\tau-1)/10)$. Since the rect function is an even function we know that $\text{rect}((5-\tau)/12) = \text{rect}((\tau-5)/12)$ and, therefore, that the product is $3\text{rect}((\tau-5)/12)\text{rect}((\tau-1)/10)$. That product is a rectangle of height 3 between -1 and 6 so the area under the product is 21.

7. A continuous-time system is described by $y(t) = \int_{-\infty}^{t+t_0} x(\lambda)d\lambda$. What range of values of t_0 guarantees that the system is causal?

Range is $t_0 \le 0$

8. A continuous-time system is described by $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$. What range of values of a guarantees that the system is causal? For causality a must be 1.

If $a > 1$ and $t = 1$ $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 < a < 1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a < -1$ and $t = -1$ $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 > a > -1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a = -1$ and $t = -1$ then $y(t) = \int_{-\infty}^{at} x(\lambda)d\lambda$ is an integral up to time $t = 1$. Not Causal

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In all problems below "x" is the excitation, "y" is the response and "h" is the impulse response.

1. A discrete-time system is described by $y[n]-0.4\,y[n-1]=4\,x[n]-5\,x[n-1]$. Fill in the table below with numbers.

n	0	1	2
$h[n]$	______	______	______

$$h[n]-0.4\,h[n-1]=4\delta[n]-5\delta[n-1]$$

$$h[0]-0.4\,h[-1]=4\delta[0]-5\delta[0-1]\Rightarrow h[0]=4$$

$$h[1]-0.4\,h[0]=4\delta[1]-5\delta[0]\Rightarrow h[1]-1.6=-5\Rightarrow h[1]=-3.4$$

$$h[2]-0.4\,h[1]=4\delta[2]-5\delta[1]\Rightarrow h[2]+1.36=0\Rightarrow h[2]=-1.36$$

2. A continuous-time system is described by $y'(t)=x(t)-2\,y(t)$. Is it stable?

   The eigenvalue is -2. Its real part is negative. The system is stable.

3. A discrete-time system is described by $y[n]=x[n]-2\,y[n-1]$. Is it stable?

   The eigenvalue is -2. Its magnitude is greater than one. The system is unstable.

4. A continuous-time system has an impulse response $\text{rect}(t)*[\delta_8(t-1)-\delta_8(t-5)]u(t)$. Is it stable?

   The impulse response is an alternating sequence of rectangular pulses of equal size. It is not absolutely integrable. It is unstable.

5. A continuous-time system is described by the differential equation $y'(t)+2\,y(t)=x'(t)$.

   (a) Write the differential equation for the special case of impulse excitation and impulse response.

   $$h'(t)+2\,h(t)=\delta'(t)$$

   (b) The impulse response is $h(t)=-2e^{-2t}\,u(t)+\delta(t)$. What is the numerical value of the integral from $t=0^-$ to $t=0^+$ of $h(t)$?

   The integral of $h(t)$ over this infinitesimal range is 1.

   (c) What is the numerical value of the integral from $t=0^-$ to $t=0^+$ of $h'(t)$?

   The integral of $h'(t)$ over this infinitesimal range is -2.

6. If $\mathrm{x}(t) = \mathrm{rect}(t/8) * 3\mathrm{rect}((t-1)/6)$ find the following numerical values.

(a) For $t = 1$, the product of $\mathrm{rect}((t-\tau)/8)$ and $3\mathrm{rect}((\tau-1)/6)$ is $3\mathrm{rect}((1-\tau)/8)\mathrm{rect}((\tau-1)/6)$. Since the rect function is an even function we know that $\mathrm{rect}((1-\tau)/8) = \mathrm{rect}((\tau-1)/8)$ and, therefore, that the product is $3\mathrm{rect}((\tau-1)/8)\mathrm{rect}((\tau-1)/6)$. That product is a rectangle of height 3 between -2 and 4 so the area under the product is 18.

(b) For $t = 5$, the product of $\mathrm{rect}((t-\tau)/8)$ and $3\mathrm{rect}((\tau-1)/6)$ is $3\mathrm{rect}((5-\tau)/8)\mathrm{rect}((\tau-1)/6)$. Since the rect function is an even function we know that $\mathrm{rect}((5-\tau)/8) = \mathrm{rect}((\tau-5)/8)$ and, therefore, that the product is $3\mathrm{rect}((\tau-5)/8)\mathrm{rect}((\tau-1)/6)$. That product is a rectangle of height 3 between 1 and 4 so the area under the product is 9.

7. A continuous-time system is described by $\mathrm{y}(t) = \int_{-\infty}^{t+t_0} \mathrm{x}(\lambda)\,d\lambda$. What range of values of t_0 guarantees that the system is causal?

Range is $t_0 \le 0$

8. A continuous-time system is described by $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$. What range of values of a guarantees that the system is causal? For causality a must be 1.

If $a > 1$ and $t = 1$ $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 < a < 1$ and $t = -1$ then $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a < -1$ and $t = -1$ $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$ is an integral up to a time greater than 1. Not Causal

If $0 > a > -1$ and $t = -1$ then $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$ is an integral up to a time greater than -1. Not Causal

If $a = -1$ and $t = -1$ then $\mathrm{y}(t) = \int_{-\infty}^{at} \mathrm{x}(\lambda)\,d\lambda$ is an integral up to time $t = 1$. Not Causal