Solution of ECE 315 Test 3 Su10

1. Provide numerical values of the constants in the CTFT pairs below.

(a) $4\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(f+f_0)-\delta(f-f_0)]$

$\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} (j/2)[\delta(f+1000)-\delta(f+1000)]$

$4\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} j2[\delta(f+1000)-\delta(f+1000)]$

$A = j2\ , f_0 = 1000$

(b) $4\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$

$4\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} j2\left[\delta\left(\frac{\omega}{2\pi}+1000\right)-\delta\left(\frac{\omega}{2\pi}-1000\right)\right]$

$4\sin(2000\pi t) \xleftrightarrow{\mathcal{F}} j4\pi[\delta(\omega+2000\pi)-\delta(\omega-2000\pi)]$

$A = j4\pi\ ,\ \omega_0 = 2000\pi$

(c) $A\,\mathrm{sinc}^2(b(t-c)) \xleftrightarrow{\mathcal{F}} 10\,\mathrm{tri}(5f)e^{j10\pi f}$

$\mathrm{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(f)$

$(1/5)\mathrm{sinc}^2(t/5) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(5f)$

$(1/5)\mathrm{sinc}^2((t+5)/5) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(5f)e^{j10\pi f}$

$2\,\mathrm{sinc}^2((t+5)/5) \xleftrightarrow{\mathcal{F}} 10\,\mathrm{tri}(5f)e^{j10\pi f}$

$A = 2\ ,\ b = 1/5\ ,\ c = -5$

(d) $\mathrm{sinc}(t/3) * \mathrm{sinc}((t-2)/4) \xleftrightarrow{\mathcal{F}} A\,\mathrm{rect}(bf)e^{cf}$

$\mathrm{sinc}(t/3) * \mathrm{sinc}((t-2)/4) \xleftrightarrow{\mathcal{F}} 3\,\mathrm{rect}(3f) \times 4\,\mathrm{rect}(4f)e^{-j4\pi f}$

$\mathrm{sinc}(t/3) * \mathrm{sinc}((t-2)/4) \xleftrightarrow{\mathcal{F}} 12\,\mathrm{rect}(3f)\mathrm{rect}(4f)e^{-j4\pi f}$

$\mathrm{sinc}(t/3) * \mathrm{sinc}((t-2)/4) \xleftrightarrow{\mathcal{F}} 12\,\mathrm{rect}(4f)e^{-j4\pi f}$

$A = 12\ ,\ b = 4\ ,\ c = -j4\pi$

2. A periodic signal $\mathrm{x}(t)$ with fundamental period 500 μs has a CTFT $\mathrm{X}(f) = 3\mathrm{sinc}(f/20000)\delta_{2000}(f)$. Its CTFS harmonic function, based on a representation time T of 500 μs, is $\mathrm{c_x}[k]$. What is the numerical value of $\mathrm{c_x}[3]$?

The CTFS harmonic function value at harmonic number k is the same as the impulse strength of the CTFT at a frequency of k/T. Therefore $\mathrm{c_x}[3] = \mathrm{X}(3/500\ \mu\mathrm{s}) = \mathrm{X}(6000\ \mathrm{Hz}) = 3\mathrm{sinc}(3/10) = 2.5752$.

3. Find the numerical signal energy of

(a) $\mathrm{x}(t) = -3\mathrm{sinc}(2t)$.

$$\mathrm{X}(f) = -(3/2)\mathrm{rect}(f/2) \Rightarrow E_\mathrm{x} = \int_{-\infty}^{\infty} \left|-(3/2)\mathrm{rect}(f/2)\right|^2 df$$

$$E_\mathrm{x} = (9/4)\int_{-1}^{1} df = 9/2 = 4.5$$

(b) $\mathrm{x}(t) = -3\mathrm{sinc}^2(2t)$

$$\mathrm{X}(f) = -(3/2)\mathrm{tri}(f/2) \Rightarrow E_\mathrm{x} = \int_{-\infty}^{\infty} \left|-(3/2)\mathrm{tri}(f/2)\right|^2 df$$

$$E_\mathrm{x} = (9/4)\int_{-2}^{2} \mathrm{tri}^2(f/2)df = (9/2)\int_{0}^{2} \mathrm{tri}^2(f/2)df = (9/2)\int_{0}^{2}\left[1-(f/2)\right]^2 df$$

$$E_\mathrm{x} = (9/2)\int_{0}^{2}\left(1+f^2/4-f\right)df = (9/2)\left[f+f^3/12-f^2/2\right]_0^2 = (9/2)(2+2/3-2) = 3$$

Solution of ECE 315 Test 3 Su10

1. Provide numerical values of the constants in the CTFT pairs below.

(a) $7\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} A\left[\delta(f+f_0)-\delta(f-f_0)\right]$

$\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} (j/2)\left[\delta(f+1500)-\delta(f+1500)\right]$

$7\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} (j7/2)\left[\delta(f+1500)-\delta(f+1500)\right]$

$A = j7/2$, $f_0 = 1500$

(b) $7\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} A\left[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right]$

$$7\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} (j7/2)\left[\delta\left(\frac{\omega}{2\pi}+1500\right)-\delta\left(\frac{\omega}{2\pi}-1500\right)\right]$$

$7\sin(3000\pi t) \xleftrightarrow{\mathcal{F}} j7\pi\left[\delta(\omega+3000\pi)-\delta(\omega-3000\pi)\right]$

$A = j7\pi$, $\omega_0 = 3000\pi$

(c) $A\,\mathrm{sinc}^2(b(t-c)) \xleftrightarrow{\mathcal{F}} 4\,\mathrm{tri}(6f)e^{j14\pi f}$

$\mathrm{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(f)$

$(1/6)\mathrm{sinc}^2(t/6) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(6f)$

$(1/6)\mathrm{sinc}^2((t+7)/6) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(6f)e^{j14\pi f}$

$(2/3)\mathrm{sinc}^2((t+7)/6) \xleftrightarrow{\mathcal{F}} 4\,\mathrm{tri}(6f)e^{j14\pi f}$

$A = 2/3$, $b = 1/6$, $c = -7$

(d) $\mathrm{sinc}(t/5) * \mathrm{sinc}((t-1)/3) \xleftrightarrow{\mathcal{F}} A\,\mathrm{rect}(bf)e^{cf}$

$\mathrm{sinc}(t/5) * \mathrm{sinc}((t-1)/3) \xleftrightarrow{\mathcal{F}} 5\,\mathrm{rect}(5f) \times 3\,\mathrm{rect}(3f)e^{-j2\pi f}$

$\mathrm{sinc}(t/5) * \mathrm{sinc}((t-1)/3) \xleftrightarrow{\mathcal{F}} 15\,\mathrm{rect}(5f)\mathrm{rect}(3f)e^{-j2\pi f}$

$\mathrm{sinc}(t/5) * \mathrm{sinc}((t-1)/3) \xleftrightarrow{\mathcal{F}} 15\,\mathrm{rect}(5f)e^{-j2\pi f}$

$A = 15$, $b = 5$, $c = -j2\pi$

2. A periodic signal $x(t)$ with fundamental period $500\ \mu s$ has a CTFT $X(f) = 3\,\mathrm{sinc}(f/20000)\delta_{2000}(f)$. Its CTFS harmonic function, based on a representation time T of $500\ \mu s$, is $c_x[k]$. What is the numerical value of $c_x[5]$?

The CTFS harmonic function value at harmonic number k is the same as the impulse strength of the CTFT at a frequency of k/T. Therefore $c_x[5] = X(5/500\ \mu s) = X(10000\ \mathrm{Hz}) = 3\,\mathrm{sinc}(1/2) = 6/\pi \cong 1.91$.

3. Find the numerical signal energy of

(a) $x(t) = -8\,\mathrm{sinc}(3t)$.

$$X(f) = -(8/3)\mathrm{rect}(f/3) \Rightarrow E_x = \int_{-\infty}^{\infty} \left|-(8/3)\mathrm{rect}(f/3)\right|^2 df$$

$$E_x = (64/9)\int_{-3/2}^{3/2} df = 64/3 \cong 21.333$$

(b) $x(t) = -8\,\mathrm{sinc}^2(3t)$

$$X(f) = -(8/3)\mathrm{tri}(f/3) \Rightarrow E_x = \int_{-\infty}^{\infty} \left|-(8/3)\mathrm{tri}(f/3)\right|^2 df$$

$$E_x = (64/9)\int_{-3}^{3} \mathrm{tri}^2(f/3)\,df = (128/9)\int_{0}^{3} \mathrm{tri}^2(f/3)\,df = (128/9)\int_{0}^{3} \left[1-(f/3)\right]^2 df$$

$$E_x = (128/9)\int_{0}^{3} \left(1 + f^2/9 - 2f/3\right) df = (128/9)\left[f + f^3/27 - f^2/3\right]_0^3 = (128/9)(3+1-3) = 128/9 \cong 14.222$$

Solution of ECE 315 Test 3 Su10

1. Provide numerical values of the constants in the CTFT pairs below.

(a) $3\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(f+f_0) - \delta(f-f_0)]$

$\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} (j/2)[\delta(f+2500) - \delta(f+2500)]$

$3\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} (j3/2)[\delta(f+2500) - \delta(f+2500)]$

$A = j3/2 \ , f_0 = 2500$

(b) $3\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$

$$3\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} (j3/2)\left[\delta\left(\frac{\omega}{2\pi}+2500\right) - \delta\left(\frac{\omega}{2\pi}-2500\right)\right]$$

$3\sin(5000\pi t) \xleftrightarrow{\mathcal{F}} j3\pi[\delta(\omega+5000\pi) - \delta(\omega-5000\pi)]$

$A = j3\pi \ , \omega_0 = 5000\pi$

(c) $A\,\mathrm{sinc}^2(b(t-c)) \xleftrightarrow{\mathcal{F}} 9\,\mathrm{tri}(3f)e^{j8\pi f}$

$\mathrm{sinc}^2(t) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(f)$

$(1/3)\mathrm{sinc}^2(t/3) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(3f)$

$(1/3)\mathrm{sinc}^2((t+4)/3) \xleftrightarrow{\mathcal{F}} \mathrm{tri}(3f)e^{j8\pi f}$

$3\,\mathrm{sinc}^2((t+4)/3) \xleftrightarrow{\mathcal{F}} 9\,\mathrm{tri}(3f)e^{j8\pi f}$

$A = 3 \ , \ b = 1/3 \ , \ c = -4$

(d) $\mathrm{sinc}(t/8) * \mathrm{sinc}((t-5)/6) \xleftrightarrow{\mathcal{F}} A\,\mathrm{rect}(bf)e^{cf}$

$\mathrm{sinc}(t/8) * \mathrm{sinc}((t-5)/6) \xleftrightarrow{\mathcal{F}} 8\,\mathrm{rect}(8f) \times 6\,\mathrm{rect}(6f)e^{-j10\pi f}$

$\mathrm{sinc}(t/8) * \mathrm{sinc}((t-5)/6) \xleftrightarrow{\mathcal{F}} 48\,\mathrm{rect}(8f)\mathrm{rect}(6f)e^{-j10\pi f}$

$\mathrm{sinc}(t/8) * \mathrm{sinc}((t-5)/6) \xleftrightarrow{\mathcal{F}} 48\,\mathrm{rect}(8f)e^{-j10\pi f}$

$A = 48 \ , \ b = 8 \ , \ c = -j10\pi$

2. A periodic signal $\mathrm{x}(t)$ with fundamental period $500\ \mu\mathrm{s}$ has a CTFT $\mathrm{X}(f) = 3\,\mathrm{sinc}(f/20000)\delta_{2000}(f)$. Its CTFS harmonic function, based on a representation time T of $500\ \mu\mathrm{s}$, is $\mathrm{c_x}[k]$. What is the numerical value of $\mathrm{c_x}[7]$?

The CTFS harmonic function value at harmonic number k is the same as the impulse strength of the CTFT at a frequency of k/T. Therefore $\mathrm{c_x}[7] = \mathrm{X}(7/500\ \mu\mathrm{s}) = \mathrm{X}(14000\ \mathrm{Hz}) = 3\,\mathrm{sinc}(7/10) = 1.1036$.

3. Find the numerical signal energy of

(a) $\mathrm{x}(t) = -12\,\mathrm{sinc}(7t)$.

$$\mathrm{X}(f) = -(12/7)\mathrm{rect}(f/7) \Rightarrow E_\mathrm{x} = \int_{-\infty}^{\infty} \left|-(12/7)\mathrm{rect}(f/7)\right|^2 df$$

$$E_\mathrm{x} = (144/49)\int_{-7/2}^{7/2} df = 144/7 = 20.5714$$

(b) $\mathrm{x}(t) = -12\,\mathrm{sinc}^2(7t)$

$$\mathrm{X}(f) = -(12/7)\mathrm{tri}(f/7) \Rightarrow E_\mathrm{x} = \int_{-\infty}^{\infty} \left|-(12/7)\mathrm{tri}(f/7)\right|^2 df$$

$$E_\mathrm{x} = (144/49)\int_{-7}^{7} \mathrm{tri}^2(f/7)df = (288/49)\int_{0}^{7} \mathrm{tri}^2(f/7)df = (288/49)\int_{0}^{7}\left[1-(f/7)\right]^2 df$$

$$E_\mathrm{x} = (288/49)\int_{0}^{7}\left(1+f^2/49-2f/7\right)df = (288/49)\left[f+f^3/147-f^2/7\right]_0^7 = (288/49)(7+2.333-7) = (288/49)2.333 = 13.7143$$