Solution of EECS 315 Test 3 F10

1. Given $x(t) \xleftrightarrow[12]{\mathcal{FS}} \text{tri}((k-1)/4) + \text{tri}((k+1)/4)$ what is the numerical average value of $x(t)$?
The average value of any periodic signal is the value of its CTFS harmonic function at $k=0$. Therefore the average value of $x(t)$ is $\text{tri}((0-1)/4) + \text{tri}((0+1)/4) = \text{tri}(-1/4) + \text{tri}(1/4) = 3/4 + 3/4 = 3/2$.

2. Given $x(t) \xleftrightarrow[8]{\mathcal{FS}} 4(u[k+3] - u[k-4])$ what is the numerical average signal power of $x(t)$?
The average signal power of a signal is the sum of the squares of the magnitudes of the harmonic function values. In this case
$$P_x = 4^2(1+1+1+1+1+1+1) = 16 \times 7 = 112$$

3. A signal $x(t)$ has a discontinuity at $t=0$. The limit of $x(t)$ approaching $t=0$ from above is 3 and the limit of $x(t)$ approaching $t=0$ from below is -4. What is the numerical value of the CTFS representation of $x(t)$ at $t=0$?
The Fourier series always goes through the mid-point of a discontinuity of the signal it is representing. In this case that would be at -1/2.

4. Partial sums of a Fourier series of the form $\sum_{k=-N}^{N} c_x[k] e^{j2\pi kt/T}$ overshoot and ring at a discontinuity. This effect is called the ____________________________ phenomenon. Gibbs

5. Find the numerical values of the constants in these CTFT pairs.
(a) $4\,\text{rect}(2t) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}(bf)$
$$4\,\text{rect}(2t) \xleftrightarrow{\mathcal{F}} 2\,\text{sinc}(f/2) \Rightarrow A = 2,\ b = 1/2$$
(b) $7\,\text{tri}(t-5) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}^2(bf) e^{cf}$
$$7\,\text{tri}(t-5) \xleftrightarrow{\mathcal{F}} 7\,\text{sinc}^2(f) e^{-j10\pi f} \Rightarrow A = 7,\ b = 1,\ c = -j10\pi$$
(c) $A\cos(bt) \xleftrightarrow{\mathcal{F}} 3\delta(f-4) + 3\delta(f+4)$
$$6\cos(8\pi t) \xleftrightarrow{\mathcal{F}} 3\delta(f-4) + 3\delta(f+4) \Rightarrow A = 6,\ b = 8\pi$$
(d) $13\sin(22\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(\omega+b) - \delta(\omega-b)]$
$$13\sin(22\pi t) \xleftrightarrow{\mathcal{F}} j13\pi[\delta(\omega+22\pi) - \delta(\omega-22\pi)] \Rightarrow A = j13\pi,\ b = 22\pi$$

6. If we know a signal $x(t)$ is periodic, what do we know for sure about its CTFT that we would not know if it were not periodic?
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We know that the CTFT consists entirely of impulses.

7. A system has a frequency response $H(j\omega) = \dfrac{100}{j\omega + 200}$.
(a) If we apply the constant signal $x(t) = 12$ to this system the response is also a constant. What is the numerical value of the response constant?
The response to a constant is that constant times the frequency response at a frequency of zero. The frequency reponse at zero is 1/2. Therefore the system response is 6.
(b) If we apply the signal $x(t) = 3\sin(14\pi t)$ to the system the response is $y(t) = A\sin(14\pi t + \theta)$. What are the numerical values of A and θ (θ in radians)?
In this case $\omega = 14\pi$ and $H(j14\pi) = \dfrac{100}{j14\pi + 200} = 0.4883\angle -0.2165$. So the response amplitude A will be $3 \times 0.4883 = 1.4649$ and the response will lag the excitation by 0.2165 radians so $\theta = -0.2165$ radians.

8. Given $x(t) \xleftrightarrow{\mathcal{F}} \delta(f-3) + \delta(f+3)$ and $x(t) \xleftrightarrow[2]{\mathcal{FS}} c_x[k] = \delta[k-a] + \delta[k+a]$, what is the numerical value of a?
$x(t)$ is a 3 Hz sinusoid. Its fundamental period is 1/3 second. Using a representation time of 2 seconds this sinusoid will lie at the 6th harmonic. Therefore $a = 6$.

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1. Given $x(t) \xleftrightarrow[12]{\mathcal{FS}} \text{tri}((k-1)/5) + \text{tri}((k+1)/5)$ what is the numerical average value of $x(t)$?
The average value of any periodic signal is the value of its CTFS harmonic function at $k = 0$. Therefore the average value of $x(t)$ is $\text{tri}((0-1)/5) + \text{tri}((0+1)/5) = \text{tri}(-1/5) + \text{tri}(1/5) = 4/5 + 4/5 = 8/5 = 1.6$.

2. Given $x(t) \xleftrightarrow[8]{\mathcal{FS}} 4(u[k+4] - u[k-5])$ what is the numerical average signal power of $x(t)$?
The average signal power of a signal is the sum of the squares of the magnitudes of the harmonic function values. In this case

$$P_x = 4^2(1+1+1+1+1+1+1+1+1) = 16 \times 9 = 144$$

3. A signal $x(t)$ has a discontinuity at $t = 0$. The limit of $x(t)$ approaching $t = 0$ from above is 5 and the limit of $x(t)$ approaching $t = 0$ from below is -4. What is the numerical value of the CTFS representation of $x(t)$ at $t = 0$?
The Fourier series always goes through the mid-point of a discontinuity of the signal it is representing. In this case that would be at 1/2.

4. Partial sums of a Fourier series of the form $\sum_{k=-N}^{N} c_x[k] e^{j2\pi kt/T}$ overshoot and ring at a discontinuity. This effect is called the ____________________________ phenomenon. Gibbs

5. Find the numerical values of the constants in these CTFT pairs.

(a) $7\,\text{rect}(3t) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}(bf)$

$$7\,\text{rect}(3t) \xleftrightarrow{\mathcal{F}} (7/3)\text{sinc}(f/3) \Rightarrow A = 7/3,\ b = 1/3$$

(b) $4\,\text{tri}(t-4) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}^2(bf)e^{cf}$

$$4\,\text{tri}(t-4) \xleftrightarrow{\mathcal{F}} 4\,\text{sinc}^2(f)e^{-j8\pi f} \Rightarrow A = 4,\ b = 1, c = -j8\pi$$

(c) $A\cos(bt) \xleftrightarrow{\mathcal{F}} 8\delta(f-3) + 8\delta(f+3)$

$$16\cos(6\pi t) \xleftrightarrow{\mathcal{F}} 8\delta(f-3) + 8\delta(f+3) \Rightarrow A = 16,\ b = 6\pi$$

(d) $11\sin(20\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(\omega+b) - \delta(\omega-b)]$

$$11\sin(20\pi t) \xleftrightarrow{\mathcal{F}} j11\pi[\delta(\omega+20\pi) - \delta(\omega-20\pi)] \Rightarrow A = j11\pi, b = 20\pi$$

6. If we know a signal $x(t)$ is periodic, what do we know for sure about its CTFT that we would not know if it were not periodic?

We know that the CTFT consists entirely of impulses.

7. A system has a frequency response $H(j\omega) = \dfrac{100}{j\omega + 300}$.

(a) If we apply the constant signal $x(t) = 12$ to this system the response is also a constant. What is the numerical value of the response constant?
The response to a constant is that constant times the frequency response at a frequency of zero. The frequency reponse at zero is 1/3. Therefore the system response is 4.

(b) If we apply the signal $x(t) = 3\sin(14\pi t)$ to the system the response is $y(t) = A\sin(14\pi t + \theta)$. What are the numerical values of A and θ (θ in radians)?
In this case $\omega = 14\pi$ and $H(j14\pi) = \dfrac{100}{j14\pi + 300} = 0.3298\angle -0.1456$. So the response amplitude A will be $3 \times 0.3298 = 0.9894$ and the response will lag the excitation by 0.1456 radians so $\theta = -0.1456$ radians.

8. Given $x(t) \xleftrightarrow{\mathcal{F}} \delta(f-5) + \delta(f+5)$ and $x(t) \xleftrightarrow[2]{\mathcal{FS}} c_x[k] = \delta[k-a] + \delta[k+a]$, what is the numerical value of a?

$x(t)$ is a 5 Hz sinusoid. Its fundamental period is 1/5 second. Using a representation time of 2 seconds this sinusoid will lie at the 10th harmonic. Therefore $a = 10$.

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1. Given $x(t) \xleftrightarrow[12]{\mathcal{FS}} \text{tri}((k-1)/3) + \text{tri}((k+1)/3)$ what is the numerical average value of $x(t)$?
The average value of any periodic signal is the value of its CTFS harmonic function at $k = 0$. Therefore the average value of $x(t)$ is $\text{tri}((0-1)/3) + \text{tri}((0+1)/3) = \text{tri}(-1/3) + \text{tri}(1/3) = 2/3 + 2/3 = 4/3 \cong 1.333$.

2. Given $x(t) \xleftrightarrow[8]{\mathcal{FS}} 4(u[k+2] - u[k-3])$ what is the numerical average signal power of $x(t)$?
The average signal power of a signal is the sum of the squares of the magnitudes of the harmonic function values. In this case

$$P_x = 4^2(1+1+1+1+1) = 16 \times 5 = 80$$

3. A signal $x(t)$ has a discontinuity at $t = 0$. The limit of $x(t)$ approaching $t = 0$ from above is 1 and the limit of $x(t)$ approaching $t = 0$ from below is -4. What is the numerical value of the CTFS representation of $x(t)$ at $t = 0$?
The Fourier series always goes through the mid-point of a discontinuity of the signal it is representing. In this case that would be at -3/2.

4. Partial sums of a Fourier series of the form $\sum_{k=-N}^{N} c_x[k] e^{j2\pi kt/T}$ overshoot and ring at a discontinuity. This effect is called the ____________________________ phenomenon. Gibbs

5. Find the numerical values of the constants in these CTFT pairs.
   (a) $11\,\text{rect}(5t) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}(bf)$

   $$11\,\text{rect}(5t) \xleftrightarrow{\mathcal{F}} (11/5)\text{sinc}(f/5) \Rightarrow A = 11/5,\ b = 1/5$$

   (b) $3\,\text{tri}(t-2) \xleftrightarrow{\mathcal{F}} A\,\text{sinc}^2(bf)e^{cf}$

   $$3\,\text{tri}(t-2) \xleftrightarrow{\mathcal{F}} 3\,\text{sinc}^2(f)e^{-j4\pi f} \Rightarrow A = 3,\ b = 1,\ c = -j4\pi$$

   (c) $A\cos(bt) \xleftrightarrow{\mathcal{F}} 12\delta(f-7) + 12\delta(f+7)$

   $$24\cos(14\pi t) \xleftrightarrow{\mathcal{F}} 12\delta(f-7) + 12\delta(f+7) \Rightarrow A = 24,\ b = 14\pi$$

   (d) $5\sin(32\pi t) \xleftrightarrow{\mathcal{F}} A[\delta(\omega+b) - \delta(\omega-b)]$

   $$5\sin(32\pi t) \xleftrightarrow{\mathcal{F}} j5\pi[\delta(\omega+32\pi) - \delta(\omega-32\pi)] \Rightarrow A = j5\pi, b = 32\pi$$

6. If we know a signal $x(t)$ is periodic, what do we know for sure about its CTFT that we would not know if it were not periodic?

   We know that the CTFT consists entirely of impulses.

7. A system has a frequency response $H(j\omega) = \dfrac{100}{j\omega + 150}$.

   (a) If we apply the constant signal $x(t) = 12$ to this system the response is also a constant. What is the numerical value of the response constant?
   The response to a constant is that constant times the frequency response at a frequency of zero. The frequency reponse at zero is 2/3. Therefore the system response is 8.

   (b) If we apply the signal $x(t) = 3\sin(14\pi t)$ to the system the response is $y(t) = A\sin(14\pi t + \theta)$. What are the numerical values of A and θ (θ in radians)?
   In this case $\omega = 14\pi$ and $H(j14\pi) = \dfrac{100}{j14\pi + 150} = 0.6397\angle -0.2852$. So the response amplitude A will be $3 \times 0.6397 = 1.9191$ and the response will lag the excitation by 0.2852 radians so $\theta = -0.2852$ radians.

8. Given $x(t) \xleftrightarrow{\mathcal{F}} \delta(f-6) + \delta(f+6)$ and $x(t) \xleftrightarrow[2]{\mathcal{FS}} c_x[k] = \delta[k-a] + \delta[k+a]$, what is the numerical value of a?

   $x(t)$ is a 6 Hz sinusoid. Its fundamental period is 1/6 second. Using a representation time of 2 seconds this sinusoid will lie at the 12th harmonic. Therefore $a = 12$.