Solution of EECS 315 Test 2 F10

In every system on this test, x is the excitation and y is the response.

1. $y(t) = \text{ramp}(x(t)) = x(t)u(x(t))$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

Let $x_1(t) = g(t)$. Then $y_1(t) = \text{ramp}(g(t))$.

Let $x_2(t) = -g(t)$. Then $y_2(t) = \text{ramp}(-g(t)) \neq -\text{ramp}(g(t)). \Rightarrow$ Non-Linear

$y(t)$ depends only on $x(t) \Rightarrow$ Static

Many different $x(t)'s$ can produce $y(t) = 0 \Rightarrow$ Non-Invertible

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = \text{ramp}(g(t - t_0)) = y_1(t - t_0) \Rightarrow$ Time-Invariant

2. $y(t) = x(t)u(t)$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

Let $x_1(t) = g(t)$. Then $y_1(t) = g(t)u(t)$.

Let $x_2(t) = K\,g(t)$. Then $y_2(t) = K\,g(t)u(t) = K\,y_1(t) \Rightarrow$ Homogeneous

Let $x_2(t) = h(t)$. Then $y_2(t) = h(t)u(t)$.

Let $x_3(t) = g(t) + h(t)$. Then $y_3(t) = [g(t) + h(t)]u(t) = y_1(t) + y_2(t). \Rightarrow$ Additive $\Rightarrow$ Linear

$y(t)$ depends only on $x(t) \Rightarrow$ Static

For $t < 0$, $y(t) = 0$ regardless of $x(t) \Rightarrow$ Non-Invertible

Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = g(t - t_0)u(t) \neq y_1(t - t_0) \Rightarrow$ Time-Variant

3. $4\,y[n] - 3\,y[n-1] = x[n]$ Circle the correct properties.

Linear	Static	Invertible	Stable
Non-Linear	Dynamic	Non-Invertible	Unstable

Standard difference equation form with constant coefficients $\Rightarrow$ Linear

Block diagram requires delays $\Rightarrow$ Dynamic

Difference equation states directly how to find $x[n] \Rightarrow$ Invertible

Eigenvalue is 3/4. Magnitude less than one. $\Rightarrow$ Stable

4. $y(t) = \sin(x(t))$ Circle the correct properties.

Linear	Causal	Invertible	Stable
Non-Linear	Non-Causal	Non-Invertible	Unstable

Let $x_1(t) = g(t)$. Then $y_1(t) = \sin(g(t))$.

Let $x_2(t) = K\,g(t)$. Then $y_2(t) = \sin(K\,g(t)) \neq K\,y_1(t) \Rightarrow$ Non-Linear

$y(t)$ depends only on $x(t) \Rightarrow$ Static $\Rightarrow$ Causal

$x(t) = \sin^{-1}(y(t))$ which is multi-valued $\Rightarrow$ Non-Invertible

$\sin(x(t))$ is never greater then one in magnitude $\Rightarrow$ Stable

5. The impulse response of a continuous-time system described by a differential equation consists of a linear combination of one or more functions of the form Ke^{st} where s is an eigenvalue and, in some cases, an impulse of the form $K_\delta \delta(t)$. How many constants of each type are needed for each system.

(a) $a\,y'(t) + b\,y(t) = c\,x(t)$ a, b and c are constants.

One K's   Zero K_δ's

(b) $a\,y'(t) + b\,y(t) = c\,x'(t)$ a, b and c are constants.

One K's   One K_δ's

(c) $a\,y'''(t) + b\,y(t) = c\,x'(t)$ a, b and c are constants.

Three K's   Zero K_δ's

(d) $a\,y''(t) + b\,y(t) = c\,x''(t) + d\,x'(t)$ a, b, c and d are constants.

Two K's   One K_δ's

6. If $y(t) = -4 + 8\,\mathrm{rect}(t/3) * 2\,\mathrm{rect}(2t)$ what is the maximum (most positive) numerical value of $y(t)$ over all time t ?

The maximum occurs when $8\,\mathrm{rect}(t/3) * 2\,\mathrm{rect}(2t)$ reaches its maximum value and that occurs when the narrower rectangle is fully within the wider rectangle. The area under the product at that time is $8 \times 2 \times 1/2 = 8$. So the maximum is 4.

7. If $y[n] = (u[n-4] - u[n-13]) * (u[n+6] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

These are two rectangular shapes, one of width 9 and one of width 7. The maximum occurs when the narrower one lies fully inside the wider one. At that time the sum of the product is 7.

(b) Over what numerical range or ranges of n is $y[n]$ <u>non-zero</u> ?

$-2 \le n < 13$ or $-2 \le n \le 12$

8. If $y[n] = (u[4-n] - u[n-13]) * (u[n+6] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

The first signal is one from n equals minus infinity through 4 and minus one from n equals 13 to positive infinity. The second signal is rectangular with 7 non-zero impulses. The maximum occurs anywhere the rectangular pulse lies wholly within the region in which the first signal is one. Then the sum of the product is 7.

(b) Over what numerical range or ranges of n is $y[n]$ <u>zero</u> ?

$n = 5,6$

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In every system on this test, x is the excitation and y is the response.

1. $y(t) = \sin(x(t))$ Circle the correct properties.

Linear	Causal	Invertible	Stable
Non-Linear	Non-Causal	Non-Invertible	Unstable

Let $x_1(t) = g(t)$. Then $y_1(t) = \sin(g(t))$.
Let $x_2(t) = K\,g(t)$. Then $y_2(t) = \sin(K\,g(t)) \neq K\,y_1(t) \Rightarrow$ Non-Linear
$y(t)$ depends only on $x(t) \Rightarrow$ Static $\Rightarrow$ Causal
$x(t) = \sin^{-1}(y(t))$ which is multi-valued $\Rightarrow$ Non-Invertible
$\sin(x(t))$ is never greater then one in magnitude $\Rightarrow$ Stable

2. $y(t) = \text{ramp}(x(t)) = x(t)\,u(x(t))$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable
Let $x_1(t) = g(t)$. Then $y_1(t) = \text{ramp}(g(t))$.
Let $x_2(t) = -g(t)$. Then $y_2(t) = \text{ramp}(-g(t)) \neq -\text{ramp}(g(t)). \Rightarrow$ Non-Linear
$y(t)$ depends only on $x(t) \Rightarrow$ Static
Many different $x(t)'s$ can produce $y(t) = 0 \Rightarrow$ Non-Invertible
Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = \text{ramp}(g(t - t_0)) = y_1(t - t_0) \Rightarrow$ Time-Invariant

3. $y(t) = x(t)\,u(t)$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable
Let $x_1(t) = g(t)$. Then $y_1(t) = g(t)\,u(t)$.
Let $x_2(t) = K\,g(t)$. Then $y_2(t) = K\,g(t)\,u(t) = K\,y_1(t) \Rightarrow$ Homogeneous
Let $x_2(t) = h(t)$. Then $y_2(t) = h(t)\,u(t)$.
Let $x_3(t) = g(t) + h(t)$. Then $y_3(t) = [g(t) + h(t)]u(t) = y_1(t) + y_2(t). \Rightarrow$ Additive $\Rightarrow$ Linear
$y(t)$ depends only on $x(t) \Rightarrow$ Static
For $t < 0$, $y(t) = 0$ regardless of $x(t) \Rightarrow$ Non-Invertible
Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = g(t - t_0)\,u(t) \neq y_1(t - t_0) \Rightarrow$ Time-Variant

4. $4\,y[n] - 3\,y[n-1] = x[n]$ Circle the correct properties.

Linear	Static	Invertible	Stable
Non-Linear	Dynamic	Non-Invertible	Unstable

Standard difference equation form with constant coefficients $\Rightarrow$ Linear
Block diagram requires delays $\Rightarrow$ Dynamic
Difference equation states directly how to find $x[n] \Rightarrow$ Invertible
Eigenvalue is 3/4. Magnitude less than one. $\Rightarrow$ Stable

5. The impulse response of a continuous-time system described by a differential equation consists of a linear combination of one or more functions of the form Ke^{st} where s is an eigenvalue and, in some cases, an impulse of the form $K_\delta \delta(t)$. How many constants of each type are needed for each system.

(a) $a\,y''(t) + b\,y(t) = c\,x''(t) + d\,x'(t)$ *a, b, c* and *d* are constants.

Two K's One K_δ's

(b) $a\,y'(t) + b\,y(t) = c\,x(t)$ *a, b* and *c* are constants.

One K's Zero K_δ's

(c) $a\,y'(t) + b\,y(t) = c\,x'(t)$ *a, b* and *c* are constants.

One K's One K_δ's

(d) $a\,y'''(t) + b\,y(t) = c\,x'(t)$ *a, b* and *c* are constants.

Three K's Zero K_δ's

6. If $y(t) = -6 + 8\,\text{rect}(t/3) * 2\,\text{rect}(2t)$ what is the maximum (most positive) numerical value of $y(t)$ over all time t ?

The maximum occurs when $8\,\text{rect}(t/3) * 2\,\text{rect}(2t)$ reaches its maximum value and that occurs when the narrower rectangle is fully within the wider rectangle. The area under the product at that time is $8 \times 2 \times 1/2 = 8$. So the maximum is 2.

7. If $y[n] = (u[n-4] - u[n-12]) * (u[n+5] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

These are two rectangular shapes, one of width 8 and one of width 6. The maximum occurs when the narrower one lies fully inside the wider one. At that time the sum of the product is 6.

(b) Over what numerical range or ranges of n is $y[n]$ <u>non-zero</u> ?

$-1 \le n < 12$ or $-1 \le n \le 11$

8. If $y[n] = (u[4-n] - u[n-12]) * (u[n+5] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

The first signal is one from n equals minus infinity through 4 and minus one from n equals 12 to positive infinity. The second signal is rectangular with 6 non-zero impulses. The maximum occurs anywhere the rectangular pulse lies wholly within the region in which the first signal is one. Then the sum of the product is 6.

(b) Over what numerical range or ranges of n is $y[n]$ <u>zero</u> ?

$n = 5,6$

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In every system on this test, x is the excitation and y is the response.

1. $\mathrm{y}(t)=\mathrm{x}(t)\mathrm{u}(t)$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\mathrm{g}(t)\mathrm{u}(t)$.

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=K\,\mathrm{g}(t)\mathrm{u}(t)=K\,\mathrm{y}_1(t)\Rightarrow$ Homogeneous

Let $\mathrm{x}_2(t)=\mathrm{h}(t)$. Then $\mathrm{y}_2(t)=\mathrm{h}(t)\mathrm{u}(t)$.

Let $\mathrm{x}_3(t)=\mathrm{g}(t)+\mathrm{h}(t)$. Then $\mathrm{y}_3(t)=\left[\mathrm{g}(t)+\mathrm{h}(t)\right]\mathrm{u}(t)=\mathrm{y}_1(t)+\mathrm{y}_2(t).\Rightarrow$ Additive $\Rightarrow$ Linear

$\mathrm{y}(t)$ depends only on $\mathrm{x}(t)\Rightarrow$ Static

For $t<0$, $\mathrm{y}(t)=0$ regardless of $\mathrm{x}(t)\Rightarrow$ Non-Invertible

Let $\mathrm{x}_2(t)=\mathrm{g}(t-t_0)$. Then $\mathrm{y}_2(t)=\mathrm{g}(t-t_0)\mathrm{u}(t)\neq\mathrm{y}_1(t-t_0)\Rightarrow$ Time-Variant

2. $4\,\mathrm{y}[n]-3\,\mathrm{y}[n-1]=\mathrm{x}[n]$ Circle the correct properties.

Linear	Static	Invertible	Stable
Non-Linear	Dynamic	Non-Invertible	Unstable

Standard difference equation form with constant coefficients $\Rightarrow$ Linear

Block diagram requires delays $\Rightarrow$ Dynamic

Difference equation states directly how to find $\mathrm{x}[n]\Rightarrow$ Invertible

Eigenvalue is 3/4. Magnitude less than one. $\Rightarrow$ Stable

3. $\mathrm{y}(t)=\sin(\mathrm{x}(t))$ Circle the correct properties.

Linear	Causal	Invertible	Stable
Non-Linear	Non-Causal	Non-Invertible	Unstable

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\sin(\mathrm{g}(t))$.

Let $\mathrm{x}_2(t)=K\,\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=\sin(K\,\mathrm{g}(t))\neq K\,\mathrm{y}_1(t)\Rightarrow$ Non-Linear

$\mathrm{y}(t)$ depends only on $\mathrm{x}(t)\Rightarrow$ Static $\Rightarrow$ Causal

$\mathrm{x}(t)=\sin^{-1}(\mathrm{y}(t))$ which is multi-valued $\Rightarrow$ Non-Invertible

$\sin(\mathrm{x}(t))$ is never greater then one in magnitude $\Rightarrow$ Stable

4. $\mathrm{y}(t)=\mathrm{ramp}(\mathrm{x}(t))=\mathrm{x}(t)\mathrm{u}(\mathrm{x}(t))$ Circle the correct properties.

Stable	Linear	Static	Invertible	Time Invariant
Unstable	Non-Linear	Dynamic	Non-Invertible	Time Variant

x bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

Let $\mathrm{x}_1(t)=\mathrm{g}(t)$. Then $\mathrm{y}_1(t)=\mathrm{ramp}(\mathrm{g}(t))$.

Let $\mathrm{x}_2(t)=-\mathrm{g}(t)$. Then $\mathrm{y}_2(t)=\mathrm{ramp}(-\mathrm{g}(t))\neq-\mathrm{ramp}(\mathrm{g}(t)).\Rightarrow$ Non-Linear

$\mathrm{y}(t)$ depends only on $\mathrm{x}(t)\Rightarrow$ Static

Many different $\mathrm{x}(t)'s$ can produce $\mathrm{y}(t)=0\Rightarrow$ Non-Invertible

Let $\mathrm{x}_2(t)=\mathrm{g}(t-t_0)$. Then $\mathrm{y}_2(t)=\mathrm{ramp}(\mathrm{g}(t-t_0))=\mathrm{y}_1(t-t_0)\Rightarrow$ Time-Invariant

5. The impulse response of a continuous-time system described by a differential equation consists of a linear combination of one or more functions of the form Ke^{st} where s is an eigenvalue and, in some cases, an impulse of the form $K_\delta\delta(t)$. How many constants of each type are needed for each system.

(a) $a\,y'(t) + b\,y(t) = c\,x'(t)$ *a, b* and *c* are constants.

One K's One K_δ's

(b) $a\,y'''(t) + b\,y(t) = c\,x'(t)$ *a, b* and *c* are constants.

Three K's Zero K_δ's

(c) $a\,y''(t) + b\,y(t) = c\,x''(t) + d\,x'(t)$ *a, b, c* and *d* are constants.

Two K's One K_δ's

(d) $a\,y'(t) + b\,y(t) = c\,x(t)$ *a, b* and *c* are constants.

One K's Zero K_δ's

6. If $y(t) = -5 + 8\,\mathrm{rect}(t/3) * 2\,\mathrm{rect}(2t)$ what is the maximum (most positive) numerical value of $y(t)$ over all time t ?

The maximum occurs when $8\,\mathrm{rect}(t/3) * 2\,\mathrm{rect}(2t)$ reaches its maximum value and that occurs when the narrower rectangle is fully within the wider rectangle. The area under the product at that time is $8 \times 2 \times 1/2 = 8$. So the maximum is 3.

7. If $y[n] = (u[n-4] - u[n-12]) * (u[n+4] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

These are two rectangular shapes, one of width 8 and one of width 5. The maximum occurs when the narrower one lies fully inside the wider one. At that time the sum of the product is 5.

(b) Over what numerical range or ranges of n is $y[n]$ <u>non-zero</u> ?

$0 \le n < 12$ or $0 \le n \le 11$

8. If $y[n] = (u[4-n] - u[n-12]) * (u[n+4] - u[n-1])$

(a) What is the maximum (most positive) numerical value of $y[n]$ over all time n ?

The first signal is one from n equals minus infinity through 4 and minus one from n equals 12 to positive infinity. The second signal is rectangular with 5 non-zero impulses. The maximum occurs anywhere the rectangular pulse lies wholly within the region in which the first signal is one. Then the sum of the product is 5.

(b) Over what numerical range or ranges of n is $y[n]$ <u>zero</u> ?

$n = 5,6,7$