Solution of EECS 315 Test 1 F11

1. With reference to the three signals graphed below,

(a) Let $g_{1e}(t)$ be the even part of $g_1(t)$. Find the numerical value of $g_{1e}(5)$.

$$g_{1e}(5) = \frac{g_1(5) + g_1(-5)}{2} = \frac{1+3}{2} = 2$$

(b) Let $g_{2o}(t)$ be the odd part of $g_2(t)$. Find the numerical value of $g_{2o}(-3)$.

$$g_{2o}(-3) = \frac{g_2(-3) - g_2(3)}{2} = \frac{2-(-3)}{2} = 5/2$$

(c) Let $g_4(t) = g_3(t)g_1(t)$. Find the numerical value of $g_4(6)$.

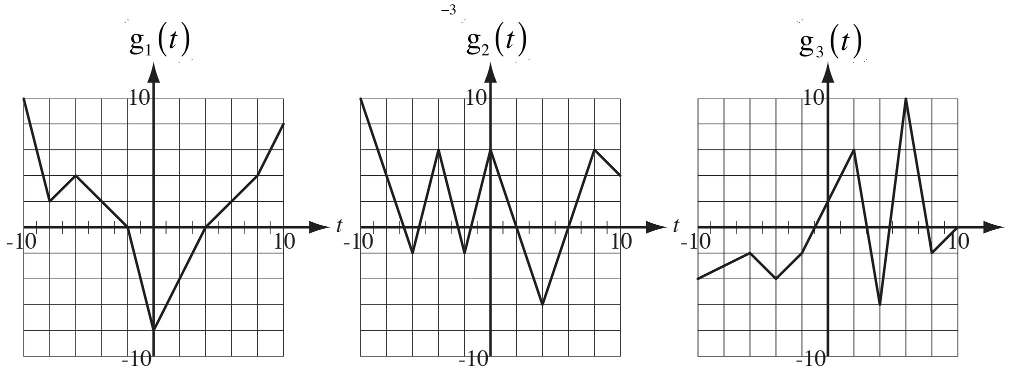
$$g_4(6) = g_3(6)g_1(6) = 10 \times 2 = 20$$

(d) Find the numerical value of the signal energy of $g_3(t)$ in the time range $0 < t < 2$.

$$E_3 = \int_0^2 (2+2t)^2\,dt = \left[4t + 4t^2 + 4t^3/3\right]_0^2 = 8 + 16 + 32/3 = \frac{24+48+32}{3} = 104/3 = 34.667$$

(e) Let $g_{1o}(t)$ be the odd part of $g_1(t)$. Find the numerical value of $\int_{-3}^{3} g_{1o}(t)\,dt$.

$$\int_{-3}^{3} g_{1o}(t)\,dt = 0$$



2. Let $\mathrm{g}(t) = -2\,\mathrm{rect}(t+1)\mathrm{ramp}(-1-t)$. Over what numerical range of time is $\mathrm{g}(t)$ non-zero?

$\mathrm{g}(t)$ is non-zero over the time range for which $-2\,\mathrm{rect}(t+1)$ and $\mathrm{ramp}(-1-t)$ are simultaneously non-zero.

$$-2\,\mathrm{rect}(t+1) \text{ is non-zero for } -1/2 < t+1 < 1/2 \text{ or } -3/2 < t < -1/2$$
$$\mathrm{ramp}(-1-t) \text{ is non-zero for } -1-t > 0 \text{ or } t+1 < 0 \text{ or } t < -1$$

So the non-zero range for $\mathrm{g}(t)$ is $-3/2 < t < -1$.

3. Let $\mathrm{g}[n] = 13\sin(13\pi n/40) + 11\cos(20\pi n/25)$. If $\mathrm{g}[n]$ is periodic, find its numerical fundamental period N_0. If it is not periodic, just write "Not periodic".

$$\mathrm{g}[n] = 13\underbrace{\sin(2\pi n(13/80))}_{N_{01}=80} + 11\underbrace{\cos(2\pi n(2/5))}_{N_{02}=5}$$

$$N_0 = \mathrm{LCM}(N_{01}, N_{02}) = \mathrm{LCM}(80,5) = 80$$

4. What is the numerical average signal power of $5(\mathrm{sgn}[n]+1)$?

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} \left|5(\mathrm{sgn}[n]+1)\right|^2 = \lim_{N\to\infty} \frac{25}{2N} \left(\sum_{n=-N}^{-1} 0^2 + 1^2 + \sum_{n=1}^{N-1} 2^2 \right) = \lim_{N\to\infty} 25\frac{0+1+4N}{2N} = \lim_{N\to\infty} 25\frac{0+1/N+4}{2} = 50$$

5. Let $g[n] = -3(3-n)u[n]u[3-n]$. Find the numerical range of discrete times over which $g[n]$ is non-zero and find the numerical signal energy of $g[n]$. (Observe carefully the inequality symbols for the time range.)

Non-zero for $0 \le n < 3$

$$E_g = \sum_{n=-\infty}^{\infty} \left|-3(3-n)u[n]u[3-n]\right|^2 = 9\sum_{n=0}^{2}(3-n)^2 = 9\left[3^2 + 2^2 + 1^2\right] = 126$$

6. Let $g[n] = 2\delta_4[n] - 3\delta_6[n]$. Find the fundamental period N_0 and the numerical signal power of $g[n]$.

$$N_0 = \text{LCM}(4,6) = 12$$

$$P = \frac{1}{12}\sum_{n=0}^{11}\left|2\delta_4[n] - 3\delta_6[n]\right|^2 = \frac{(-1)^2 + 0^2 + 0^2 + 0^2 + 2^2 + 0^2 + (-3)^2 + 0^2 + 2^2 + 0^2 + 0^2 + 0^2}{12} = 1.5$$

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1. With reference to the three signals graphed below,

(a) Let $g_{1e}(t)$ be the even part of $g_1(t)$. Find the numerical value of $g_{1e}(3)$.

$$g_{1e}(3) = \frac{g_1(3) + g_1(-3)}{2} = \frac{-2+1}{2} = -1/2$$

(b) Let $g_{2o}(t)$ be the odd part of $g_2(t)$. Find the numerical value of $g_{2o}(-7)$.

$$g_{2o}(-7) = \frac{g_2(-7) - g_2(7)}{2} = \frac{1-3}{2} = -1$$

(c) Let $g_4(t) = g_3(t)g_1(t)$. Find the numerical value of $g_4(2)$.

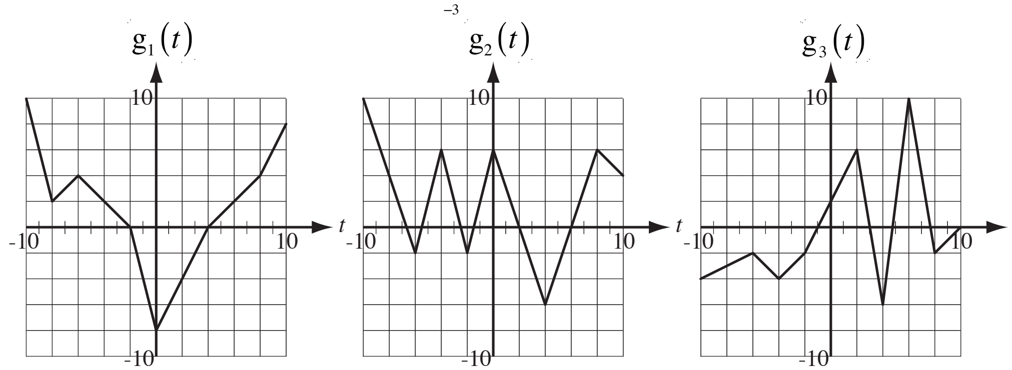
$$g_4(2) = g_3(2)g_1(2) = 6 \times (-4) = -24$$

(d) (2 pts) Find the numerical value of the signal energy of $g_2(t)$ in the time range $0 < t < 2$.

$$E_2 = \int_0^2 (6-3t)^2\,dt = \left[36t - 18t^2 + 3t^3\right]_0^2 = 72 - 72 + 24 = 24$$

(e) Let $g_{1o}(t)$ be the odd part of $g_1(t)$. Find the numerical value of $\int_{-3}^{3} g_{1o}(t)\,dt$.

$$\int_{-3}^{3} g_{1o}(t)\,dt = 0$$



2. Let $\mathrm{g}(t) = -2\,\mathrm{rect}(t-1)\,\mathrm{ramp}(1-t)$. Over what numerical range of time is $\mathrm{g}(t)$ non-zero?

$\mathrm{g}(t)$ is non-zero over the time range for which $-2\,\mathrm{rect}(t-1)$ and $\mathrm{ramp}(1-t)$ are simultaneously non-zero.

$$-2\,\mathrm{rect}(t-1) \text{ is non-zero for } -1/2 < t-1 < 1/2 \text{ or } 1/2 < t < 3/2$$
$$\mathrm{ramp}(1-t) \text{ is non-zero for } 1-t > 0 \text{ or } t-1 < 0 \text{ or } t < 1$$

So the non-zero range for $\mathrm{g}(t)$ is $1/2 < t < 1$.

3. Let $\mathrm{g}[n] = 13\sin(13\pi n/30) + 11\cos(20\pi n/25)$. If $\mathrm{g}[n]$ is periodic, find its numerical fundamental period N_0. If it is not periodic, just write "Not periodic".

$$\mathrm{g}[n] = 13\underbrace{\sin(2\pi n(13/60))}_{N_{01}=60} + 11\underbrace{\cos(2\pi n(2/5))}_{N_{02}=5}$$

$$N_0 = \mathrm{LCM}(N_{01}, N_{02}) = \mathrm{LCM}(60,5) = 60$$

4. What is the numerical average signal power of $5(\mathrm{sgn}[n]+2)$?

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} \left|5(\mathrm{sgn}[n]+2)\right|^2 = \lim_{N\to\infty} \frac{25}{2N}\left(\sum_{n=-N}^{-1} 1^2 + 2^2 + \sum_{n=1}^{N-1} 3^2\right) = \lim_{N\to\infty} 25\frac{N+4+9N}{2N} = \lim_{N\to\infty} 25\frac{1+4/N+9}{2} = 125$$

5. Let $g[n] = -3(4-n)u[n]u[4-n]$. Find the numerical range of discrete times over which $g[n]$ is non-zero and find the numerical signal energy of $g[n]$. (Observe carefully the inequality symbols for the time range.)

Non-zero for $0 \le n < 4$

$$E_g = \sum_{n=-\infty}^{\infty} \left| -3(4-n)u[n]u[4-n] \right|^2 = 9\sum_{n=0}^{3}(4-n)^2 = 9\left[4^2 + 3^2 + 2^2 + 1^2\right] = 270$$

6. Let $g[n] = 4\delta_4[n] - 3\delta_6[n]$. Find the fundamental period N_0 and the numerical signal power of $g[n]$.

$$N_0 = \text{LCM}(4,6) = 12$$

$$P = \frac{1}{12}\sum_{n=0}^{11}\left|4\delta_4[n] - 3\delta_6[n]\right|^2 = \frac{1^2 + 0^2 + 0^2 + 0^2 + 4^2 + 0^2 + (-3)^2 + 0^2 + 4^2 + 0^2 + 0^2 + 0^2}{12} = \frac{42}{12} = 3.5$$

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$$g_{3e}(5) = \frac{g_3(5)+g_3(-5)}{2} = \frac{2-3}{2} = -1/2$$

(b) Let $g_{1o}(t)$ be the odd part of $g_1(t)$. Find the numerical value of $g_{1o}(-3)$.

$$g_{1o}(-3) = \frac{g_1(-3)-g_1(3)}{2} = \frac{1-(-2)}{2} = 1.5$$

(c) Let $g_4(t) = g_3(t)g_2(t)$. Find the numerical value of $g_4(5)$.

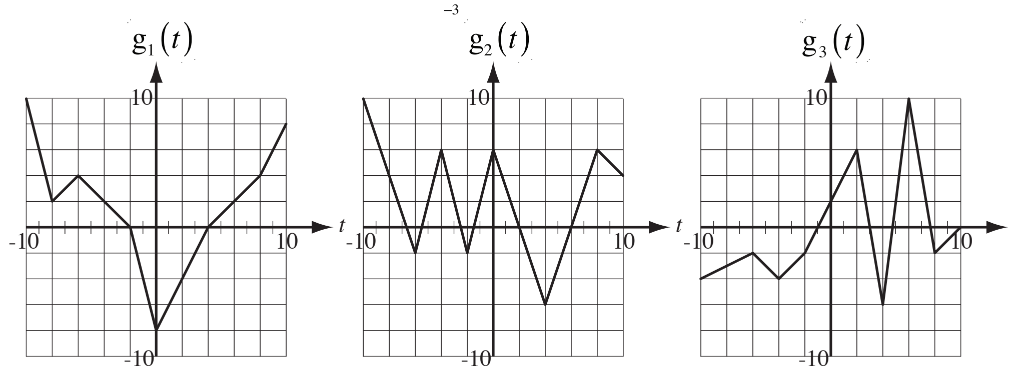
$$g_4(5) = g_3(5)g_2(5) = 2\times(-3) = -6$$

(d) Find the numerical value of the signal energy of $g_1(t)$ in the time range $0 < t < 2$.

$$E_1 = \int_0^2 (-8+2t)^2\,dt = \left[64t - 16t^2 + 4t^3/3\right]_0^2 = 128 - 64 + 32/3 = \frac{384-192+32}{3} = 224/3 = 74.667$$

(e) Let $g_{1o}(t)$ be the odd part of $g_1(t)$. Find the numerical value of $\int_{-3}^{3} g_{1o}(t)\,dt$.

$$\int_{-3}^{3} g_{1o}(t)\,dt = 0$$



2. Let $\mathrm{g}(t) = -2\,\mathrm{rect}(t+1)\,\mathrm{ramp}(-1-t)$. Over what numerical range of time is $\mathrm{g}(t)$ non-zero?

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$$\mathrm{ramp}(-1-t) \text{ is non-zero for } -1-t > 0 \text{ or } t+1 < 0 \text{ or } t < -1$$

So the non-zero range for $\mathrm{g}(t)$ is $-3/2 < t < -1$.

3. Let $\mathrm{g}[n] = 13\sin(13\pi n/80) + 11\cos(20\pi n/50)$. If $\mathrm{g}[n]$ is periodic, find its numerical fundamental period N_0. If it is not periodic, just write "Not periodic".

$$\mathrm{g}[n] = 13\underbrace{\sin(2\pi n(13/160))}_{N_{01}=160} + 11\underbrace{\cos(2\pi n(1/5))}_{N_{02}=5}$$

$$N_0 = \mathrm{LCM}(N_{01}, N_{02}) = \mathrm{LCM}(160,5) = 160$$

4. What is the numerical average signal power of $3(\mathrm{sgn}[n]+1)$?

$$P = \lim_{N\to\infty} \frac{1}{2N} \sum_{n=-N}^{N-1} \left|3(\mathrm{sgn}[n]+1)\right|^2 = \lim_{N\to\infty} \frac{9}{2N}\left(\sum_{n=-N}^{-1} 0^2 + 1^2 + \sum_{n=1}^{N-1} 2^2\right) = \lim_{N\to\infty} 9\,\frac{0+1+4N}{2N} = \lim_{N\to\infty} 9\,\frac{0+1/N+4}{2} = 18$$

5. Let $\mathrm{g}[n] = -3(2-n)\mathrm{u}[n]\mathrm{u}[2-n]$. Find the numerical range of discrete times over which $\mathrm{g}[n]$ is non-zero and find the numerical signal energy of $\mathrm{g}[n]$. (Observe carefully the inequality symbols for the time range.)

Non-zero for $0 \le n < 2$

$$E_{\mathrm{g}} = \sum_{n=-\infty}^{\infty} \left| -3(2-n)\mathrm{u}[n]\mathrm{u}[2-n] \right|^2 = 9\sum_{n=0}^{2}(2-n)^2 = 9\left[2^2 + 1^2\right] = 45$$

6. Let $\mathrm{g}[n] = 2\delta_4[n] - 8\delta_6[n]$. Find the fundamental period N_0 and the numerical signal power of $\mathrm{g}[n]$.

$$N_0 = \mathrm{LCM}(4,6) = 12$$

$$P = \frac{1}{12}\sum_{n=0}^{11}\left|2\delta_4[n] - 8\delta_6[n]\right|^2 = \frac{(-6)^2 + 0^2 + 0^2 + 0^2 + 2^2 + 0^2 + (-8)^2 + 0^2 + 2^2 + 0^2 + 0^2 + 0^2}{12} = \frac{108}{12} = 9$$