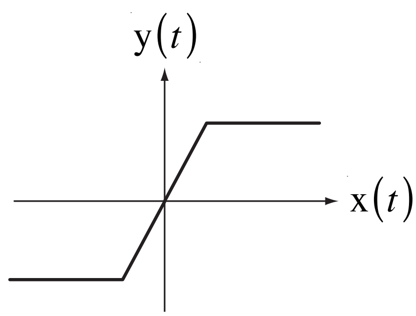
Solution of EECS 315 Test 2 F11

1. A static continuous-time system has the relation between $x(t)$ and $y(t)$ illustrated below. Circle the correct system properties.



Non-Linear, Stable, Non-Invertible, Causal

2. A discrete-time system is described by $y[n]-y[n-1]=x[n]$, where x is the excitation and y is the response. Circle the correct system properties.

Linear, Unstable, Invertible, Causal, Dynamic

3. A continuous-time system is described by $y''(t) - 2y'(t) + 5y(t) = 4x(t)$.

(a) Find the numerical eigenvalues.

$s_{1,2} = 1 \pm j2$

(b) Is it stable or unstable? (Circle the correct answer.)

Unstable

4. A discrete-time system is described by $y[n] + 1.6y[n-1] + 1.28y[n-2] = 3x[n]$.

(a) Find the numerical eigenvalues.

$z_{1,2} = -0.8 \pm j0.8$

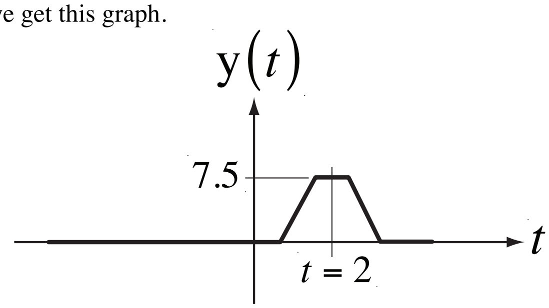
Unstable

(c) What is the numerical value of its impulse response at time $n = 1$, $h[1]$?

$h[0] = 3$, $h[1] = -4.8$

5. If $\mathrm{x}(t) = 3\,\mathrm{rect}(t-2)$ and $\mathrm{h}(t) = 5\,\mathrm{rect}(2t)$ and $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t)$, find the numerical value of $\mathrm{y}(2)$.

Convolving graphically we get this graph.



Alternate Solution:

$$\mathrm{y}(t) = 15\int_{-\infty}^{\infty} \mathrm{rect}(2\tau)\mathrm{rect}(t-\tau-2)\,d\tau \Rightarrow \mathrm{y}(2) = 15\int_{-\infty}^{\infty} \mathrm{rect}(2\tau)\mathrm{rect}(2-\tau-2)\,d\tau = 15\int_{-\infty}^{\infty} \mathrm{rect}(2\tau)\mathrm{rect}(\tau)\,d\tau = 15\int_{-1/4}^{1/4} d\tau = 7.5$$

Alternate Solution:

$$\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) \Rightarrow \mathrm{y}(t+2) = \mathrm{x}(t+2) * \mathrm{h}(t) = 3\,\mathrm{rect}(t) * 5\,\mathrm{rect}(2t)$$

$\mathrm{y}(t+2) = 15\,\mathrm{rect}(t) * \mathrm{rect}(2t)$, a trapezoid of height 7.5 centered at $t = 0$

Therefore $\mathrm{y}(t)$ is a trapezoid of height 7.5 centered at $t = 2$ and $\mathrm{y}(2) = 7.5$

6. Let $x[n] = 5(u[n] - u[n-4])$ and $h[n] = \delta[n+1] - 2\delta[n-1]$ and $y[n] = x[n] * h[n]$.

(a) Find the numerical maximum value of $y[n]$ over all n.

(b) Find the numerical minimum value of $y[n]$ over all n.

(c) Find the range of n's for which $y[n]$ is non-zero. (Carefully observe the inequality signs.)

(d) Find the signal energy of $y[n]$.

$$y[n] = 5(u[n] - u[n-4]) * (\delta[n+1] - 2\delta[n-1]) = 5(u[n+1] - u[n-3] - 2(u[n-1] - u[n-5]))$$

$$y[n] = 5(u[n+1] - u[n-3]) - 10(u[n-1] - u[n-5])$$

Maximum is 5, Minimum is -10, Non-zero range is $-1 \le n < 5$

$$E_y = 5^2 + 5^2 + (-5)^2 + (-5)^2 + (-10)^2 + (-10)^2 = 300$$

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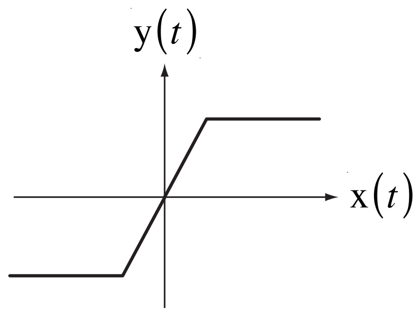


Non-Linear, Stable, Non-Invertible, Causal

2. A discrete-time system is described by $\mathrm{y}[n]-\mathrm{y}[n-1]=\mathrm{x}[n]$, where x is the excitation and y is the response. Circle the correct system properties.

Linear, Unstable, Invertible, Causal, Dynamic

3. A continuous-time system is described by $y''(t) + 2y'(t) + 5y(t) = 4x(t)$.

(a) Find the numerical eigenvalues.

$s_{1,2} = -1 \pm j2$

(b) Is it stable or unstable? (Circle the correct answer.)

Stable

4. A discrete-time system is described by $y[n] + 1.8y[n-1] + 1.4y[n-2] = 3x[n]$.

(a) Find the numerical eigenvalues.

$z_{1,2} = -0.9 \pm j0.7681$

(b) Is it stable or unstable? (Circle the correct answer.)

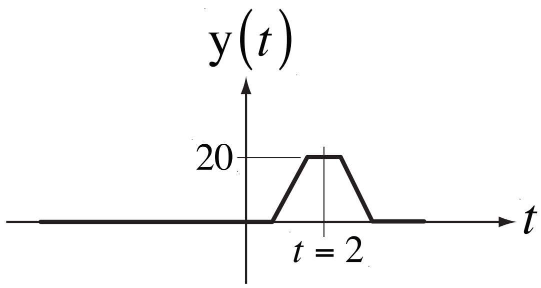
Unstable

(c) What is the numerical value of its impulse response at time $n = 1$, $h[1]$?

$h[0] = 3$, $h[1] = -5.4$

5. If $x(t) = 8\,\text{rect}(t-2)$ and $h(t) = 5\,\text{rect}(2t)$ and $y(t) = x(t) * h(t)$, find the numerical value of $y(2)$.

Convolving graphically we get this graph.



Alternate Solution:

$$y(t) = 40\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(t-\tau-2)\,d\tau \Rightarrow y(2) = 40\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(2-\tau-2)\,d\tau = 40\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(\tau)\,d\tau = 40\int_{-1/4}^{1/4} d\tau = 20$$

Alternate Solution:

$$y(t) = x(t) * h(t) \Rightarrow y(t+2) = x(t+2) * h(t) = 8\,\text{rect}(t) * 5\,\text{rect}(2t)$$

$y(t+2) = 40\,\text{rect}(t) * \text{rect}(2t)$, a trapezoid of height 20 centered at $t = 0$

Therefore $y(t)$ is a trapezoid of height 20 centered at $t = 2$ and $y(2) = 20$

6. Let $\mathrm{x}[n] = 7(\mathrm{u}[n] - \mathrm{u}[n-3])$ and $\mathrm{h}[n] = \delta[n+1] - 2\delta[n-1]$ and $\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n]$.

(a) Find the numerical maximum value of $\mathrm{y}[n]$ over all n.

(b) Find the numerical minimum value of $\mathrm{y}[n]$ over all n.

(c) Find the range of n's for which $\mathrm{y}[n]$ is non-zero. (Carefully observe the inequality signs.)

(d) Find the signal energy of $\mathrm{y}[n]$.

$$\mathrm{y}[n] = 7(\mathrm{u}[n] - \mathrm{u}[n-3]) * (\delta[n+1] - 2\delta[n-1]) = 7(\mathrm{u}[n+1] - \mathrm{u}[n-2] - 2(\mathrm{u}[n-1] - \mathrm{u}[n-4]))$$

$$\mathrm{y}[n] = 7(\mathrm{u}[n+1] - \mathrm{u}[n-2]) - 14(\mathrm{u}[n-1] - \mathrm{u}[n-4])$$

Maximum is 7, Minimum is -14, Non-zero range is $-1 \le n < 4$

$$E_{\mathrm{y}} = 7^2 + 7^2 + (-7)^2 + (-14)^2 + (-14)^2 = 539$$

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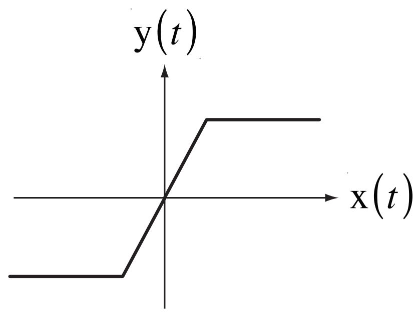


Non-Linear, Stable, Non-Invertible, Causal

2. A discrete-time system is described by $y[n]-y[n-1]=x[n]$, where x is the excitation and y is the response. Circle the correct system properties.

Linear, Unstable, Invertible, Causal, Dynamic

3. A continuous-time system is described by $y''(t) - 4y'(t) + 5y(t) = 4x(t)$.

(a) Find the numerical eigenvalues.

$s_{1,2} = 2 \pm j$

(b) Is it stable or unstable? (Circle the correct answer.)

Unstable

4. A discrete-time system is described by $y[n] + 1.7y[n-1] + 1.3y[n-2] = 3x[n]$.

(a) Find the numerical eigenvalues.

$z_{1,2} = -0.85 \pm j0.7599$

(b) Is it stable or unstable? (Circle the correct answer.)

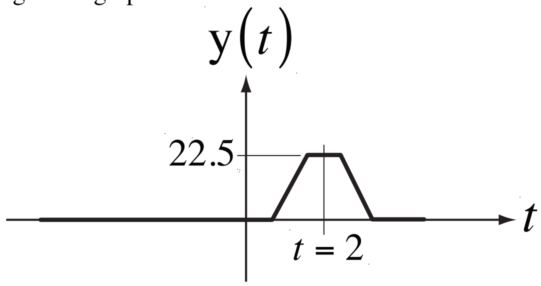
Unstable

(c) What is the numerical value of its impulse response at time $n = 1$, $h[1]$?

$h[0] = 3$, $h[1] = -5.1$

5. If $x(t) = 3\,\text{rect}(t-2)$ and $h(t) = 15\,\text{rect}(2t)$ and $y(t) = x(t) * h(t)$, find the numerical value of $y(2)$.

Convolving graphically we get this graph.



Alternate Solution:

$$y(t) = 45\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(t-\tau-2)\,d\tau \Rightarrow y(2) = 45\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(2-\tau-2)\,d\tau = 45\int_{-\infty}^{\infty} \text{rect}(2\tau)\text{rect}(\tau)\,d\tau = 45\int_{-1/4}^{1/4} d\tau = 22.5$$

Alternate Solution:

$$y(t) = x(t) * h(t) \Rightarrow y(t+2) = x(t+2) * h(t) = 3\,\text{rect}(t) * 15\,\text{rect}(2t)$$

$y(t+2) = 45\,\text{rect}(t) * \text{rect}(2t)$, a trapezoid of height 22.5 centered at $t = 0$

Therefore $y(t)$ is a trapezoid of height 22.5 centered at $t = 2$ and $y(2) = 22.5$

6. Let $\mathrm{x}[n]=5(\mathrm{u}[n]-\mathrm{u}[n-4])$ and $\mathrm{h}[n]=\delta[n+1]-3\delta[n-2]$ and $\mathrm{y}[n]=\mathrm{x}[n]*\mathrm{h}[n]$.

(a) Find the numerical maximum value of $\mathrm{y}[n]$ over all n.

(b) Find the numerical minimum value of $\mathrm{y}[n]$ over all n.

(c) Find the range of n's for which $\mathrm{y}[n]$ is non-zero. (Carefully observe the inequality signs.)

(d) Find the signal energy of $\mathrm{y}[n]$.

$$\mathrm{y}[n]=5(\mathrm{u}[n]-\mathrm{u}[n-4])*(\delta[n+1]-3\delta[n-2])=5(\mathrm{u}[n+1]-\mathrm{u}[n-3]-3(\mathrm{u}[n-2]-\mathrm{u}[n-6]))$$

$$\mathrm{y}[n]=5(\mathrm{u}[n+1]-\mathrm{u}[n-3])-15(\mathrm{u}[n-2]-\mathrm{u}[n-6])$$

Maximum is 5, Minimum is -15, Non-zero range is $-1\le n<6$

$$E_{\mathrm{y}}=5^2+5^2+5^2+(-10)^2+(-15)^2+(-15)^2+(-15)^2=850$$