Solution of ECE 315 Test 3 F11

1. Find the numerical values of the constants.

$$x(t) = 15\,\text{rect}(t/4) * \delta_5(t) \xleftrightarrow{\;\mathcal{FS}\;}_5 c_x[k] = A\,\text{sinc}(ak)$$

Using $\text{rect}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\;\mathcal{FS}\;}_{mT_0} (w/T_0)\text{sinc}(wk/mT_0)\delta_m[k]$

$$x(t) = 15\,\text{rect}(t/4) * \delta_5(t) \xleftrightarrow{\;\mathcal{FS}\;}_5 c_x[k] = 15(4/5)\text{sinc}((4/5)k) = 12\,\text{sinc}((4/5)k)$$

$$A = 12 \;,\; a = 4/5 \text{ or } 0.8$$

2. If $x(t) \xleftrightarrow{\;\mathcal{FS}\;}_6 \delta[k-1] + \delta[k+1]$ and $x(t) \xleftrightarrow{\;\mathcal{FS}\;}_{18} \delta[k-a] + \delta[k+a]$, what is the numerical value of a?

Using $\quad$ If $x(t) \xleftrightarrow{\;\mathcal{FS}\;}_T c_x[k]$ $\quad$ then $c_{xm}[k] = \begin{cases} c_x[k/m] \;,\; k/m \text{ an integer} \\ 0 \qquad\quad,\; \text{otherwise} \end{cases}$
$\qquad\qquad$ and $x(t) \xleftrightarrow{\;\mathcal{FS}\;}_{mT} c_{xm}[k]$

$m = 3$. Therefore $x(t) \xleftrightarrow{\;\mathcal{FS}\;}_{18} \begin{cases} \delta[k/3-1] + \delta[k/3+1] \;,\; k/3 \text{ an integer} \\ 0 \qquad\qquad\qquad,\; \text{otherwise} \end{cases} = \delta[k-3] + \delta[k+3]$

Alternate Solution:

Using $\cos(2\pi t/T_0) \xleftrightarrow{\;\mathcal{FS}\;}_{mT_0} (1/2)(\delta[k-m] + \delta[k+m])$

$$\cos(2\pi t/6) \xleftrightarrow{\;\mathcal{FS}\;}_6 \delta[k-1] + \delta[k+1]$$

and

$$\cos(2\pi t/6) \xleftrightarrow{\;\mathcal{FS}\;}_{18} \delta[k-3] + \delta[k+3]$$

$a = 3$

3.   Find the numerical values of the constants.

(a) $-11\,\mathrm{sinc}(5t)\xleftrightarrow{\mathscr{F}}A\,\mathrm{rect}(af)$

$-11\,\mathrm{sinc}(5t)\xleftrightarrow{\mathscr{F}}-(11/5)\mathrm{rect}(f/5)$

$A = $ -11/5 or -2.2 , $a = $ 1/5 or 0.2

(b) $28\delta(t-5)\xleftrightarrow{\mathscr{F}}Ae^{af}$

$28\delta(t-5)\xleftrightarrow{\mathscr{F}}28e^{-j10\pi f}$

$A = 28$, $a = -j10\pi$ or $-j31.416$

or $j18.85$

$(7/3)\mathrm{tri}((t+2)/3)\xleftrightarrow{\mathscr{F}}7\,\mathrm{sinc}^2(3f)e^{j4\pi f}$
$(7/3)\mathrm{tri}(t/3+2/3)\xleftrightarrow{\mathscr{F}}7\,\mathrm{sinc}^2(3f)e^{j4\pi f}$

$A = $ 7/3 or 2.333 , $a = $ 1/3 or 0.333 , $b = $ 2/3 or 0.667

(e) $A\sin(at)\longleftrightarrow^{\mathscr{F}} j19\left[\delta(f+4)-\delta(f-4)\right]$

$38\sin(8\pi t)\longleftrightarrow^{\mathscr{F}} j19\left[\delta(f+4)-\delta(f-4)\right]$

$A = 38$, $a = 8\pi$ or 25.133

(f) $5\operatorname{sgn}(8t)\longleftrightarrow^{\mathscr{F}} A/\omega$

Using $\operatorname{sgn}(t)\longleftrightarrow^{\mathscr{F}}\dfrac{1}{j\pi f}$, $5\operatorname{sgn}(8t)\longleftrightarrow^{\mathscr{F}} 5\times(1/8)\dfrac{1}{j\pi f/8}=\dfrac{5}{j\pi f}=\dfrac{10}{j\omega}$

$A = 10/j = -j10$

(g) $\dfrac{d}{dt}\left(2\operatorname{rect}(4t)\right)\longleftrightarrow^{\mathscr{F}} Af\operatorname{sinc}(af)=B\sin(bf)$

$A = j\pi$, $a = 1/4$ or 0.25 , $B = j4$, $b = \pi/4$ or 0.7854

(h) $33\cos(12\pi t)\cos(200\pi t)\longleftrightarrow^{\mathscr{F}} A\left[\delta(f-a)+\delta(f+b)+\delta(f+a)+\delta(f-b)\right]$

$33\cos(12\pi t)\cos(200\pi t)\longleftrightarrow^{\mathscr{F}} 33(1/2)\left[\delta(f-6)+\delta(f+6)\right]*(1/2)\left[\delta(f-100)+\delta(f+100)\right]$

$33\cos(12\pi t)\cos(200\pi t)\longleftrightarrow^{\mathscr{F}} (33/4)\left[\delta(f-106)+\delta(f+94)+\delta(f-94)+\delta(f+106)\right]$

$A = 33/4$ or 8.25 , $a = 106$, $b = 94$

or $A = 33/4$ or 8.25 , $a = -106$, $b = 94$

or $A = 33/4$ or 8.25 , $a = -106$, $b = -94$

or $A = 33/4$ or 8.25 , $a = -106$, $b = -94$

Solution of ECE 315 Test 3 F11

1. Find the numerical values of the constants.

$$x(t) = 15\operatorname{rect}(t/14) * \delta_6(t) \xleftrightarrow[6]{\mathcal{FS}} c_x[k] = A\operatorname{sinc}(ak)$$

Using $\operatorname{rect}(t/w) * \delta_{T_0}(t) \xleftrightarrow[mT_0]{\mathcal{FS}} (w/T_0)\operatorname{sinc}(wk/mT_0)\delta_m[k]$

$$x(t) = 15\operatorname{rect}(t/14) * \delta_6(t) \xleftrightarrow[6]{\mathcal{FS}} c_x[k] = 15(14/6)\operatorname{sinc}((14/6)k) = 35\operatorname{sinc}((7/3)k)$$

$$A = 35 \ , \ a = 7/3 \text{ or } 2.333$$

2. If $x(t) \xleftrightarrow[2]{\mathcal{FS}} \delta[k-1] + \delta[k+1]$ and $x(t) \xleftrightarrow[18]{\mathcal{FS}} \delta[k-a] + \delta[k+a]$, what is the numerical value of a?

Using
$$\begin{array}{l} \text{If } x(t) \xleftrightarrow[T]{\mathcal{FS}} c_x[k] \\ \text{and } x(t) \xleftrightarrow[mT]{\mathcal{FS}} c_{xm}[k] \end{array} \quad \text{then } c_{xm}[k] = \begin{cases} c_x[k/m] \ , & k/m \text{ an integer} \\ 0 \ , & \text{otherwise} \end{cases}$$

$m = 9$. Therefore $x(t) \xleftrightarrow[18]{\mathcal{FS}} \begin{cases} \delta[k/9-1] + \delta[k/9+1] \ , & k/9 \text{ an integer} \\ 0 \ , & \text{otherwise} \end{cases} = \delta[k-9] + \delta[k+9]$

Alternate Solution:

Using $\cos(2\pi t/T_0) \xleftrightarrow[mT_0]{\mathcal{FS}} (1/2)(\delta[k-m] + \delta[k+m])$

$$\cos(2\pi t/6) \xleftrightarrow[2]{\mathcal{FS}} \delta[k-1] + \delta[k+1]$$

and

$$\cos(2\pi t/6) \xleftrightarrow[18]{\mathcal{FS}} \delta[k-9] + \delta[k+9]$$

$a = 9$

3. Find the numerical values of the constants.

(a) $-6\,\text{sinc}(13t)\xleftrightarrow{\;\mathcal{F}\;}A\,\text{rect}(af)$

$-6\,\text{sinc}(13t)\xleftrightarrow{\;\mathcal{F}\;}-(6/13)\,\text{rect}(f/13)$

$A = -6/13$ or -0.4615 , $a = 1/13$ or 0.0769

(b) $22\delta(t+5)\xleftrightarrow{\;\mathcal{F}\;}Ae^{af}$

$22\delta(t+5)\xleftrightarrow{\;\mathcal{F}\;}22e^{j10\pi f}$

$A = 22$, $a = j10\pi$ or $j31.416$

(c) $17\,\text{tri}(9(t-3))\xleftrightarrow{\;\mathcal{F}\;}A\,\text{sinc}^2(af)e^{bf}$

$17\,\text{tri}(9(t-3))\xleftrightarrow{\;\mathcal{F}\;}(17/9)\,\text{sinc}^2(f/9)e^{-j6\pi f}$

$A = 17/9$ or 1.889 , $a = 1/9$ or 0.1111 , $b = -j6\pi$ or $-j18.85$

(d) $A\,\text{tri}(at+b)\xleftrightarrow{\;\mathcal{F}\;}4\,\text{sinc}^2(5f)e^{j6\pi f}$

$(4/5)\,\text{tri}((t+3)/5)\xleftrightarrow{\;\mathcal{F}\;}4\,\text{sinc}^2(5f)e^{j6\pi f}$

$(4/5)\,\text{tri}(t/5+3/5)\xleftrightarrow{\;\mathcal{F}\;}4\,\text{sinc}^2(5f)e^{j6\pi f}$

$A = 4/5$ or 0.8 , $a = 1/5$ or 0.2 , $b = 3/5$ or 0.6

(e)　$A\sin(at)\xleftrightarrow{\mathscr{I}} j15\big[\delta(f+7)-\delta(f-7)\big]$

$30\sin(14\pi t)\xleftrightarrow{\mathscr{I}} j15\big[\delta(f+7)-\delta(f-7)\big]$

$A=30$, $a=14\pi$ or 43.98

(f)　$24\,\mathrm{sgn}(4t)\xleftrightarrow{\mathscr{I}} A/\omega$

Using $\mathrm{sgn}(t)\xleftrightarrow{\mathscr{I}}\dfrac{1}{j\pi f}$, $\quad 24\,\mathrm{sgn}(4t)\xleftrightarrow{\mathscr{I}} 24\times(1/4)\dfrac{1}{j\pi f/4}=\dfrac{24}{j\pi f}=\dfrac{48}{j\omega}$

$A=48/j=-j48$

(g)　$\dfrac{d}{dt}\big(7\,\mathrm{rect}(9t)\big)\xleftrightarrow{\mathscr{I}} Af\,\mathrm{sinc}(af)=B\sin(bf)$

$\dfrac{d}{dt}\big(7\,\mathrm{rect}(9t)\big)\xleftrightarrow{\mathscr{I}} j2\pi f(7/9)\mathrm{sinc}(f/9)=j(14/9)\pi f\,\mathrm{sinc}(f/9)=j14\sin(\pi f/9)$

$A=j14\pi/9$ or $j4.887$, $a=1/9$ or 0.1111 , $B=j14$, $b=\pi/9$ or 0.3491

(h)　$21\cos(8\pi t)\cos(160\pi t)\xleftrightarrow{\mathscr{I}} A\big[\delta(f-a)+\delta(f+b)+\delta(f+a)+\delta(f-b)\big]$

$21\cos(8\pi t)\cos(160\pi t)\xleftrightarrow{\mathscr{I}} 21(1/2)\big[\delta(f-4)+\delta(f+4)\big]*(1/2)\big[\delta(f-80)+\delta(f+80)\big]$

$24\cos(8\pi t)\cos(160\pi t)\xleftrightarrow{\mathscr{I}} (21/4)\big[\delta(f-84)+\delta(f+76)+\delta(f-76)+\delta(f+84)\big]$

$A=21/4$ or 5.25 , $a=84$, $b=76$

or　$A=21/4$ or 5.25 , $a=-84$, $b=76$

or　$A=21/4$ or 5.25 , $a=-84$, $b=-76$

or　$A=21/4$ or 5.25 , $a=-84$, $b=-76$

Solution of ECE 315 Test 3 F11

1. Find the numerical values of the constants.

    $$x(t) = 15 \operatorname{rect}(t/3) * \delta_8(t) \overset{\mathcal{FS}}{\underset{8}{\longleftrightarrow}} c_x[k] = A \operatorname{sinc}(ak)$$

    Using $\operatorname{rect}(t/w) * \delta_{T_0}(t) \overset{\mathcal{FS}}{\underset{mT_0}{\longleftrightarrow}} (w/T_0) \operatorname{sinc}(wk/mT_0) \delta_m[k]$

    $$x(t) = 15 \operatorname{rect}(t/3) * \delta_8(t) \overset{\mathcal{FS}}{\underset{8}{\longleftrightarrow}} c_x[k] = 15(3/8) \operatorname{sinc}((3/8)k) = 5.625 \operatorname{sinc}((3/8)k)$$

    $A = 45/8$ or 5.625 , $a = 3/8$ or 0.375

2. If $x(t) \overset{\mathcal{FS}}{\underset{6}{\longleftrightarrow}} \delta[k-1] + \delta[k+1]$ and $x(t) \overset{\mathcal{FS}}{\underset{30}{\longleftrightarrow}} \delta[k-a] + \delta[k+a]$, what is the numerical value of a?

    Using $\quad$ If $x(t) \overset{\mathcal{FS}}{\underset{T}{\longleftrightarrow}} c_x[k]$

    and $x(t) \overset{\mathcal{FS}}{\underset{mT}{\longleftrightarrow}} c_{xm}[k]$ $\quad$ then $c_{xm}[k] = \begin{cases} c_x[k/m] , & k/m \text{ an integer} \\ 0 & , \text{ otherwise} \end{cases}$

    $m = 5$. Therefore $x(t) \overset{\mathcal{FS}}{\underset{30}{\longleftrightarrow}} \begin{cases} \delta[k/5-1] + \delta[k/5+1] , & k/3 \text{ an integer} \\ 0 & , \text{ otherwise} \end{cases} = \delta[k-5] + \delta[k+5]$

    Alternate Solution:

    Using $\cos(2\pi t/T_0) \overset{\mathcal{FS}}{\underset{mT_0}{\longleftrightarrow}} (1/2)(\delta[k-m] + \delta[k+m])$

    $$\cos(2\pi t/6) \overset{\mathcal{FS}}{\underset{6}{\longleftrightarrow}} \delta[k-1] + \delta[k+1]$$

    and

    $$\cos(2\pi t/6) \overset{\mathcal{FS}}{\underset{30}{\longleftrightarrow}} \delta[k-5] + \delta[k+5]$$

    $a = 5$

3.   Find the numerical values of the constants.

(a) $-3\operatorname{sinc}(10t)\xleftrightarrow{\mathscr{F}} A\operatorname{rect}(af)$

$-3\operatorname{sinc}(10t)\xleftrightarrow{\mathscr{F}}-(3/10)\operatorname{rect}(f/10)$

$A = $ -3/10 or -0.3 , $a = $ 1/10 or 0.1

(b) $8\delta(t-3)\xleftrightarrow{\mathscr{F}} Ae^{af}$

$8\delta(t-3)\xleftrightarrow{\mathscr{F}} 28e^{-j6\pi f}$

$A = 8$, $a = -j6\pi$ or $-j18.85$

(c) $-12\operatorname{tri}(7(t+1))\xleftrightarrow{\mathscr{F}} A\operatorname{sinc}^2(af)e^{bf}$

$-12\operatorname{tri}(7(t+1))\xleftrightarrow{\mathscr{F}}-(12/7)\operatorname{sinc}^2(f/7)e^{j2\pi f}$

$A = $ -12/7 or -1.714 , $a = $ 1/7 or 0.1429 , $b = j2\pi$ or $j6.28$

(d) $A\operatorname{tri}(at+b)\xleftrightarrow{\mathscr{F}} 31\operatorname{sinc}^2(12f)e^{j14\pi f}$

$(31/12)\operatorname{tri}((t+7)/12)\xleftrightarrow{\mathscr{F}} 31\operatorname{sinc}^2(12f)e^{j14\pi f}$

$(31/12)\operatorname{tri}(t/12+7/12)\xleftrightarrow{\mathscr{F}} 31\operatorname{sinc}^2(12f)e^{j14\pi f}$

$A = $ 31/12 or 2.5833 , $a = $ 1/12 or 0.0833 , $b = $ 7/12 or 0.5833

(e) $A\sin(at) \xleftrightarrow{\mathscr{F}} j17[\delta(f+22)-\delta(f-22)]$

$34\sin(44\pi t) \xleftrightarrow{\mathscr{F}} j17[\delta(f+22)-\delta(f-22)]$

$A = 34$, $a = 44\pi$ or 138.23

(f) $26\operatorname{sgn}(15t) \xleftrightarrow{\mathscr{F}} A/\omega$

Using $\operatorname{sgn}(t) \xleftrightarrow{\mathscr{F}} \dfrac{1}{j\pi f}$, $26\operatorname{sgn}(15t) \xleftrightarrow{\mathscr{F}} 26\times(1/15)\dfrac{1}{j\pi f/15} = \dfrac{26}{j\pi f} = \dfrac{52}{j\omega}$

$A = 52/j = -j52$

(g) $\dfrac{d}{dt}\big(16\operatorname{rect}(9t)\big) \xleftrightarrow{\mathscr{F}} Af\operatorname{sinc}(af) = B\sin(bf)$

$\dfrac{d}{dt}\big(16\operatorname{rect}(9t)\big) \xleftrightarrow{\mathscr{F}} j2\pi f(16/9)\operatorname{sinc}(f/9) = j(32/9)\pi f\operatorname{sinc}(f/9) = j32\sin(\pi f/9)$

$A = j32\pi/9$ or $j11.1701$, $a = 1/9$ or 0.1111 , $B = j32$, $b = \pi/9$ or 0.3491

(h) $14\cos(22\pi t)\cos(240\pi t) \xleftrightarrow{\mathscr{F}} A[\delta(f-a)+\delta(f+b)+\delta(f+a)+\delta(f-b)]$

$14\cos(22\pi t)\cos(240\pi t) \xleftrightarrow{\mathscr{F}} 14(1/2)[\delta(f-11)+\delta(f+11)]*(1/2)[\delta(f-120)+\delta(f+120)]$

$14\cos(22\pi t)\cos(240\pi t) \xleftrightarrow{\mathscr{F}} (14/4)[\delta(f-131)+\delta(f+109)+\delta(f-109)+\delta(f+131)]$

$A = 14/4$ or 3.5 , $a = 131$, $b = 109$

or $A = 14/4$ or 3.5 , $a = -131$, $b = 109$

or $A = 14/4$ or 3.5 , $a = -131$, $b = -109$

or $A = 14/4$ or 3.5 , $a = -131$, $b = -109$