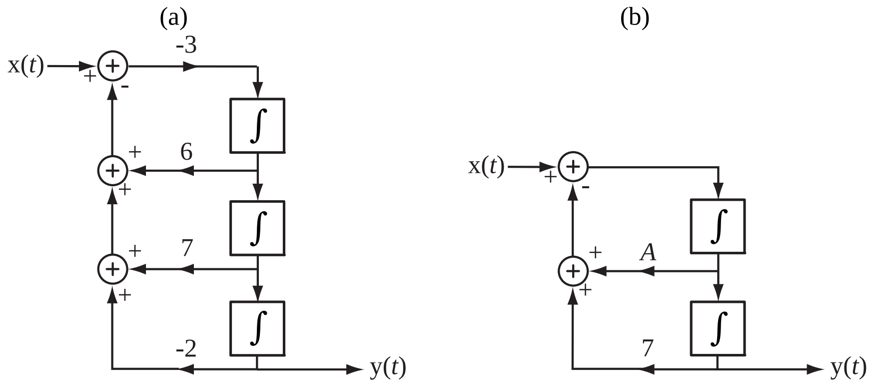
Solution of ECE 315 Test 2 Su08

1. The systems represented by these block diagrams can each be described by a differential equation of the form,

$$a_N \frac{d^N}{dt^N}\left(y(t)\right) + a_{N-1}\frac{d^{N-1}}{dt^{N-1}}y(t) + \cdots + a_2\frac{d^2}{dt^2}y(t) + a_1\frac{d}{dt}y(t) + a_0\,y(t) = x(t)\ .$$

For each system what is the numerical value of *N*?
For each system what are the *a* coefficients, starting with a_N and going down to a_0?



(a) $N = 3$

$$y'''(t) = -3\{x(t) - [6y''(t) + 7y'(t) - 2y(t)]\}$$

$$-(1/3)y'''(t) + 6y''(t) + 7y'(t) - 2y(t) = x(t)$$

(b) $N = 2$

$$y''(t) = x(t) - [A\,y'(t) + 7\,y(t)]$$

$$y''(t) + A\,y'(t) + 7\,y(t) = x(t)$$

In the (b) part, what range of values of *A* will make the system stable?

Eigenvalues are

$$\lambda_{1,2} = \frac{-A \pm \sqrt{A^2 - 28}}{2}$$

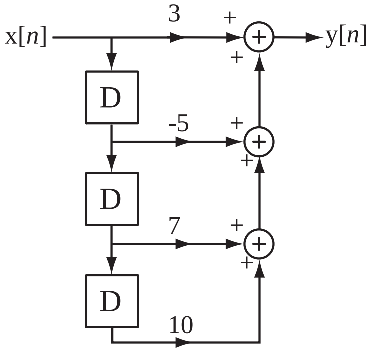
If $A^2 < 28$, the eigenvalues are complex and their real parts are simply $-A/2$. Therefore for $A^2 < 28$ and $A > 0$ the system is stable.
If $A^2 \geq 28$, the eigenvalues are real and $\sqrt{A^2 - 28} < |A|$. If $A \geq \sqrt{28}$ then both eigenvalues are negative real and the system is stable. If $A < -\sqrt{28}$ then $-A > \sqrt{A^2 - 28} > 0$ and both eigenvalues are positive real and the system is unstable.

Therefore, overall, if $A > 0$, the system is stable.

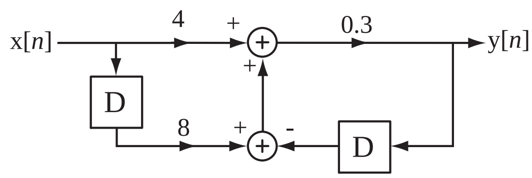
2. The systems represented by these block diagrams can each be described by a difference equation of the form,

$$a_N \, y[n] + a_{N-1} \, y[n-1] + \cdots + a_2 \, y[n-(N-2)] + a_1 \, y[n-(N-1)] + a_0 \, y[n-N] =$$
$$b_M \, x[n] + b_{M-1} \, x[n-1] + \cdots + b_2 \, x[n-(M-2)] + b_1 \, x[n-(M-1)] + b_0 [n-M] \; .$$

(a)



(b)



For each system what is the numerical value of *N*?
For each system what are the numerical *a* coefficients, starting with a_N and going down to a_0?
For each system what is the numerical value of *M*?
For each system what are the numerical *b* coefficients, starting with b_M and going down to b_0?
For each system, is it stable?

(a) $N = 0$, $M = 3$, Stable

$$y[n] = 3x[n] - 5x[n-1] + 7x[n-2] + 10x[n-3]$$

Eigenvalue from the homogeneous system equation ($y[n] = 0$) is $\alpha = 0$ which is less in magnitude than 1. Therefore the system is stable. Another way of seeing this is if x is bounded then

$$3x[n] - 5x[n-1] + 7x[n-2] + 10x[n-3]$$

is also bounded. Therefore y is bounded and the system is stable.

(b) $N = 1$, $M = 1$, Stable

$$y[n] = 0.3\{4x[n] + (8x[n-1] - y[n-1])\}$$

$$y[n]/0.3 + y[n-1] = 4x[n] + 8x[n-1]$$

The eigenvalue is -0.3. $|-0.3| < 1$ so the system is stable.

3. Indicate the properties of these systems for which x is the excitation and y is the response.

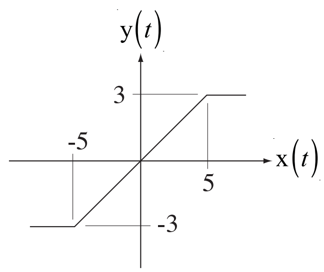
(a) $y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda$

Linear, Unstable, Dynamic, Time Invariant, Causal, Invertible

(b) $y[n] = \begin{cases} n\,x[n] & , \; n > 0 \\ 0 & , \; \text{otherwise} \end{cases}$

Linear, Unstable, Static, Time Variant, Causal, Non-Invertible

(c) A system whose behavior at any time *t* is determined by this graph.



Non-Linear, Stable, Static, Time Invariant, Causal, Non-Invertible

4. Find the total numerical solution of this difference equation with initial conditions.

$$y[n] - 0.1y[n-1] - 0.2y[n-2] = 5 \ , \ y[0] = 1 \ , \ y[1] = 4$$

The characteristic equation is $\alpha^2 - 0.1\alpha - 0.2 = 0$ and the solutions, which are the eigenvalues, are

$$\alpha_{1,2} = 0.5, -0.4 \ .$$

So the homogeneous solution is

$$y_h[n] = K_1(0.5)^n + K_2(-0.4)^n \ .$$

The forcing function is a constant so the particular solution is also a constant K_p.

$$K_p - 0.1K_p - 0.2K_p = 5 \Rightarrow 0.7K_p = 5 \Rightarrow K_p = 7.1429 \ .$$

The total solution is then

$$y[n] = K_1(0.5)^n + K_2(-0.4)^n + 7.1429 \ .$$

Using the initial conditions,

$$y[0] = K_1 + K_2 + 7.1429 = 1$$

$$y[1] = 0.5K_1 - 0.4K_2 + 7.1429 = 4$$

or, in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 0.5 & -0.4 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -6.1429 \\ -3.1429 \end{bmatrix}$$

Solving,

$$K_1 = -6.2223 \ , \ K_2 = 0.0794$$

and, finally,

$$y[n] = -6.2223(0.5)^n + 0.0794(-0.4)^n + 7.1429$$

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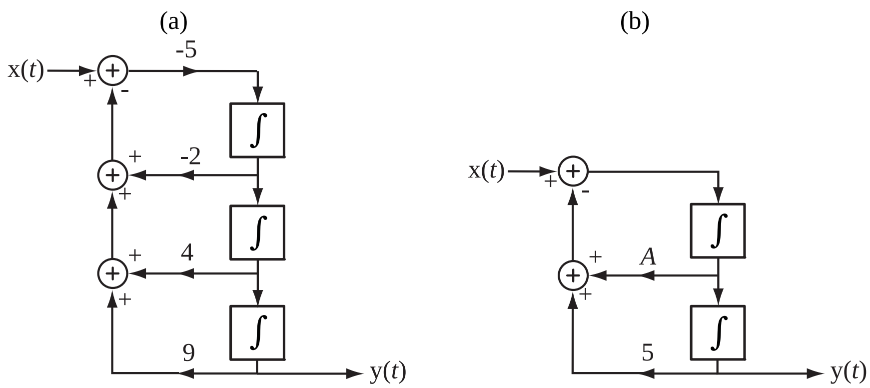
1. The systems represented by these block diagrams can each be described by a differential equation of the form,

$$a_N \frac{d^N}{dt^N}\left(y(t)\right) + a_{N-1}\frac{d^{N-1}}{dt^{N-1}}y(t) + \cdots + a_2 \frac{d^2}{dt^2}y(t) + a_1 \frac{d}{dt}y(t) + a_0\, y(t) = x(t) \ .$$

For each system what is the numerical value of *N*?
For each system what are the *a* coefficients, starting with a_N and going down to a_0?



(a) *N* = 3

$$y'''(t) = -5\left\{x(t) - \left[-2y''(t) + 4y'(t) + 9y(t)\right]\right\}$$

$$-(1/5)y'''(t) - 2y''(t) + 4y'(t) + 9y(t) = x(t)$$

(b) *N* = 2

$$y''(t) = x(t) - \left[Ay'(t) + 5y(t)\right]$$

$$y''(t) + Ay'(t) + 5y(t) = x(t)$$

In the (b) part, what range of values of *A* will make the system stable?

Eigenvalues are

$$\lambda_{1,2} = \frac{-A \pm \sqrt{A^2 - 20}}{2}$$

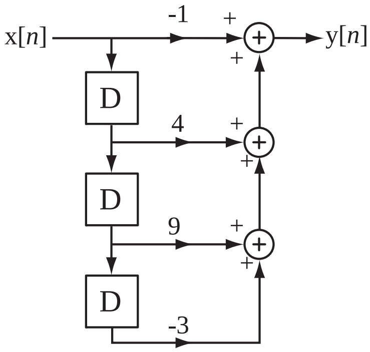
If $A^2 < 20$, the eigenvalues are complex and their real parts are simply $-A/2$. Therefore for $A^2 < 20$ and $A > 0$ the system is stable.
If $A^2 \geq 20$, the eigenvalues are real and $\sqrt{A^2 - 20} < |A|$. If $A \geq \sqrt{20}$ then both eigenvalues are negative real and the system is stable. If $A < -\sqrt{20}$ then $-A > \sqrt{A^2 - 20} > 0$ and both eigenvalues are positive real and the system is unstable.

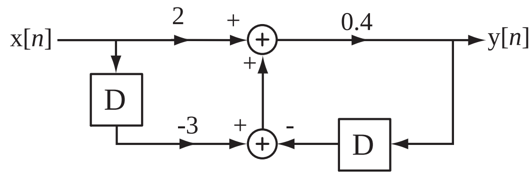
Therefore, overall, if $A > 0$, the system is stable.

2. The systems represented by these block diagrams can each be described by a difference equation of the form,

$$a_N \, y[n] + a_{N-1} \, y[n-1] + \cdots + a_2 \, y[n-(N-2)] + a_1 \, y[n-(N-1)] + a_0 \, y[n-N] =$$
$$b_M \, x[n] + b_{M-1} \, x[n-1] + \cdots + b_2 \, x[n-(M-2)] + b_1 \, x[n-(M-1)] + b_0 [n-M] \, .$$

(a) (b)





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For each system what are the numerical *b* coefficients, starting with b_M and going down to b_0?
For each system, is it stable?

(a) $N = 0$, $M = 3$, Stable

$$y[n] = -x[n] + 4x[n-1] + 9x[n-2] - 3x[n-3]$$

Eigenvalue from the homogeneous system equation ($y[n] = 0$) is $\alpha = 0$ which is less in magnitude than 1. Therefore the system is stable. Another way of seeing this is if x is bounded then

$$-x[n] + 4x[n-1] + 9x[n-2] - 3x[n-3]$$

is also bounded. Therefore y is bounded and the system is stable.

(b) $N = 1$, $M = 1$, Stable

$$y[n] = 0.4\{2x[n] + (-3x[n-1] - y[n-1])\}$$

$$y[n]/0.4 + y[n-1] = 2x[n] - 3x[n-1]$$

The eigenvalue is -0.4. $|-0.4| < 1$ so the system is stable.

3. Indicate the properties of these systems for which x is the excitation and y is the response.

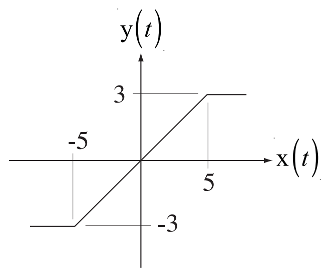
(a) $y(t) = \int_{-\infty}^{t} x(\lambda) d\lambda$

Linear, Unstable, Dynamic, Time Invariant, Causal, Invertible

(b) $y[n] = \begin{cases} n x[n] & , \ n > 0 \\ 0 & , \ \text{otherwise} \end{cases}$

Linear, Unstable, Static, Time Variant, Causal, Non-Invertible

(c) A system whose behavior at any time *t* is determined by this graph.



Non-Linear, Stable, Static, Time Invariant, Causal, Non-Invertible

4. Find the total numerical solution of this difference equation with initial conditions.

$$y[n]-0.2y[n-1]-0.24y[n-2]=5 \ , \ y[0]=1 \ , \ y[1]=4$$

The characteristic equation is $\alpha^2-0.2\alpha-0.24=0$ and the solutions, which are the eigenvalues, are

$$\alpha_{1,2}=0.6,-0.4 \ .$$

So the homogeneous solution is

$$y_h[n]=K_1(0.6)^n+K_2(-0.4)^n .$$

The forcing function is a constant so the particular solution is also a constant K_p.

$$K_p-0.2K_p-0.24K_p=5 \Rightarrow 0.56K_p=5 \Rightarrow K_p=8.9286 \ .$$

The total solution is then

$$y[n]=K_1(0.6)^n+K_2(-0.4)^n+8.9286 .$$

Using the initial conditions,

$$y[0]=K_1+K_2+8.9286=1$$

$$y[1]=0.6K_1-0.4K_2+8.9286=4$$

or, in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 0.6 & -0.4 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}=\begin{bmatrix} -7.9286 \\ -4.9286 \end{bmatrix}$$

Solving,

$$K_1=-8.1 \ , \ K_2=0.1714$$

and, finally,

$$y[n]=-8.1(0.6)^n+0.1714(-0.4)^n+8.9286$$

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1. The systems represented by these block diagrams can each be described by a differential equation of the form,

$$a_N \frac{d^N}{dt^N}\left(\mathrm{y}(t)\right) + a_{N-1}\frac{d^{N-1}}{dt^{N-1}}\mathrm{y}(t) + \cdots + a_2 \frac{d^2}{dt^2}\mathrm{y}(t) + a_1 \frac{d}{dt}\mathrm{y}(t) + a_0\,\mathrm{y}(t) = \mathrm{x}(t) \ .$$

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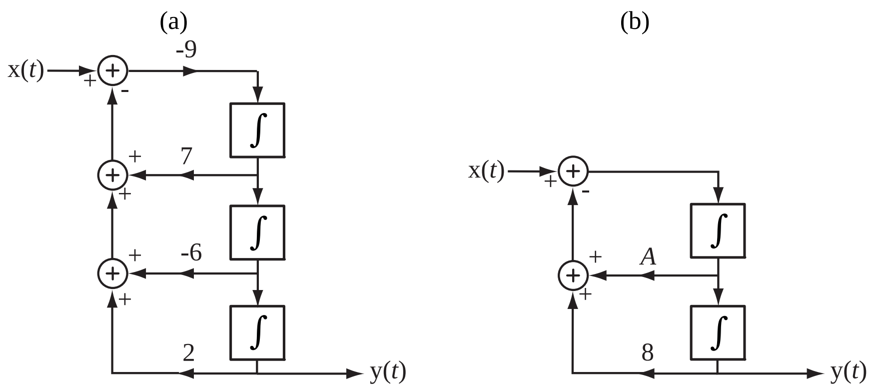


(a) $N = 3$

$$\mathrm{y}'''(t) = -9\left\{\mathrm{x}(t) - \left[7\,\mathrm{y}''(t) - 6\,\mathrm{y}'(t) + 2\,\mathrm{y}(t)\right]\right\}$$

$$-(1/9)\,\mathrm{y}'''(t) + 7\,\mathrm{y}''(t) - 6\,\mathrm{y}'(t) + 2\,\mathrm{y}(t) = \mathrm{x}(t)$$

(b) $N = 2$

$$\mathrm{y}''(t) = \mathrm{x}(t) - \left[A\,\mathrm{y}'(t) + 8\,\mathrm{y}(t)\right]$$

$$\mathrm{y}''(t) + A\,\mathrm{y}'(t) + 8\,\mathrm{y}(t) = \mathrm{x}(t)$$

In the (b) part, what range of values of *A* will make the system stable?

Eigenvalues are

$$\lambda_{1,2} = \frac{-A \pm \sqrt{A^2 - 32}}{2}$$

If $A^2 < 32$, the eigenvalues are complex and their real parts are simply $-A/2$. Therefore for $A^2 < 32$ and $A > 0$ the system is stable.

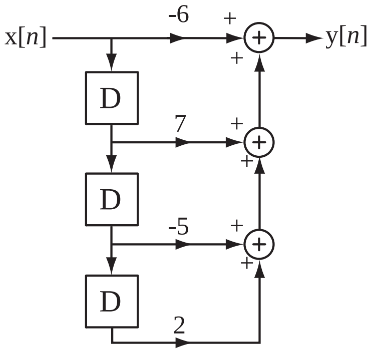
If $A^2 \geq 32$, the eigenvalues are real and $\sqrt{A^2 - 32} < |A|$. If $A \geq \sqrt{32}$ then both eigenvalues are negative real and the system is stable. If $A < -\sqrt{32}$ then $-A > \sqrt{A^2 - 32} > 0$ and both eigenvalues are positive real and the system is unstable.

Therefore, overall, if $A > 0$, the system is stable.

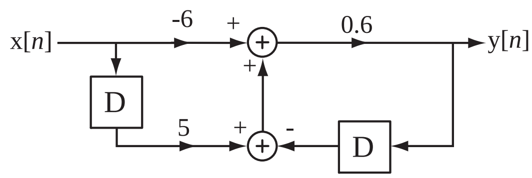
2. The systems represented by these block diagrams can each be described by a difference equation of the form,

$$a_N \mathrm{y}[n] + a_{N-1} \mathrm{y}[n-1] + \cdots + a_2 \mathrm{y}[n-(N-2)] + a_1 \mathrm{y}[n-(N-1)] + a_0 \mathrm{y}[n-N] =$$
$$b_M \mathrm{x}[n] + b_{M-1} \mathrm{x}[n-1] + \cdots + b_2 \mathrm{x}[n-(M-2)] + b_1 \mathrm{x}[n-(M-1)] + b_0[n-M] .$$

(a)



(b)



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For each system, is it stable?

Stable Unstable

(a) $N = 0$, $M = 3$, Stable

$$\mathrm{y}[n] = -6\mathrm{x}[n] + 7\mathrm{x}[n-1] - 5\mathrm{x}[n-2] + 2\mathrm{x}[n-3]$$

Eigenvalue from the homogeneous system equation ($\mathrm{y}[n] = 0$) is $\alpha = 0$ which is less in magnitude than 1. Therefore the system is stable. Another way of seeing this is if x is bounded then

$$-6\mathrm{x}[n] + 7\mathrm{x}[n-1] - 5\mathrm{x}[n-2] + 2\mathrm{x}[n-3]$$

is also bounded. Therefore y is bounded and the system is stable.

(b) $N = 1$, $M = 1$, Stable

$$\mathrm{y}[n] = 0.6\{-6\mathrm{x}[n] + (5\mathrm{x}[n-1] - \mathrm{y}[n-1])\}$$

$$\mathrm{y}[n]/0.6 + \mathrm{y}[n-1] = -6\mathrm{x}[n] + 5\mathrm{x}[n-1]$$

The eigenvalue is -0.6. $|-0.6| < 1$ so the system is stable.

3. Indicate the properties of these systems for which x is the excitation and y is the response.

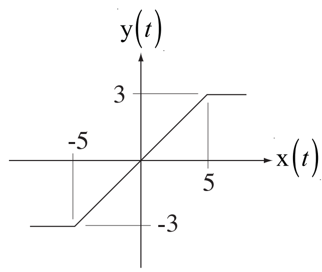
(a) $y(t) = \int_{-\infty}^{t} x(\lambda) d\lambda$

Linear, Unstable, Dynamic, Time Invariant, Causal, Invertible

(b) $y[n] = \begin{cases} n x[n] & , \ n > 0 \\ 0 & , \ \text{otherwise} \end{cases}$

Linear, Unstable, Static, Time Variant, Causal, Non-Invertible

(c) A system whose behavior at any time *t* is determined by this graph.



Non-Linear, Stable, Static, Time Invariant, Causal, Non-Invertible

4. Find the total numerical solution of this difference equation with initial conditions.

$$y[n]-0.2y[n-1]-0.35y[n-2]=5 \ , \ y[0]=1 \ , \ y[1]=4$$

The characteristic equation is $\alpha^2-0.2\alpha-0.35=0$ and the solutions, which are the eigenvalues, are

$$\alpha_{1,2}=0.7,-0.5 \ .$$

So the homogeneous solution is

$$y_h[n]=K_1(0.7)^n+K_2(-0.5)^n .$$

The forcing function is a constant so the particular solution is also a constant K_p.

$$K_p-0.2K_p-0.35K_p=5 \Rightarrow 0.45K_p=5 \Rightarrow K_p=11.111 \ .$$

The total solution is then

$$y[n]=K_1(0.7)^n+K_2(-0.5)^n+11.111.$$

Using the initial conditions,

$$y[0]=K_1+K_2+11.111=1$$

$$y[1]=0.7K_1-0.5K_2+11.111=4$$

or, in matrix form,

$$\begin{bmatrix} 1 & 1 \\ 0.7 & -0.5 \end{bmatrix}\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}=\begin{bmatrix} -10.111 \\ -7.111 \end{bmatrix}$$

Solving,

$$K_1=-10.1387 \ , K_2=0.0277$$

and, finally,

$$y[n]=-10.1387(0.7)^n+0.0277(-0.5)^n+11.111$$