Solution of ECE 315 Test 1 Su09

1. A signal $\mathrm{x}(t)$ is non-zero only in the time range $-2 < t < 3$. If $\mathrm{y}(t) = \mathrm{x}(2t+4)$ what is the numerical range of time over which $\mathrm{y}(t)$ is non-zero?

   The operation $t \to t+4$ shifts x to the left by 4. So the new limits would be at -6 and -1.
   The operation $t \to 2t$ applied to $t \to t+4$ yields $t \to 2t+4$ and compresses the shifted x in time by a factor of two. So the new limits now would be at -3 and -1/2. The validity of this analysis can be confirmed by letting $t = -1/2$ in $\mathrm{y}(t)$ yielding $\mathrm{y}(-1/2) = \mathrm{x}(2(-1/2)+4) = \mathrm{x}(+3)$ and $\mathrm{y}(-3) = \mathrm{x}(2(-3)+4) = \mathrm{x}(-2)$. So the range is $-3 < t < -1/2$.

2. A signal $\mathrm{x}[n]$ is non-zero only in the range $1 \le n < 14$. If $\mathrm{y}[n] = \mathrm{x}[3n]$ how many of the non- zero values of x appear in y?

   The only values of n in $\mathrm{y}[n]$ that correspond to non-zero values of $\mathrm{x}[n]$ in the range $1 \le n < 14$ are $n = 1 \Rightarrow 3n = 3,\ n = 2 \Rightarrow 3n = 6,\ n = 3 \Rightarrow 3n = 9,\ n = 4 \Rightarrow 3n = 12$. So the answer is four.

3. What is the numerical signal energy of

   (a) $\mathrm{x}[n] = -5\,\mathrm{u}[n+3]\mathrm{u}[4-n]$?

   $\mathrm{x}[n]$ is zero for $n < -3$ and for $n > 4$. Therefore we can express it as $\mathrm{x}[n] = \begin{cases} 0 & ,\ n < -3 \\ -5 & ,\ -3 \le n \le 4 \\ 0 & ,\ n > 4 \end{cases}$. Its energy is the sum of the squares of the magnitudes of its values which is $(-5)^2 \times 8 = 200$.

   (b) $\mathrm{x}[n] = -5\,\mathrm{u}[n-3]\mathrm{u}[4+n]$?

   For $n \ge 3$ $\mathrm{x}[n]$ is one. Therefore the signal energy is infinite. This is a power signal.

4. For each of these signals decide whether it is periodic and, if it is, find the numerical fundamental period.

(a) $\mathrm{x}(t)=\mathrm{x}_1(t)+\mathrm{x}_2(t)$ and $\mathrm{x}_1(t)$ is periodic with fundamental period 14 and $\mathrm{x}_2(t)$ is periodic with fundamental period 8.

The LCM of 14 and 8 is 56. So the fundamental period is 56.

(b) $\mathrm{x}(t)=4\cos(20t)-7\sin(32t)$

The two fundamental frequencies are $10/\pi$ and $16/\pi$. The greatest common divisor of those two numbers is $2/\pi$. So the fundamental frequency of x is $2/\pi$ and its fundamental period is $\pi/2$.

(c) $\mathrm{x}(t)=4\cos(8\pi t)-7\sin(12t)$

The two fundamental frequencies are 4 and $6/\pi$. These two numbers do not have a non-zero greatest common divisor. Therefore x is not periodic.

(d) $\mathrm{x}[n]=\cos(n/3)$

$\mathrm{x}[n]=\cos(2\pi(1/6\pi)n)$. $1/6\pi$ is not a rational number. Therefore x is not periodic.

(e) $\mathrm{x}[n]=\cos(7\pi n/3)-4\sin(11\pi n/5)$

$$\mathrm{x}[n]=\underbrace{\cos(2\pi(7/6)n)}_{N_{01}=6}-4\underbrace{\sin(2\pi(11/10)n)}_{N_{02}=10}$$

$$N_0=\mathrm{LCM}(N_{01},N_{02})=\mathrm{LCM}(6,10)=30$$

5. Find the numerical signal power of $\mathrm{x}[n]=\delta_3[n]-3\delta_9[n]-1$.

This signal is periodic with fundamental period 9.

$$P_{\mathrm{x}}=\frac{1}{N}\sum_{n=\langle N\rangle}\left|\mathrm{x}[n]\right|^2=\frac{1}{9}\sum_{n=0}^{8}\left(\delta_3[n]-3\delta_9[n]-1\right)^2$$
$$=\frac{1}{9}\sum_{n=0}^{8}\left(\delta_3^2[n]+9\delta_9^2[n]+1-6\delta_3[n]\delta_9[n]+6\delta_9[n]-2\delta_3[n]\right)$$

$$P_{\mathrm{x}}=\frac{1}{9}(3+9+9-6+6-6)=5/3$$

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1. A signal $\mathrm{x}(t)$ is non-zero only in the time range $-2 < t < 3$. If $\mathrm{y}(t) = \mathrm{x}(2t-4)$ what is the numerical range of time over which $\mathrm{y}(t)$ is non-zero?

   The operation $t \to t-4$ shifts x to the right by 4. So the new limits would be at +2 and +7.
   The operation $t \to 2t$ applied to $t \to t-4$ yields $t \to 2t-4$ and compresses the shifted x in time by a factor of two. So the new limits now would be at +1 and +7/2. The validity of this analysis can be confirmed by letting $t = +1$ in $\mathrm{y}(t)$ yielding $\mathrm{y}(1) = \mathrm{x}(2(1)-4) = \mathrm{x}(-2)$ and $\mathrm{y}(+7/2) = \mathrm{x}(2(+7/2)-4) = \mathrm{x}(+3)$. So the range is $+2 < t < +7/2$.

2. A signal $\mathrm{x}[n]$ is non-zero only in the range $0 \le n < 14$. If $\mathrm{y}[n] = \mathrm{x}[3n]$ how many of the non- zero values of x appear in y?

   The only values of n in $\mathrm{y}[n]$ that correspond to non-zero values of $\mathrm{x}[n]$ in the range $1 \le n < 14$ are $n = 0 \Rightarrow 2n = 0,\ n = 1 \Rightarrow 3n = 3,\ n = 2 \Rightarrow 3n = 6,\ n = 3 \Rightarrow 3n = 9,\ n = 4 \Rightarrow 3n = 12$. So the answer is five.

3. What is the numerical signal energy of

   (a) $\mathrm{x}[n] = -5\,\mathrm{u}[n+5]\,\mathrm{u}[4-n]$?

   $\mathrm{x}[n]$ is zero for $n < -5$ and for $n > 4$. Therefore we can express it as $\mathrm{x}[n] = \begin{cases} 0 & , n < -5 \\ -5 & , -3 \le n \le 4 \\ 0 & , n > 4 \end{cases}$. Its energy is the sum of the squares of the magnitudes of its values which is $(-5)^2 \times 10 = 250$.

   (b) $\mathrm{x}[n] = -5\,\mathrm{u}[n-5]\,\mathrm{u}[4+n]$?

   For $n \ge 5$ $\mathrm{x}[n]$ is one. Therefore the signal energy is infinite. This is a power signal.

4. For each of these signals decide whether it is periodic and, if it is, find the numerical fundamental period.

(a) $x(t) = x_1(t) + x_2(t)$ and $x_1(t)$ is periodic with fundamental period 16 and $x_2(t)$ is periodic with fundamental period 6.

The LCM of 16 and 6 is 48. So the fundamental period is 48.

(b) $x(t) = 4\cos(22t) - 7\sin(32t)$

The two fundamental frequencies are $11/\pi$ and $16/\pi$. The greatest common divisor of those two numbers is $1/\pi$. So the fundamental frequency of x is $1/\pi$ and its fundamental period is π.

(c) $x(t) = 4\cos(4\pi t) - 7\sin(12t)$

The two fundamental frequencies are 2 and $6/\pi$. These two numbers do not have a non-zero greatest common divisor. Therefore x is not periodic.

(d) $x[n] = \cos(n/5)$

$x[n] = \cos(2\pi(1/10\pi)n)$. $1/10\pi$ is not a rational number. Therefore x is not periodic.

(e) $x[n] = \cos(7\pi n/4) - 4\sin(11\pi n/5)$

$$x[n] = \underbrace{\cos(2\pi(7/8)n)}_{N_{01}=8} - 4\underbrace{\sin(2\pi(11/10)n)}_{N_{02}=10}$$

$$N_0 = \text{LCM}(N_{01}, N_{02}) = \text{LCM}(8,10) = 40$$

5. Find the numerical signal power of $x[n] = \delta_2[n] - 4\delta_8[n] - 1$.

This signal is periodic with fundamental period 8.

$$P_x = \frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \frac{1}{8}\sum_{n=0}^{7}(\delta_2[n] - 4\delta_8[n] - 1)^2$$

$$= \frac{1}{8}\sum_{n=0}^{7}(\delta_2^2[n] + 16\delta_8^2[n] + 1 - 8\delta_2[n]\delta_8[n] + 8\delta_8[n] - 2\delta_2[n])$$

$$P_x = \frac{1}{8}\sum_{n=0}^{7}(4 + 16 + 8 - 8 + 8 - 8) = 5/2$$

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1. A signal $x(t)$ is non-zero only in the time range $-2 < t < 3$. If $y(t) = x(3t+4)$ what is the numerical range of time over which $y(t)$ is non-zero?

   The operation $t \rightarrow t+4$ shifts x to the left by 4. So the new limits would be at -6 and -1.
   The operation $t \rightarrow 3t$ applied to $t \rightarrow t+4$ yields $t \rightarrow 3t+4$ and compresses the shifted x in time by a factor of three. So the new limits now would be at -2 and -1/3. The validity of this analysis can be confirmed by letting $t = -1/3$ in $y(t)$ yielding $y(-1/3) = x(3(-1/3)+4) = x(+3)$ and $y(-2) = x(3(-2)+4) = x(-2)$. So the range is $-2 < t < -1/3$.

2. A signal $x[n]$ is non-zero only in the range $4 \le n < 14$. If $y[n] = x[3n]$ how many of the non- zero values of x appear in y?

   The only values of n in $y[n]$ that correspond to non-zero values of $x[n]$ in the range $4 \le n < 14$ are $n = 2 \Rightarrow 3n = 6,\ n = 3 \Rightarrow 3n = 9,\ n = 4 \Rightarrow 3n = 12$. So the answer is three.

3. What is the numerical signal energy of

   (a) $x[n] = -5\,u[n+3]\,u[2-n]$?

   $x[n]$ is zero for $n < -3$ and for $n > 2$. Therefore we can express it as $x[n] = \begin{cases} 0 & , n < -3 \\ -5 & , -3 \le n \le 2 \\ 0 & , n > 2 \end{cases}$. Its energy is the sum of the squares of the magnitudes of its values which is $(-5)^2 \times 6 = 150$.

   (b) $x[n] = -5\,u[n-3]\,u[2+n]$?

   For $n \ge 3$ $x[n]$ is one. Therefore the signal energy is infinite. This is a power signal.

4. For each of these signals decide whether it is periodic and, if it is, find the numerical fundamental period.

(a) $x(t) = x_1(t) + x_2(t)$ and $x_1(t)$ is periodic with fundamental period 12 and $x_2(t)$ is periodic with fundamental period 20.

The LCM of 12 and 10 is 60. So the fundamental period is 60.

(b) $x(t) = 4\cos(24t) - 7\sin(32t)$

The two fundamental frequencies are $12/\pi$ and $16/\pi$. The greatest common divisor of those two numbers is $4/\pi$. So the fundamental frequency of x is $4/\pi$ and its fundamental period is $\pi/4$.

(c) $x(t) = 4\cos(12\pi t) - 7\sin(8t)$

The two fundamental frequencies are 6 and $4/\pi$. These two numbers do not have a non-zero greatest common divisor. Therefore x is not periodic.

(d) $x[n] = \cos(n/12)$

$x[n] = \cos(2\pi(1/24\pi)n)$. $1/24\pi$ is not a rational number. Therefore x is not periodic.

(e) $x[n] = \cos(7\pi n/8) - 4\sin(11\pi n/5)$

$$x[n] = \underbrace{\cos(2\pi(7/16)n)}_{N_{01}=16} - 4\underbrace{\sin(2\pi(11/10)n)}_{N_{02}=10}$$

$$N_0 = \text{LCM}(N_{01}, N_{02}) = \text{LCM}(16,10) = 80$$

5. Find the numerical signal power of $x[n] = \delta_5[n] - 2\delta_{11}[n] - 1$.

This signal is periodic with fundamental period 55.

$$P_x = \frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \frac{1}{55}\sum_{n=0}^{54}(\delta_5[n] - 2\delta_{11}[n] - 1)^2$$

$$= \frac{1}{55}\sum_{n=0}^{54}(\delta_5^2[n] + 4\delta_{11}^2[n] + 1 - 4\delta_5[n]\delta_{11}[n] + 4\delta_{11}[n] - 2\delta_5[n])$$

$$P_x = \frac{1}{55}\sum_{n=0}^{54}(11 + 20 + 55 - 4 + 20 - 22) = 16/11 \cong 1.4545$$