Solution of ECE 315 Test 2 Su09

In the system descriptions below x is always the excitation and y is always the response.

1. A system is described by $y[n]=\begin{cases}4 & , x[n]>3\\ x[n] & , -1\le x[n]<4\\ -1 & , x[n]<-1\end{cases}$. Circle the correct properties for this system.

If $x[n]=3$, $y[n]=3$ and if $x[n]=6$, $y[n]=4$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1[n]=g[n]$, then $y_1[n]=\begin{cases}4 & , g[n]>3\\ g[n] & , -1\le g[n]<4\\ -1 & , g[n]<-1\end{cases}$

If $x_2[n]=g[n-n_0]$, then $y_2[n]=\left\{\begin{array}{ll}4 & , g[n-n_0]>3\\ g[n-n_0] & , -1\le g[n-n_0]<4\\ -1 & , g[n-n_0]<-1\end{array}\right\}=y_1[n-n_0]\Rightarrow$ Time Invariant

The response never exceeds 4 in magnitude, regardless of the excitation. $\Rightarrow$ Stable

The response at time n depends only on the excitation at time n. $\Rightarrow$ Static

All static systems are causal. $\Rightarrow$ Causal

If the response is 4 or -1 there is no way to know what the excitation is. $\Rightarrow$ Not Invertible

2. A system is described by $y'(t+1)+x(t)=6$. Circle the correct properties for this system.

If $x(t)=0$, $y'(t+1)=6$ and if $x(t)=2$, $y'(t+1)=8$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1(t)=g(t)$, then $y_1'(t+1)=6-g(t)$

If $x_2(t)=g(t-t_0)$, then $y_2'(t+1)=6-g(t-t_0)=y_1'(t-t_0+1)\Rightarrow$ Time Invariant

If $x(t)$ is a non-zero constant then $y'(t+1)$ is also and $y(t)$ is not bounded. $\Rightarrow$ Unstable

The response at time $t+1$ depends on the excitation at time t. $\Rightarrow$ Dynamic

The response at time $t+1$ depends only on the excitation at the previous time t. $\Rightarrow$ Causal

$x(t)=6-y'(t+1)\Rightarrow$ Invertible

3. A system is described by $y[n]+1.8\,y[n-1]+1.62\,y[n-2]=x[n]$. Circle the correct properties for this system.

This equation is in the standard form for linear time invariant discrete-time systems therefore it is linear and time invariant.

The eigenvalues are $-0.9\pm j0.9$ with magntitude $1.2728>1.\Rightarrow$ Unstable

The response at time n depends on responses at previous times. $\Rightarrow$ Dynamic

The response at time n depends on responses at previous times. $\Rightarrow$ Causal

$y[n]+1.8\,y[n-1]+1.62\,y[n-2]=x[n]$ So knowing y allows us to compute x $\Rightarrow$ Invertible

4. If $\mathrm{h}(t) = 7\,\mathrm{ramp}(2t)$ and $\mathrm{x}(t) = 3\mathrm{u}(t)$ and $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t)$ find these numerical values.

(a) $\mathrm{x}(1) = 3 \;,\; \mathrm{h}(1) = 7\,\mathrm{ramp}(2) = 14 \Rightarrow \mathrm{x}(1)\mathrm{h}(1) = 42$

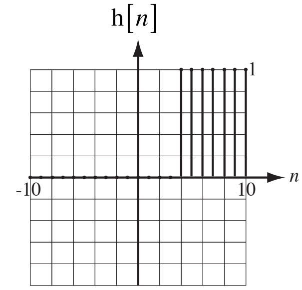
(b) $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau = 21\int_{-\infty}^{\infty} \mathrm{u}(\tau)\mathrm{ramp}(2(t-\tau))\,d\tau$

$$\mathrm{y}(t) = \left\{ \begin{array}{l} 0 \;,\; t<0 \\ 21\int_0^t 2(t-\tau)\,d\tau \;\;,\; t>0 \end{array} \right\} = \left\{ \begin{array}{l} 0 \;,\; t<0 \\ 21\left[2t\tau-\tau^2\right]_0^t \;\;,\; t>0 \end{array} \right\} = \left\{ \begin{array}{l} 0 \;,\; t<0 \\ 21t^2 \;\;,\; t>0 \end{array} \right\}$$

$$\mathrm{y}(1) = \left\{ \begin{array}{l} 0 \;,\; t<0 \\ 21(1)^2 \;\;,\; t>0 \end{array} \right\} = 21$$

Although this analytical solution is correct, it is probably easier to solve this problem graphically.

5. If a discrete-time system is described by $\mathrm{y}[n] = \sum_{m=-\infty}^{n-4} \mathrm{x}[m]$ graph its impulse response $\mathrm{h}[n]$ in the space provided. (Put a vertical scale on the graph so actual numerical values could be read.)



$$\mathrm{h}[n] = \sum_{m=-\infty}^{n-4} \delta[m] = \begin{Bmatrix} 0 & , & n < 4 \\ 1 & , & n \geq 4 \end{Bmatrix} = \mathrm{u}[n-4]$$

6. If a discrete-time system has an impulse response $\mathrm{h}[n] = -3\,\mathrm{ramp}[n-2]$ find its response to the excitation $\mathrm{x}[n] = \delta[n+3] - 2\delta[n+1]$ and fill in the table below with numbers.

$$\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n] = -3\,\mathrm{ramp}[n-2] * (\delta[n+3] - 2\delta[n+1])$$

$$\mathrm{y}[n] = -3\,\mathrm{ramp}[n+1] + 6\,\mathrm{ramp}[n-1]$$

n	0	1	2	3
$\mathrm{y}[n]$	-3	-6	-3	0

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In the system descriptions below x is always the excitation and y is always the response.

1. A system is described by $y[n-1]=\begin{cases}4 & , \ x[n]>3 \\ x[n] & , \ -1\le x[n]<4 \\ -1 & , \ x[n]<-1\end{cases}$. Circle the correct properties for this system.

If $x[n]=3$, $y[n-1]=3$ and if $x[n]=6$, $y[n-1]=4$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1[n]=g[n]$, then $y_1[n-1]=\begin{cases}4 & , \ g[n]>3 \\ g[n] & , \ -1\le g[n]<4 \\ -1 & , \ g[n]<-1\end{cases}$

If $x_2[n]=g[n-n_0]$, then $y_2[n-1]=\left\{\begin{matrix}4 & , \ g[n-n_0]>3 \\ g[n-n_0] & , \ -1\le g[n-n_0]<4 \\ -1 & , \ g[n-n_0]<-1\end{matrix}\right\}=y_1[n-n_0-1]\Rightarrow$ Time Invariant

The response never exceeds 4 in magnitude, regardless of the excitation. $\Rightarrow$ Stable

The response at time $n-1$ depends on the excitation at time n. $\Rightarrow$ Dynamic

The response at time $n-1$ depends on the excitation at time n. $\Rightarrow$ Non-Causal

If the response is 4 or -1 there is no way to know what the excitation is. $\Rightarrow$ Not Invertible

2. A system is described by $y'(t+1)+x(t)=6$. Circle the correct properties for this system.

If $x(t)=0$, $y'(t+1)=6$ and if $x(t)=2$, $y'(t+1)=8$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1(t)=g(t)$, then $y_1'(t+1)=6-g(t)$

If $x_2(t)=g(t-t_0)$, then $y_2'(t+1)=6-g(t-t_0)=y_1'(t-t_0+1)\Rightarrow$ Time Invariant

If $x(t)$ is a non-zero constant then $y'(t+1)$ is also and $y(t)$ is not bounded. $\Rightarrow$ Unstable

The response at time $t+1$ depends on the excitation at time t. $\Rightarrow$ Dynamic

The response at time $t+1$ depends only on the excitation at the previous time t. $\Rightarrow$ Causal

$x(t)=6-y'(t+1)\Rightarrow$ Invertible

3. A system is described by $y[n]-1.8\,y[n-1]+1.62\,y[n-2]=x[n]$. Circle the correct properties for this system.

This equation is in the standard form for linear time invariant discrete-time systems therefore it is linear and time invariant.

The eigenvalues are $0.9\pm j0.9$ with magntitude $1.2728>1.\Rightarrow$ Unstable

The response at time n depends on responses at previous times. $\Rightarrow$ Dynamic

The response at time n depends on responses at previous times. $\Rightarrow$ Causal

$y[n]-1.8\,y[n-1]+1.62\,y[n-2]=x[n]$ So knowing y allows us to compute x $\Rightarrow$ Invertible

4. If $\mathrm{h}(t) = 7\,\mathrm{ramp}(3t)$ and $\mathrm{x}(t) = 3\mathrm{u}(t)$ and $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t)$ find these numerical values.

(a) $\mathrm{x}(1) = 3$, $\mathrm{h}(1) = 7\,\mathrm{ramp}(3) = 21 \Rightarrow \mathrm{x}(1)\mathrm{h}(1) = 63$

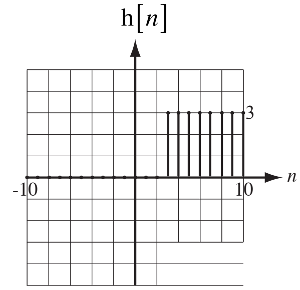
(b) $$\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau = 21\int_{-\infty}^{\infty} \mathrm{u}(\tau)\mathrm{ramp}(3(t-\tau))\,d\tau$$

$$\mathrm{y}(t) = \left\{\begin{matrix} 0 \ , \ t<0 \\ 21\int_0^t 3(t-\tau)\,d\tau \ , \ t>0 \end{matrix}\right\} = \left\{\begin{matrix} 0 \ , \ t<0 \\ 21\left[3t\tau - (3/2)\tau^2\right]_0^t \ , \ t>0 \end{matrix}\right\} = \left\{\begin{matrix} 0 \ , \ t<0 \\ 31.5t^2 \ , \ t>0 \end{matrix}\right\}$$

$$\mathrm{y}(1) = \left\{\begin{matrix} 0 \ , \ t<0 \\ 31.5(1)^2 \ , \ t>0 \end{matrix}\right\} = 31.5$$

Although this analytical solution is correct, it is probably easier to solve this problem graphically.

5. If a discrete-time system is described by $y[n] = 3\sum_{m=-\infty}^{n-3} x[m]$ graph its impulse response $h[n]$ in the space provided. (Put a vertical scale on the graph so actual numerical values could be read.)



$$h[n] = 3\sum_{m=-\infty}^{n-3} \delta[m] = \left\{ \begin{matrix} 0 & , n < 3 \\ 3 & , n \geq 3 \end{matrix} \right\} = 3u[n-3]$$

6. If a discrete-time system has an impulse response $h[n] = -5\,\text{ramp}[n-2]$ find its response to the excitation $x[n] = 2\delta[n+3] - 3\delta[n+1]$ and fill in the table below with numbers.

$$y[n] = x[n] * h[n] = -5\,\text{ramp}[n-2] * \left(2\delta[n+3] - 3\delta[n+1]\right)$$

$$y[n] = -10\,\text{ramp}[n+1] + 15\,\text{ramp}[n-1]$$

n	0	1	2	3
$y[n]$	-10	-20	-15	-10

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In the system descriptions below x is always the excitation and y is always the response.

1. A system is described by $y[n+1] = \begin{cases} 4 & , \ x[n] > 3 \\ x[n] & , \ -1 \le x[n] < 4 \\ -1 & , \ x[n] < -1 \end{cases}$. Circle the correct properties for this system.

If $x[n] = 3$, $y[n+1] = 3$ and if $x[n] = 6$, $y[n+1] = 4$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1[n] = g[n]$, then $y_1[n+1] = \begin{cases} 4 & , \ g[n] > 3 \\ g[n] & , \ -1 \le g[n] < 4 \\ -1 & , \ g[n] < -1 \end{cases}$

If $x_2[n] = g[n-n_0]$, then $y_2[n+1] = \left\{ \begin{matrix} 4 & , \ g[n-n_0] > 3 \\ g[n-n_0] & , \ -1 \le g[n-n_0] < 4 \\ -1 & , \ g[n-n_0] < -1 \end{matrix} \right\} = y_1[n-n_0+1] \Rightarrow$ Time Invariant

The response never exceeds 4 in magnitude, regardless of the excitation. $\Rightarrow$ Stable

The response at time $n+1$ depends on the excitation at time n. $\Rightarrow$ Dynamic

The response at time $n+1$ depends on the excitation at time n. $\Rightarrow$ Causal

If the response is 4 or -1 there is no way to know what the excitation is. $\Rightarrow$ Not Invertible

2. A system is described by $y'(t-1) + x(t) = 6$. Circle the correct properties for this system.

If $x(t) = 0$, $y'(t-1) = 6$ and if $x(t) = 2$, $y'(t-1) = 8$. Not homogeneous $\Rightarrow$ Not Linear.

If $x_1(t) = g(t)$, then $y_1'(t-1) = 6 - g(t)$

If $x_2(t) = g(t-t_0)$, then $y_2'(t-1) = 6 - g(t-t_0) = y_1'(t-t_0-1) \Rightarrow$ Time Invariant

If $x(t)$ is a non-zero constant then $y'(t-1)$ is also and $y(t)$ is not bounded. $\Rightarrow$ Unstable

The response at time $t-1$ depends on the excitation at time t. $\Rightarrow$ Dynamic

The response at time $t-1$ depends on the excitation at future time t. $\Rightarrow$ Non-Causal

$x(t) = 6 - y'(t-1) \Rightarrow$ Invertible

3. A system is described by $y[n] + 1.8\,y[n-1] + 1.62\,y[n-2] = x[n]$. Circle the correct properties for this system.

This equation is in the standard form for linear time invariant discrete-time systems therefore it is linear and time invariant.

The eigenvalues are $-0.9 \pm j0.9$ with magntitude $1.2728 > 1. \Rightarrow$ Unstable

The response at time n depends on responses at previous times. $\Rightarrow$ Dynamic

The response at time n depends on responses at previous times. $\Rightarrow$ Causal

$y[n] + 1.8\,y[n-1] + 1.62\,y[n-2] = x[n]$ So knowing y allows us to compute x $\Rightarrow$ Invertible

4. If $\mathrm{h}(t) = 7\,\mathrm{ramp}(2t)$ and $\mathrm{x}(t) = 5\,\mathrm{u}(t)$ and $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t)$ find these numerical values.

(a) $\mathrm{x}(1)\mathrm{h}(1) =$ ________________

$$\mathrm{x}(1) = 5 \;\;,\;\; \mathrm{h}(1) = 7\,\mathrm{ramp}(2) = 14 \Rightarrow \mathrm{x}(1)\mathrm{h}(1) = 70$$

(b) $\mathrm{y}(1) =$ ________________

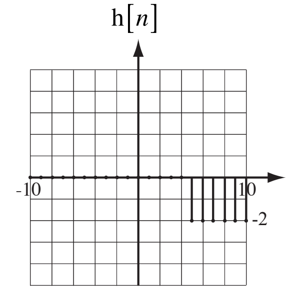
$$\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) = \int_{-\infty}^{\infty} \mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau = 35\int_{-\infty}^{\infty} \mathrm{u}(\tau)\mathrm{ramp}(2(t-\tau))\,d\tau$$

$$\mathrm{y}(t) = \left\{\begin{matrix} 0 \;\;,\;\; t<0 \\ 35\int_0^t 2(t-\tau)\,d\tau \;\;,\;\; t>0 \end{matrix}\right\} = \left\{\begin{matrix} 0 \;\;,\;\; t<0 \\ 35\left[2t\tau - \tau^2\right]_0^t \;\;,\;\; t>0 \end{matrix}\right\} = \left\{\begin{matrix} 0 \;\;,\;\; t<0 \\ 35t^2 \;\;,\;\; t>0 \end{matrix}\right\}$$

$$\mathrm{y}(1) = \left\{\begin{matrix} 0 \;\;,\;\; t<0 \\ 35(1)^2 \;\;,\;\; t>0 \end{matrix}\right\} = 35$$

Although this analytical solution is correct, it is probably easier to solve this problem graphically.

5. If a discrete-time system is described by $\mathrm{y}[n] = -2\sum_{m=-\infty}^{n-5} \mathrm{x}[m]$ graph its impulse response $\mathrm{h}[n]$ in the space provided. (Put a vertical scale on the graph so actual numerical values could be read.)



$$\mathrm{h}[n] = -2\sum_{m=-\infty}^{n-5} \delta[m] = \begin{Bmatrix} 0 & , & n < 5 \\ -2 & , & n \geq 5 \end{Bmatrix} = -2\,\mathrm{u}[n-5]$$

6. If a discrete-time system has an impulse response $\mathrm{h}[n] = -3\,\mathrm{ramp}[n-1]$ find its response to the excitation $\mathrm{x}[n] = \delta[n+3] - 2\delta[n+1]$ and fill in the table below with numbers.

$$\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n] = -3\,\mathrm{ramp}[n-1] * \left(\delta[n+3] - 2\delta[n+1]\right)$$

$$\mathrm{y}[n] = -3\,\mathrm{ramp}[n+2] + 6\,\mathrm{ramp}[n]$$

n	0	1	2	3
$\mathrm{y}[n]$				

n	0	1	2	3
$\mathrm{y}[n]$	-6	-3	0	3