Solution of ECE 315 Test 3 Su09

1. Supply the numerical value of the constant or constants in each Laplace transform pair.

(a) $4\,\mathrm{u}(t-3) \xleftrightarrow{\mathcal{L}} 4e^{as}/s \;\;,\;\; \sigma > 0$

$4\,\mathrm{u}(t-3) \xleftrightarrow{\mathcal{L}} 4e^{-3s}/s \;\;,\;\; \sigma > 0 \Rightarrow a = -3$

(b) $-2\sin(10\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{A}{s^2+as+b} \;\;,\;\; \sigma < 0$

$-2\sin(10\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{20\pi}{s^2+(10\pi)^2} \;\;,\;\; \sigma < 0 \Rightarrow A = 20\pi \;\;,\;\; a = 0 \;\;, b = 100\pi^2$

(c) $7\cos(4t-1)\mathrm{u}(t-1/4) \xleftrightarrow{\mathcal{L}} \dfrac{As}{s^2+as+b}e^{cs} \;\;,\;\; \sigma > 0$

$7\cos(4t-1)\mathrm{u}(t-1/4) = 7\cos(4(t-1/4))\mathrm{u}(t-1/4)$

$7\cos(4(t-1/4))\mathrm{u}(t-1/4) \xleftrightarrow{\mathcal{L}} \dfrac{7se^{-s/4}}{s^2+16} \;\;,\;\; \sigma > 0 \Rightarrow A = 7 \;, a = 0 \;,\; b = 16 \;, c = -1/4$

(d) $7e^{3t}\sin(2\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{A}{s^2+as+b} \;\;,\;\; \sigma < c$

$7e^{3t}\sin(2\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{-7(2\pi)}{(s-3)^2+(2\pi)^2} = \dfrac{-14\pi}{s^2-6s+9+(2\pi)^2} \;\;,\;\; \sigma < 3$

$A = -14\pi,\; a = -6 \;, b = 9+(2\pi)^2 \cong 48.48 \;\;,\;\; c = 3$

(e) $\left[A\,\mathrm{ramp}(at)+B+Ce^{ct}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 12\dfrac{s-1}{s^2(s+3)} \;\;,\;\; \sigma > 0$

$$12\frac{s-1}{s^2(s+3)} = 12\left[\frac{-1/3}{s^2}+\frac{4/9}{s}-\frac{4/9}{s+3}\right]$$

$12\left[-1/3\,\mathrm{ramp}(t)+(4/9)-(4/9)e^{-3t}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 12\dfrac{s-1}{s^2(s+3)} \;\;,\;\; \sigma > 0$

$\left[-4\,\mathrm{ramp}(t)+(16/3)-(16/3)e^{-3t}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 12\dfrac{s-1}{s^2(s+3)} \;\;,\;\; \sigma > 0$

$A = -4 \;, a = 1 \;, B = 16/3 \;, C = -16/3 \;, c = -3$

2. Supply the numerical value of the constants in each Fourier (CTFT) transform pair.

(a) $$Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} \frac{8}{j\omega+2}$$

$$8e^{-2t}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} \frac{8}{j\omega+2} \Rightarrow A = 8\ ,\ a = -2$$

(b) $$Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j8\omega+2}$$

$$Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{F}} \frac{1}{j8\omega+2} = \frac{1/8}{j\omega+1/4} \Rightarrow A = 1/8\ ,\ a = -1/4$$

(c) $$3\delta_4(t) \xleftrightarrow{\mathcal{F}} A\delta_a(f)$$

$$3\delta_4(t) \xleftrightarrow{\mathcal{F}} (3/4)\delta_{1/4}(f) \Rightarrow A = 3/4\ \ ,\ a = 1/4$$

(d) $$4\delta_3(5t) \xleftrightarrow{\mathcal{F}} A\delta_a(bf)$$

$$4\delta_3(t) \xleftrightarrow{\mathcal{F}} (4/3)\delta_{1/3}(f)$$
$$4\delta_3(5t) \xleftrightarrow{\mathcal{F}} (1/5)(4/3)\delta_{1/3}(f/5) = (4/15)\delta_{1/3}(f/5)$$
$$A = 4/15\ \ ,\ a = 1/3\ \ ,\ b = 1/5$$

(e) $$4\delta_3(5t) \xleftrightarrow{\mathcal{F}} A\delta_a(f)$$

(Hint: Use the definition of the continuous-time periodic impulse and continuous-time impulse properties.)

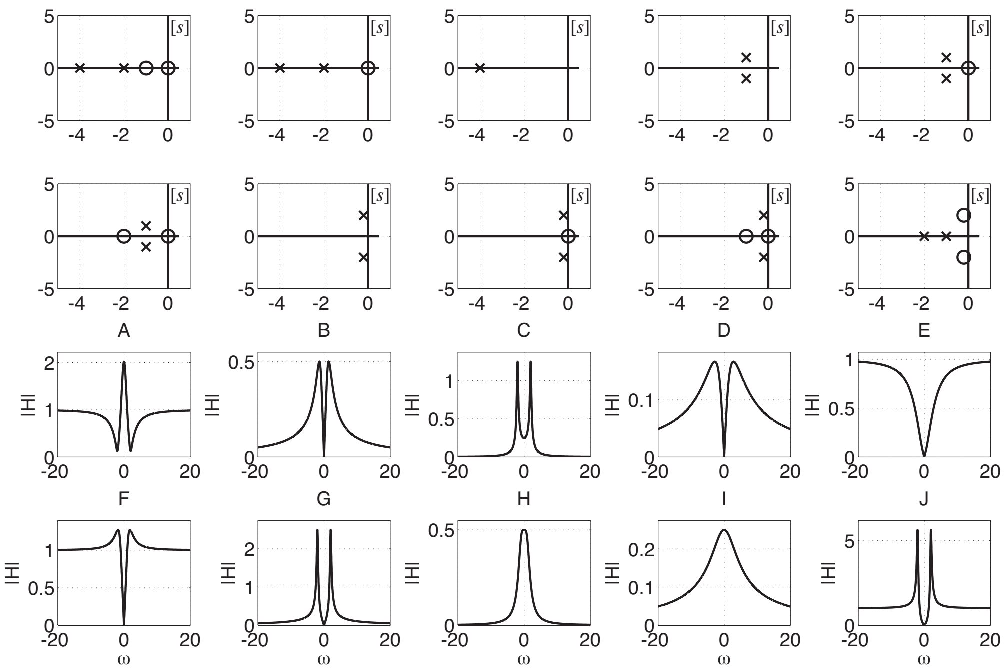
$$4\delta_3(5t) \xleftrightarrow{\mathcal{F}} (4/15)\delta_{1/3}(f/5) = (4/15)\sum_{k=-\infty}^{\infty}\delta(f/5 - k/3)$$

$$4\delta_3(5t) \xleftrightarrow{\mathcal{F}} (20/15)\sum_{k=-\infty}^{\infty}\delta(f-(5/3)k) = (4/3)\delta_{5/3}(f)$$

$$A = 4/3\ \ ,\ a = 5/3$$

3. Below are some pole-zero graphs and some magnitude frequency responses $\left|H(j\omega)\right|$. The transfer functions are all of the form $H(s) = A\dfrac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$ where $A = 1$. Match each pole-zero graph to its corresponding frequency response by writing the appropriate letter designation above the pole-zero graph.

E	D	I	H	B
F	C	G	J	A



Solution of ECE 315 Test 3 Su09

1. Supply the numerical value of the constant or constants in each Laplace transform pair.

(a) $4\,\mathrm{u}(t+1) \xleftrightarrow{\mathcal{L}} 4e^{as}/s \;,\; \sigma > 0$

$4\,\mathrm{u}(t+1) \xleftrightarrow{\mathcal{L}} 4e^{s}/s \;,\; \sigma > 0 \Rightarrow a = 1$

(b) $5\sin(30\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{A}{s^2+as+b} \;,\; \sigma < 0$

$5\sin(30\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{-150\pi}{s^2+(30\pi)^2} \;,\; \sigma < 0 \Rightarrow A = -150\pi \;,\; a = 0 \;,\; b = 900\pi^2$

(c) $3\cos(6t-1)\mathrm{u}(t-1/6) \xleftrightarrow{\mathcal{L}} \dfrac{As}{s^2+as+b}e^{cs} \;,\; \sigma > 0$

$3\cos(6t-1)\mathrm{u}(t-1/6) = 3\cos(6(t-1/6))\mathrm{u}(t-1/4)$

$3\cos(6(t-1/6))\mathrm{u}(t-1/6) \xleftrightarrow{\mathcal{L}} \dfrac{3se^{-s/6}}{s^2+36} \;,\; \sigma > 0 \Rightarrow A = 3 \;,\; a = 0 \;,\; b = 36 \;,\; c = -1/6$

(d) (10 pts) $11e^{-3t}\sin(\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{A}{s^2+as+b} \;,\; \sigma < c$

$11e^{-3t}\sin(\pi t)\mathrm{u}(-t) \xleftrightarrow{\mathcal{L}} \dfrac{-11(\pi)}{(s+3)^2+(\pi)^2} = \dfrac{-11\pi}{s^2+6s+9+(\pi)^2} \;,\; \sigma < -3$

$A = -11\pi,\; a = 6 \;,\; b = 9+(\pi)^2 \cong 18.87 \;,\; c = -3$

(e) $\left[A\,\mathrm{ramp}(at) + B + Ce^{ct}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 8\dfrac{s-3}{s^2(s+7)} \;,\; \sigma > 0$

$8\dfrac{s-3}{s^2(s+7)} = 8\left[\dfrac{-3/7}{s^2} + \dfrac{10/49}{s} - \dfrac{10/49}{s+3}\right]$

$8\left[-(3/7)\mathrm{ramp}(t) + (10/49) - (10/49)e^{-7t}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 8\dfrac{s-3}{s^2(s+7)} \;,\; \sigma > 0$

$\left[-(24/7)\mathrm{ramp}(t) + (80/49) - (80/49)e^{-7t}\right]\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} 8\dfrac{s-3}{s^2(s+7)} \;,\; \sigma > 0$

$A = -24/7 \;,\; a = 1 \;,\; B = 80/49 \;,\; C = -80/49 \;,\; c = -7$

2. Supply the numerical value of the constants in each Fourier (CTFT) transform pair.

(a) $Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \dfrac{17}{j\omega+6}$

$$8e^{-2t}\,\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \frac{17}{j\omega+6} \Rightarrow A = 17\ ,\ a = -6$$

(b) $Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \dfrac{1}{j17\omega+6}$

$$Ae^{at}\,\mathrm{u}(t) \xleftrightarrow{\mathscr{F}} \frac{1}{j17\omega+6} = \frac{1/17}{j\omega+6/17} \Rightarrow A = 1/17\ ,\ a = -6/17$$

(c) $9\delta_6(t) \xleftrightarrow{\mathscr{F}} A\delta_a(f)$

$$9\delta_6(t) \xleftrightarrow{\mathscr{F}} (9/6)\delta_{1/6}(f) \Rightarrow A = 3/2\ \ ,\ a = 1/6$$

(d) $5\delta_2(8t) \xleftrightarrow{\mathscr{F}} A\delta_a(bf)$

$$5\delta_2(t) \xleftrightarrow{\mathscr{F}} (5/2)\delta_{1/2}(f)$$
$$5\delta_2(8t) \xleftrightarrow{\mathscr{F}} (1/8)(5/2)\delta_{1/2}(f/8) = (5/16)\delta_{1/2}(f/8)$$
$$A = 5/16\ \ ,\ a = 1/5\ \ ,\ b = 1/8$$

(e) $5\delta_2(8t) \xleftrightarrow{\mathscr{F}} A\delta_a(f)$

(Hint: Use the definition of the continuous-time periodic impulse and continuous-time impulse properties.)

$$5\delta_2(8t) \xleftrightarrow{\mathscr{F}} = (5/16)\delta_{1/2}(f/8) = (5/16)\sum_{k=-\infty}^{\infty}\delta(f/8 - k/2)$$
$$5\delta_2(8t) \xleftrightarrow{\mathscr{F}} (5/2)\sum_{k=-\infty}^{\infty}\delta(f-4k) = (5/2)\delta_4(f)$$
$$A = 5/2\ \ ,\ a = 4$$

3. Below are some pole-zero graphs and some magnitude frequency responses $\left|H(j\omega)\right|$. The transfer functions are all of the form $H(s) = A\dfrac{(s-z_1)(s-z_2)\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$ where $A=1$. Match each pole-zero graph to its corresponding frequency response by writing the appropriate letter designation above the pole-zero graph.

C H G B F

A J E I D