

## Solution of ECE 315 Test 1 F05

1. Let  $X(f) = \frac{jf}{jf + 10}$ .

- (a) Find the numerical values of the magnitude  $|X(4)|$  and the angle  $\angle X(4)$  in radians

$$X(f) = \frac{jf}{jf + 10} \Rightarrow X(4) = \frac{j4}{j4 + 10} = \frac{4e^{j\pi/2}}{10.77e^{j0.3805}} = 0.3714e^{j1.19}$$

$$|X(4)| = 0.3714 \quad \angle X(4) = 1.19$$

- (b) What numerical value (in radians) does  $\angle X(f)$  approach as  $f$  approaches zero from the positive side?

$$\angle X(f) = \lim_{f \rightarrow 0^+} [\angle jf - \angle (jf + 10)] = \pi/2 - 0 = \pi/2$$

2. Let  $e^{(a-jb)x} = \alpha^x$ . Write the real and imaginary parts of  $\alpha$  in terms of  $a$  and  $b$ .

$$\underline{\text{Re}(\alpha) = e^a \cos(-b)} \quad \underline{\text{Im}(\alpha) = e^a \sin(-b)}$$

$$e^{(a-jb)x} = \alpha^x \Rightarrow \alpha = e^{a-jb} = e^a [\cos(-b) + j \sin(-b)]$$

$$\alpha = e^a \cos(-b) + je^a \sin(-b) \Rightarrow \text{Re}(\alpha) = e^a \cos(-b) \quad , \quad \text{Im}(\alpha) = e^a \sin(-b)$$

3. What are the numerical eigenvalues of the difference equation

$$4y[n] - 2y[n-1] + y[n-2] = \cos(2\pi n/12)?$$

Characteristic equation is  $4a^2 - 2a + 1 = 0$ . Therefore the eigenvalues are the solutions of the equation which are

$$a_{1,2} = \frac{2 \pm \sqrt{4-16}}{8} = 0.25 \pm j0.433 = 0.5e^{j\pm 1.0472}$$

## Solution of ECE 315 Test 1 F05

1. Let  $X(f) = \frac{jf}{jf + 3}$ .

- (a) Find the numerical values of the magnitude  $|X(4)|$  and the angle  $\angle X(4)$  in radians

$$X(f) = \frac{jf}{jf + 3} \Rightarrow X(4) = \frac{j4}{j4 + 3} = \frac{4e^{j\pi/2}}{5e^{j0.927}} = 0.8e^{j0.644}$$

$$|X(4)| = 0.8 \quad \angle X(4) = 0.644$$

- (b) What numerical value (in radians) does  $\angle X(f)$  approach as  $f$  approaches zero from the negative side?

$$\lim_{f \rightarrow 0^-} \angle X(f) = \underline{\hspace{2cm}}$$

$$\angle X(f) = \lim_{f \rightarrow 0^-} [\angle jf - \angle (jf + 3)] = -\pi/2 - 0 = -\pi/2$$

2. Let  $e^{(a-jb)x} = \alpha^x$ . Write the real and imaginary parts of  $\alpha$  in terms of  $a$  and  $b$ .

$$\underline{\text{Re}(\alpha) = e^a \cos(-b)} \quad \underline{\text{Im}(\alpha) = e^a \sin(-b)}$$

$$e^{(a-jb)x} = \alpha^x \Rightarrow \alpha = e^{a-jb} = e^a [\cos(-b) + j \sin(-b)]$$

$$\alpha = e^a \cos(-b) + je^a \sin(-b) \Rightarrow \text{Re}(\alpha) = e^a \cos(-b) \quad , \quad \text{Im}(\alpha) = e^a \sin(-b)$$

3. What are the numerical eigenvalues of the difference equation

$$2y[n] - 4y[n-1] + y[n-2] = \cos(2\pi n/12)?$$

Characteristic equation is  $2a^2 - 4a + 1 = 0$ . Therefore the eigenvalues are the solutions of the equation which are

$$a_{1,2} = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm 0.707 = 1.707, 0.293$$