Solution of ECE 315 Test 1 F06

1. A discrete-time system is described by the difference equation

$$7\,\mathrm{y}[n]-3\mathrm{y}[n-1]+\mathrm{y}[n-2]=11\,.$$

(a) The eigenvalues of this difference equation can be expressed in the polar form $Ae^{j\theta}$ where A is the magnitude and θ is the angle or phase. Find the numerical values of A and θ.

A = <u>0.378</u> θ = <u>0.968</u> (radians)

A = <u>0.378</u> θ = <u>-0.968</u> (radians)

The characteristic equation is $7\alpha^2-3\alpha+1=0$. The eigenvalues are the solutions to this equation which are

$$\frac{3\pm\sqrt{9-28}}{14}=\frac{3\pm j\sqrt{19}}{14}=0.2143\pm j0.3113=0.378e^{\pm j0.968}\,.$$

(b) The homogeneous solution approaches zero as $n\to\infty$. What numerical value does $\mathrm{y}[n]$ approach as $n\to\infty$?

$\mathrm{y}[\infty]$ = <u>2.2</u>

Since the forcing function is a constant the forced solution is also a constant K. Therefore

$7K-3K+K=11\Rightarrow K=2.2$ and the final value of y is also 2.2.

2. A complex number z has five fifth roots, $\{z_1,z_2,z_3,z_4,z_5\}$ and $z_1=Ae^{j3\pi/4}$.

(a) The number z can be expressed in the rectangular form $z=x+jy$. Find the value of x and y in terms of the unknown A.

x = <u>$0.707A^5$</u> , y = <u>$-0.707A^5$</u>

$$z=z_1^5=\left(Ae^{j3\pi/4}\right)^5=A^5e^{j15\pi/4}=A^5e^{-j\pi/4}e^{j16\pi/4}=A^5\cos(-\pi/4)+jA^5\sin(-\pi/4)=0.707A^5-j0.707A^5$$

$$x=0.707A^5 \text{ and } y=-0.707A^5$$

(b) The other four roots $\{z_2,z_3,z_4,z_5\}$ can be expressed in the polar form $Ae^{j\theta}$. Find the numerical values of the angles θ of the other four roots (all in radians).

θ = <u>3.6128</u> , <u>4.8695</u> , <u>6.1261</u> , <u>7.3827</u>

The spacing between the roots must be $2\pi/5$ radians. So the angles of the other four roots are

$$3\pi/4+2\pi/5=23\pi/20=3.6128 \text{ or } -2.6074 \text{ or}$$

$$3\pi / 4 + 4\pi / 5 = 31\pi / 20 = 4.8695 \text{ or } -1.4137 \text{ or}$$

$$3\pi / 4 + 6\pi / 5 = 39\pi / 20 = 6.1261 \text{ or } -0.1571 \text{ or}$$

$$3\pi / 4 + 8\pi / 5 = 47\pi / 20 = 7.3827 \text{ or } 1.0996 \text{ or } -5.1836 \text{ or}$$

Solution of ECE 315 Test 1 F06

1. A discrete-time system is described by the difference equation

$$8\,\mathrm{y}[n] - 2\,\mathrm{y}[n-1] + \mathrm{y}[n-2] = 11 .$$

(a) The eigenvalues of this difference equation can be expressed in the polar form $Ae^{j\theta}$ where A is the magnitude and θ is the angle or phase. Find the numerical values of A and θ.

A = <u>0.3536</u> θ = <u>1.2094</u> (radians)

A = <u>0.3536</u> θ = <u>-1.2094</u> (radians)

The characteristic equation is $8\alpha^2 - 2\alpha + 1 = 0$. The eigenvalues are the solutions to this equation which are

$$\frac{2 \pm \sqrt{4-32}}{16} = \frac{2 \pm j2\sqrt{7}}{16} = 0.125 \pm j0.3307 = 0.3536 e^{\pm j1.2094} .$$

(b) The homogeneous solution approaches zero as $n \to \infty$. What numerical value does $\mathrm{y}[n]$ approach as $n \to \infty$?

$\mathrm{y}[\infty]$ = <u>1.5714</u>

Since the forcing function is a constant the forced solution is also a constant K. Therefore

$8K - 2K + K = 11 \Rightarrow K = 11/7 = 1.5714$ and the final value of y is also 1.5714.

2. A complex number z has five fifth roots, $\{z_1, z_2, z_3, z_4, z_5\}$ and $z_1 = Ae^{j3\pi/8}$.

(a) The number z can be expressed in the rectangular form $z = x + jy$. Find the value of x and y in terms of the unknown A.

x = <u>$0.9239A^5$</u>, y = <u>$-0.3827A^5$</u>

$$z = z_1^5 = \left(Ae^{j3\pi/8}\right)^5 = A^5 e^{j15\pi/8} = A^5 e^{-j\pi/8} e^{j16\pi/8} = A^5 \cos(-\pi/8) + jA^5 \sin(-\pi/8) = 0.9239A^5 - j0.3827A^5$$

$$x = 0.9239A^5 \text{ and } y = -0.3827A^5$$

(b) The other four roots $\{z_2, z_3, z_4, z_5\}$ can be expressed in the polar form $Ae^{j\theta}$. Find the numerical values of the angles θ of the other four roots (all in radians).

θ = <u>2.4347</u> , <u>3.6914</u> , <u>4.948</u> , <u>6.2046</u>

The spacing between the roots must be $2\pi/5$ radians. So the angles of the other four roots are

$$3\pi / 8 + 2\pi / 5 = 31\pi / 40 = 2.4347 \text{ or } -3.8485 \text{ or}$$

$$3\pi / 8 + 4\pi / 5 = 47\pi / 40 = 3.6914 \text{ or } -2.5918 \text{ or}$$

$$3\pi / 8 + 6\pi / 5 = 63\pi / 40 = 4.948 \text{ or } -1.3352 \text{ or}$$

$$3\pi / 8 + 8\pi / 5 = 79\pi / 40 = 6.2046 \text{ or } -0.0785 \text{ or } -6.3617 \text{ or}$$

Solution of ECE 315 Test 1 F06

1. A discrete-time system is described by the difference equation

$$8\,\mathrm{y}[n]-3\,\mathrm{y}[n-1]+2\,\mathrm{y}[n-2]=14\,.$$

(a) The eigenvalues of this difference equation can be expressed in the polar form $Ae^{j\theta}$ where A is the magnitude and θ is the angle or phase. Find the numerical values of A and θ.

A = <u>0.5</u> θ = <u>1.1864</u> (radians)

A = <u>0.5</u> θ = <u>-1.1864</u> (radians)

The characteristic equation is $8\alpha^2-3\alpha+2=0$. The eigenvalues are the solutions to this equation which are

$$\frac{3\pm\sqrt{9-64}}{16}=\frac{3\pm j\sqrt{55}}{16}=0.1875\pm j0.4635=0.5e^{\pm j1.1864}\,.$$

(b) The homogeneous solution approaches zero as $n\to\infty$. What numerical value does $\mathrm{y}[n]$ approach as $n\to\infty$?

$\mathrm{y}[\infty]$ = <u>2</u>

Since the forcing function is a constant the forced solution is also a constant K. Therefore

$8K-3K+2K=14\Rightarrow K=14/7=2$ and the final value of y is also 2.

2. A complex number z has five fifth roots, $\{z_1,z_2,z_3,z_4,z_5\}$ and $z_1=Ae^{-j3\pi/8}$.

(a) The number z can be expressed in the rectangular form $z=x+jy$. Find the value of x and y in terms of the unknown A.

x = <u>$0.9239A^5$</u>, y = <u>$0.3827A^5$</u>

$$z=z_1^5=\left(Ae^{-j3\pi/8}\right)^5=A^5e^{-j15\pi/8}=A^5e^{j\pi/8}e^{-j16\pi/8}=A^5\cos(\pi/8)+jA^5\sin(\pi/8)=0.9239A^5+j0.3827A^5$$

$$x=0.9239A^5 \text{ and } y=0.3827A^5$$

(b) The other four roots $\{z_2,z_3,z_4,z_5\}$ can be expressed in the polar form $Ae^{j\theta}$. Find the numerical values of the angles θ of the other four roots (all in radians).

θ = <u>0.0785</u> , <u>1.3352</u> , <u>2.5918</u> , <u>3.8485</u>

The spacing between the roots must be $2\pi/5$ radians. So the angles of the other four roots are

$-3\pi / 8 + 2\pi / 5 = \pi / 40 = 0.0785$ or -6.2046 or

$-3\pi / 8 + 4\pi / 5 = 17\pi / 40 = 1.3352$ or -4.948 or

$-3\pi / 8 + 6\pi / 5 = 33\pi / 40 = 2.5918$ or -3.6914 or

$-3\pi / 8 + 8\pi / 5 = 49\pi / 40 = 3.8485$ or -2.4347 or