Solution of ECE 315 Test 12 F08

1. A continuous-time signal is zero for all time before $t = -2$, rises linearly from 0 to 3 between $t = -2$ and $t = 4$ and is zero for all time after that. This signal can be expressed in the form

$$\mathrm{x}(t) = A\,\mathrm{rect}\left(\frac{t - t_{01}}{w_1}\right)\mathrm{tri}\left(\frac{t - t_{02}}{w_2}\right).$$

(a) Find the numerical values of the constants.

$$\mathrm{x}(t) = 3\,\mathrm{rect}\left(\frac{t-1}{6}\right)\mathrm{tri}\left(\frac{t-4}{6}\right)$$

(b) Find the numerical value of the signal energy of this signal.

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty} |\mathrm{x}(t)|^2\,dt = \int_{-2}^{4} \left(\frac{t+2}{2}\right)^2 dt = (1/4)\int_{-2}^{4} \left(t^2 + 4t + 4\right)dt$$

$$E_{\mathrm{x}} = (1/4)\left[\frac{t^3}{3} + 2t^2 + 4t\right]_{-2}^{4} = (1/4)\left[\frac{64}{3} + 32 + 16 + \frac{8}{3} - 8 + 8\right] = 18$$

2. A discrete-time signal has the following values for times $n = -8$ to $n = 8$ and is zero for all other times.

n	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
$\mathrm{x}[n]$	9	4	9	9	4	9	9	4	9	9	4	9	9	4	9	9	4

This signal can be expressed in the form $\mathrm{x}[n] = \left(A - B\delta_{N_1}[n - n_0]\right)\mathrm{rect}_{N_2}[n]$.

(a) Find the numerical values of the constants.

$$\mathrm{x}[n] = \left(9 - 5\delta_3[n+1]\right)\mathrm{rect}_8[n]$$

or

$$\mathrm{x}[n] = \left(9 - 5\delta_3[n-2]\right)\mathrm{rect}_8[n]$$

(b) Let $\mathrm{y}[n]$ be a periodic signal with fundamental period 17 and let

$$\mathrm{y}[n] = \mathrm{x}[n] \quad , \quad -8 \le n \le 8$$

Find the numerical value of the signal power of $\mathrm{y}[n]$.

$$P_{\mathrm{y}} = (1/N_0)\sum_{n=\langle N_0\rangle} |\mathrm{y}[n]|^2 = (1/17)\left[9^2 \times 11 + 4^2 \times 6\right] = 58.06$$

3. A discrete-time system is described by the difference equation $\mathrm{y}[n]-0.95\,\mathrm{y}[n-2]=\mathrm{x}[n]$ where $\mathrm{x}[n]$ is the excitation and $\mathrm{y}[n]$ is the response.

(a) Fill in the table below with numbers.

n	0	1	2	3	4
$\mathrm{h}[n]$	1	0	0.95	0	0.9025

(b) What is the numerical value of $\mathrm{h}[64]$?

The eigenvalues are $\pm\sqrt{0.95}$.

$$\mathrm{h}[n]=\left[K_1\left(\sqrt{0.95}\right)^n+K_2\left(-\sqrt{0.95}\right)^n\right]\mathrm{u}[n]$$

$$\begin{array}{c}\mathrm{h}[0]=1=K_1+K_2\\ \mathrm{h}[1]=0=K_1\sqrt{0.95}-K_2\sqrt{0.95}\end{array}\Rightarrow K_1=K_2=0.5$$

$$\mathrm{h}[n]=0.5\left[\left(\sqrt{0.95}\right)^n+\left(-\sqrt{0.95}\right)^n\right]\mathrm{u}[n]$$

$$\mathrm{h}[64]=0.5\left[\left(\sqrt{0.95}\right)^{64}+\left(-\sqrt{0.95}\right)^{64}\right]\mathrm{u}[n]=0.95^{32}=0.194$$

4. When a continuous-time system with impulse response $\mathrm{h}(t)=5\,\mathrm{rect}(t)$ is excited by $\mathrm{x}(t)=4\,\mathrm{rect}(2t)$ the response is $\mathrm{y}(t)=15\,\mathrm{tri}(4t/3)-5\,\mathrm{tri}(4t)$.

(a) Change the excitation to $\mathrm{x}_a(t)=\mathrm{x}(t-1)$ and keep the same impulse response. What is the numerical value of the new response $\mathrm{y}_a(t)$ at time $t=1/2$?

Shifting the excitation shifts the response by the same time. Therefore

$$\mathrm{y}_a(t)=15\,\mathrm{tri}\left(4(t-1)/3\right)-5\,\mathrm{tri}\left(4(t-1)\right)$$

$$\mathrm{y}_a(-1/2)=15\underbrace{\mathrm{tri}(-2/3)}_{=1/3}-5\underbrace{\mathrm{tri}(-2)}_{=0}=5$$

(b) Change the excitation to $\mathrm{x}_b(t)=\frac{d}{dt}\mathrm{x}(t)$ and keep the same impulse response. What is the numerical value of the new response $\mathrm{y}_b(t)$ at time $t=1/2$?

Differentiating the excitation differentiates the response. Therefore

$$\mathrm{y}_b(t)=20\,\mathrm{rect}\left(2(t+1/2)\right)-20\,\mathrm{rect}\left(2(t-1/2)\right)$$

$$\mathrm{y}_b(1/2)=20\,\mathrm{rect}(2)-20\,\mathrm{rect}(0)=-20$$

5. A signal $\mathrm{x}(t)$ has a CTFT $\mathrm{X}(f)=3\,\mathrm{sinc}(2f)$.

(a) If $\mathrm{y}(t)=\mathrm{x}(t/2)$ write $\mathrm{Y}(f)$.

$\mathrm{Y}(f) = 6\,\mathrm{sinc}(4f)$

(b) If $\mathrm{y}(t) = \frac{d}{dt}\mathrm{x}(t)$ write $\mathrm{Y}(f)$.

$\mathrm{Y}(f) = j2\pi f \times 3\,\mathrm{sinc}(2f) = j6\pi f\,\mathrm{sinc}(2f)$

(c) If $\mathrm{y}(t) = \mathrm{x}(t+2)$ write $\mathrm{Y}(j\omega)$.

$\mathrm{Y}(f) = 3\,\mathrm{sinc}(2f)e^{j4\pi f} \Rightarrow \mathrm{Y}(j\omega) = 3\,\mathrm{sinc}(2\omega/2\pi)e^{j2\omega} = 3\,\mathrm{sinc}(\omega/\pi)e^{j2\omega}$

6. A signal $\mathrm{x}[n]$ has a DTFT $\mathrm{X}(F)$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	-2	-1	0	1	2	3	4	5	6
$\mathrm{x}[n]$	-8	2	1	-5	7	9	8	2	3

Let $\mathrm{Y}(F) = \mathrm{X}(2F)$ with $\mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{Y}(F)$. Fill in numerical values of $\mathrm{y}[n]$ in the table below.

Using

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}[n/m] & , \ n/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases} \quad , \quad \mathrm{z}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(mF)$$

n	-2	-1	0	1	2	3
$\mathrm{y}[n]$	2	0	1	0	-5	0

7. An LTI system has a frequency response $\mathrm{H}(j\omega) = \dfrac{1}{j\omega - j6} + \dfrac{1}{j\omega + j6}$.

(a) Find an expression for its impulse response $\mathrm{h}(t)$ which does not contain the square root of minus one (j).

$$\mathrm{h}(t) = e^{j6t}\,\mathrm{u}(t) + e^{-j6t}\,\mathrm{u}(t) = 2\cos(6t)\mathrm{u}(t)$$

(b) Is this system stable?

No

Explain how you know.

The impulse response is not absolutely integrable.

8. A discrete-time signal $\mathrm{x}[n]$ with fundamental period $N_0 = 4$ has a DTFS harmonic function $\mathrm{X}[k]$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	2	3	4	5
$\mathrm{x}[n]$	–1	5	2	–3

(a) Using $\mathrm{X}[k] = \dfrac{1}{N_0} \sum_{n=\langle N_0 \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_0}$ fill in the numerical values of $\mathrm{X}[k]$ in the table below.

$$\mathrm{X}[0] = \frac{1}{4}\sum_{n=\langle 4 \rangle} \mathrm{x}[n] = \frac{2-3-1+5}{4} = 3/4$$

$$\mathrm{X}[1] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\big(2 + (-3)(-j) + (-1)(-1) + 5(j)\big) = \frac{3+j8}{4} = 0.75 + j2$$

$$\mathrm{X}[2] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\big(2 + (-3)(-1) + (-1)(1) + 5(-1)\big) = -1/4$$

$$\mathrm{X}[1] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = \frac{1}{4}\big(2 + (-3)(j) + (-1)(-1) + 5(-j)\big) = \frac{3-j8}{4} = 0.75 - j2$$

k	0	1	2	3
$\mathrm{X}[k]$	$3/4 = 0.75$	$0.75 + j2$	$-1/4 = -0.25$	$0.75 - j2$

(b) If $\mathrm{y}[n] = \mathrm{x}[n-8]$, what is the numerical value of the harmonic function $\mathrm{Y}[k]$ of $\mathrm{y}[n]$ at $k = 1$?

$\mathrm{Y}[1] = \mathrm{X}[1] = 0.75 + j2$ because a shift of x by exactly two fundamental periods does not change it.

Solution of ECE 315 Test 12 F08

1. A continuous-time signal is zero for all time before $t=-2$, rises linearly from 0 to 4 between $t=-2$ and $t=6$ and is zero for all time after that. This signal can be expressed in the form

$$\mathrm{x}(t)=A\,\mathrm{rect}\left(\frac{t-t_{01}}{w_1}\right)\mathrm{tri}\left(\frac{t-t_{02}}{w_2}\right).$$

(a) Find the numerical values of the constants.

$$\mathrm{x}(t)=4\,\mathrm{rect}\left(\frac{t-2}{8}\right)\mathrm{tri}\left(\frac{t-6}{8}\right)$$

(b) Find the numerical value of the signal energy of this signal.

$$E_{\mathrm{x}}=\int_{-\infty}^{\infty}|\mathrm{x}(t)|^2\,dt=\int_{-2}^{6}\left(\frac{t+2}{2}\right)^2 dt=(1/4)\int_{-2}^{6}\left(t^2+4t+4\right)dt$$

$$E_{\mathrm{x}}=(1/4)\left[\frac{t^3}{3}+2t^2+4t\right]_{-2}^{6}=(1/4)\left[\frac{216}{3}+72+24+\frac{8}{3}-8+8\right]=\frac{170.667}{4}=42.67$$

2. A discrete-time signal has the following values for times $n=-8$ to $n=8$ and is zero for all other times.

n	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
$\mathrm{x}[n]$	1	7	7	1	7	7	1	7	7	1	7	7	1	7	7	1	7

This signal can be expressed in the form $\mathrm{x}[n]=\left(A-B\delta_{N_1}[n-n_0]\right)\mathrm{rect}_{N_2}[n]$.

(a) Find the numerical values of the constants.

$$\mathrm{x}[n]=\left(7-6\delta_3[n+2]\right)\mathrm{rect}_8[n]$$

or

$$\mathrm{x}[n]=\left(7-6\delta_3[n-1]\right)\mathrm{rect}_8[n]$$

(b) Let $\mathrm{y}[n]$ be a periodic signal with fundamental period 17 and let

$$\mathrm{y}[n]=\mathrm{x}[n]\quad,\quad -8\le n\le 8$$

Find the numerical value of the signal power of $\mathrm{y}[n]$.

$$P_{\mathrm{y}}=(1/N_0)\sum_{n=\langle N_0\rangle}|\mathrm{y}[n]|^2=(1/17)\left[7^2\times 11+1^2\times 6\right]=32.06$$

3. A discrete-time system is described by the difference equation $\mathrm{y}[n]-0.93\mathrm{y}[n-2]=\mathrm{x}[n]$ where $\mathrm{x}[n]$ is the excitation and $\mathrm{y}[n]$ is the response.

(a) Fill in the table below with numbers.

n	0	1	2	3	4
$\mathrm{h}[n]$	1	0	0.93	0	0.8649

(b) What is the numerical value of $\mathrm{h}[64]$?

The eigenvalues are $\pm\sqrt{0.93}$.

$$\mathrm{h}[n]=\left[K_1\left(\sqrt{0.93}\right)^n+K_2\left(-\sqrt{0.93}\right)^n\right]\mathrm{u}[n]$$

$$\begin{matrix}\mathrm{h}[0]=1=K_1+K_2 \\ \mathrm{h}[1]=0=K_1\sqrt{0.93}-K_2\sqrt{0.93}\end{matrix} \quad \Rightarrow K_1=K_2=0.5$$

$$\mathrm{h}[n]=0.5\left[\left(\sqrt{0.93}\right)^n+\left(-\sqrt{0.93}\right)^n\right]\mathrm{u}[n]$$

$$\mathrm{h}[64]=0.5\left[\left(\sqrt{0.93}\right)^{64}+\left(-\sqrt{0.93}\right)^{64}\right]\mathrm{u}[n]=0.93^{32}=0.0981$$

4. When a continuous-time system with impulse response $\mathrm{h}(t)=4\,\mathrm{rect}(t)$ is excited by $\mathrm{x}(t)=4\,\mathrm{rect}(2t)$ the response is $\mathrm{y}(t)=12\,\mathrm{tri}(4t/3)-4\,\mathrm{tri}(4t)$.

(a) Change the excitation to $\mathrm{x}_a(t)=\mathrm{x}(t-1)$ and keep the same impulse response. What is the numerical value of the new response $\mathrm{y}_a(t)$ at time $t=1/2$?

Shifting the excitation shifts the response by the same time. Therefore

$$\mathrm{y}_a(t)=12\,\mathrm{tri}(4(t-1)/3)-4\,\mathrm{tri}(4(t-1))$$

$$\mathrm{y}_a(-1/2)=12\underbrace{\mathrm{tri}(-2/3)}_{=1/3}-4\underbrace{\mathrm{tri}(-2)}_{=0}=4$$

(b) Change the excitation to $\mathrm{x}_b(t)=\frac{d}{dt}\mathrm{x}(t)$ and keep the same impulse response. What is the numerical value of the new response $\mathrm{y}_b(t)$ at time $t=1/2$?

Differentiating the excitation differentiates the response. Therefore

$$\mathrm{y}_b(t)=16\,\mathrm{rect}(2(t+1/2))-16\,\mathrm{rect}(2(t-1/2))$$

$$\mathrm{y}_b(1/2)=16\,\mathrm{rect}(2)-16\,\mathrm{rect}(0)=-16$$

5. A signal $\mathrm{x}(t)$ has a CTFT $\mathrm{X}(f)=8\,\mathrm{sinc}(3f)$.

(a) If $\mathrm{y}(t)=\mathrm{x}(t/2)$ write $\mathrm{Y}(f)$.

$\mathrm{Y}(f) = 16\,\mathrm{sinc}(6f)$

(b) If $\mathrm{y}(t) = \frac{d}{dt}\mathrm{x}(t)$ write $\mathrm{Y}(f)$.

$\mathrm{Y}(f) = j2\pi f \times 8\,\mathrm{sinc}(3f) = j16\pi f\,\mathrm{sinc}(3f)$

(c) If $\mathrm{y}(t) = \mathrm{x}(t+2)$ write $\mathrm{Y}(j\omega)$.

$\mathrm{Y}(f) = 8\,\mathrm{sinc}(3f)e^{j4\pi f} \Rightarrow \mathrm{Y}(j\omega) = 8\,\mathrm{sinc}(3\omega/2\pi)e^{j2\omega}$

6. A signal $\mathrm{x}[n]$ has a DTFT $\mathrm{X}(F)$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	−2	−1	0	1	2	3	4	5	6
$\mathrm{x}[n]$	2	8	−5	9	2	7	−8	2	3

Let $\mathrm{Y}(F) = \mathrm{X}(2F)$ with $\mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{Y}(F)$. Fill in numerical values of $\mathrm{y}[n]$ in the table below.

Using

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}[n/m] & , \; n/m \text{ an integer} \\ 0 & , \; \text{otherwise} \end{cases} \quad , \quad \mathrm{z}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(mF)$$

n	−2	−1	0	1	2	3
$\mathrm{y}[n]$	8	0	− 5	0	9	0

7. An LTI system has a frequency response $\mathrm{H}(j\omega) = \frac{1}{j\omega - j3} + \frac{1}{j\omega + j3}$.

(a) Find an expression for its impulse response $\mathrm{h}(t)$ which does not contain the square root of minus one (j).

$$\mathrm{h}(t) = e^{j3t}\,\mathrm{u}(t) + e^{-j3t}\,\mathrm{u}(t) = 2\cos(3t)\mathrm{u}(t)$$

(b) Is this system stable?

No

Explain how you know.

The impulse response is not absolutely integrable.

8. A discrete-time signal $\mathrm{x}[n]$ with fundamental period $N_0 = 4$ has a DTFS harmonic function $\mathrm{X}[k]$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	2	3	4	5
$\mathrm{x}[n]$	-3	7	1	4

(a) Using $\mathrm{X}[k] = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_0}$ fill in the numerical values of $\mathrm{X}[k]$ in the table below.

$$\mathrm{X}[0] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] = \frac{1+4-3+7}{4} = 9/4$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\big(1 + (4)(-j) + (-3)(-1) + 7(j)\big) = \frac{4+j3}{4} = 1 + j0.75$$

$$\mathrm{X}[2] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\big(1 + (4)(-1) + (-3)(1) + 7(-1)\big) = -13/4 \text{ or } -3.25$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = \frac{1}{4}\big(1 + (4)(j) + (-3)(-1) + 7(-j)\big) = \frac{4-j3}{4} = 1 - j0.75$$

k	0	1	2	3
$\mathrm{X}[k]$	$9/4 = 2.25$	$1 + j0.75$	$-13/4 = -3.25$	$1 - j0.75$

(b) If $\mathrm{y}[n] = \mathrm{x}[n-8]$, what is the numerical value of the harmonic function $\mathrm{Y}[k]$ of $\mathrm{y}[n]$ at $k = 1$?

$\mathrm{Y}[1] = \mathrm{X}[1] = 1 + j0.75$ because a shift of x by exactly two fundamental periods does not change it.

Solution of ECE 315 Test 12 F08

1. A continuous-time signal is zero for all time before $t=-2$, rises linearly from 0 to 2 between $t=-2$ and $t=2$ and is zero for all time after that. This signal can be expressed in the form

$$\mathrm{x}(t) = A\,\mathrm{rect}\left(\frac{t-t_{01}}{w_1}\right)\mathrm{tri}\left(\frac{t-t_{02}}{w_2}\right).$$

(a) Find the numerical values of the constants.

$$\mathrm{x}(t) = 2\,\mathrm{rect}\left(\frac{t}{4}\right)\mathrm{tri}\left(\frac{t-2}{4}\right)$$

(b) Find the numerical value of the signal energy of this signal.

$$E_{\mathrm{x}} = \int_{-\infty}^{\infty}|\mathrm{x}(t)|^2\,dt = \int_{-2}^{2}\left(\frac{t+2}{2}\right)^2 dt = (1/4)\int_{-2}^{2}\left(t^2+4t+4\right)dt$$

$$E_{\mathrm{x}} = (1/4)\left[\frac{t^3}{3}+2t^2+4t\right]_{-2}^{2} = (1/4)\left[\frac{8}{3}+8+8+\frac{8}{3}-8+8\right] = \frac{64}{12} = 5.333$$

2. A discrete-time signal has the following values for times $n=-8$ to $n=8$ and is zero for all other times.

n	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8
$\mathrm{x}[n]$	3	5	5	5	3	5	5	5	3	5	5	5	3	5	5	5	3

This signal can be expressed in the form $\mathrm{x}[n] = \left(A - B\delta_{N_1}[n-n_0]\right)\mathrm{rect}_{N_2}[n]$.

(a) Find the numerical values of the constants.

$$\mathrm{x}[n] = \left(5 - 2\delta_4[n]\right)\mathrm{rect}_8[n]$$

(b) Let $\mathrm{y}[n]$ be a periodic signal with fundamental period 17 and let

$$\mathrm{y}[n] = \mathrm{x}[n] \quad , \quad -8 \le n \le 8$$

Find the numerical value of the signal power of $\mathrm{y}[n]$.

$$P_{\mathrm{y}} = (1/N_0)\sum_{n=\langle N_0\rangle}|\mathrm{y}[n]|^2 = (1/17)\left[5^2\times 12 + 3^2\times 5\right] = 20.29$$

3. A discrete-time system is described by the difference equation $y[n]-0.98\,y[n-2]=x[n]$ where $x[n]$ is the excitation and $y[n]$ is the response.

(a) Fill in the table below with numbers.

n	0	1	2	3	4
$h[n]$	1	0	0.98	0	0.9604

(b) What is the numerical value of $h[64]$?

The eigenvalues are $\pm\sqrt{0.98}$.

$$h[n]=\left[K_1\left(\sqrt{0.98}\right)^n+K_2\left(-\sqrt{0.98}\right)^n\right]u[n]$$

$$\begin{array}{r} h[0]=1=K_1+K_2 \\ h[1]=0=K_1\sqrt{0.98}-K_2\sqrt{0.98} \end{array} \Rightarrow K_1=K_2=0.5$$

$$h[n]=0.5\left[\left(\sqrt{0.98}\right)^n+\left(-\sqrt{0.98}\right)^n\right]u[n]$$

$$h[64]=0.5\left[\left(\sqrt{0.98}\right)^{64}+\left(-\sqrt{0.98}\right)^{64}\right]u[n]=0.98^{32}=0.524$$

4. When a continuous-time system with impulse response $h(t)=3\,\mathrm{rect}(t)$ is excited by $x(t)=4\,\mathrm{rect}(2t)$ the response is $y(t)=9\,\mathrm{tri}(4t/3)-3\,\mathrm{tri}(4t)$.

(a) Change the excitation to $x_a(t)=x(t-1)$ and keep the same impulse response. What is the numerical value of the new response $y_a(t)$ at time $t=1/2$?

Shifting the excitation shifts the response by the same time. Therefore

$$y_a(t)=9\,\mathrm{tri}(4(t-1)/3)-3\,\mathrm{tri}(4(t-1))$$

$$y_a(-1/2)=9\underbrace{\mathrm{tri}(-2/3)}_{=1/3}-3\underbrace{\mathrm{tri}(-2)}_{=0}=3$$

(b) Change the excitation to $x_b(t)=\frac{d}{dt}x(t)$ and keep the same impulse response. What is the numerical value of the new response $y_b(t)$ at time $t=1/2$?

Differentiating the excitation differentiates the response. Therefore

$$y_b(t)=12\,\mathrm{rect}(2(t+1/2))-12\,\mathrm{rect}(2(t-1/2))$$

$$y_b(1/2)=12\,\mathrm{rect}(2)-12\,\mathrm{rect}(0)=-12$$

5. A signal $x(t)$ has a CTFT $X(f)=7\,\mathrm{sinc}(5f)$.

(a) If $y(t)=x(t/2)$ write $Y(f)$.

$\mathrm{Y}(f) = 14\,\mathrm{sinc}(10f)$

(b) If $\mathrm{y}(t) = \frac{d}{dt}\mathrm{x}(t)$ write $\mathrm{Y}(f)$.

$\mathrm{Y}(f) = j2\pi f \times 7\,\mathrm{sinc}(5f) = j14\pi f\,\mathrm{sinc}(5f)$

(c) If $\mathrm{y}(t) = \mathrm{x}(t+2)$ write $\mathrm{Y}(j\omega)$.

$\mathrm{Y}(f) = 7\,\mathrm{sinc}(5f)e^{j4\pi f} \Rightarrow \mathrm{Y}(j\omega) = 7\,\mathrm{sinc}(5\omega/2\pi)e^{j2\omega}$

6. A signal $\mathrm{x}[n]$ has a DTFT $\mathrm{X}(F)$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	-2	-1	0	1	2	3	4	5	6
$\mathrm{x}[n]$	3	7	-2	1	1	8	6	9	4

Let $\mathrm{Y}(F) = \mathrm{X}(2F)$ with $\mathrm{y}[n] \xleftrightarrow{\mathcal{F}} \mathrm{Y}(F)$. Fill in numerical values of $\mathrm{y}[n]$ in the table below.

Using

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}[n/m] & , \ n/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases} \quad , \quad \mathrm{z}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(mF)$$

n	-2	-1	0	1	2	3
$\mathrm{y}[n]$	7	0	-2	0	1	0

7. An LTI system has a frequency response $\mathrm{H}(j\omega) = \frac{1}{j\omega - j10} + \frac{1}{j\omega + j10}$.

(a) Find an expression for its impulse response $\mathrm{h}(t)$ which does not contain the square root of minus one (j).

$$\mathrm{h}(t) = e^{j10t}\,\mathrm{u}(t) + e^{-j10t}\,\mathrm{u}(t) = 2\cos(10t)\,\mathrm{u}(t)$$

(b) Is this system stable?

No

Explain how you know.

The impulse response is not absolutely integrable.

8. A discrete-time signal $\mathrm{x}[n]$ with fundamental period $N_0 = 4$ has a DTFS harmonic function $\mathrm{X}[k]$. Some of the values of $\mathrm{x}[n]$ are given in the table below.

n	2	3	4	5
$\mathrm{x}[n]$	1	4	7	–2

(a) Using $\mathrm{X}[k] = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_0}$ fill in the numerical values of $\mathrm{X}[k]$ in the table below.

$$\mathrm{X}[0] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] = \frac{7-2+1+4}{4} = 5/2$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left(7 + (-2)(-j) + (1)(-1) + 4(j)\right) = \frac{6+j6}{4} = 1.5 + j1.5$$

$$\mathrm{X}[2] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left(7 + (-2)(-1) + (1)(1) + 4(-1)\right) = 3/2$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j3\pi n/2} = \frac{1}{4}\left(7 + (-2)(j) + (1)(-1) + 4(-j)\right) = \frac{6-j6}{4} = 1.5 - j1.5$$

k	0	1	2	3
$\mathrm{X}[k]$	$5/2 = 2.5$	$1.5 + j1.5$	$3/2 = 1.5$	$1.5 - j1.5$

(b) If $\mathrm{y}[n] = \mathrm{x}[n-8]$, what is the numerical value of the harmonic function $\mathrm{Y}[k]$ of $\mathrm{y}[n]$ at $k = 1$?

$\mathrm{Y}[1] = \mathrm{X}[1] = 1.5 + j1.5$ because a shift of x by exactly two fundamental periods does not change it.