Student Identification Number □□□ – □□ – □□□□ (No name please)

Solution of ECE 315 Final Examination Su09

1. Find the numerical values of the constants in the z-transform pairs below.

(a) $(-2)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} A\dfrac{z}{z+a} \quad , \ |z| > |b|$

$$(-2)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z+2} \quad , \ |z| > 2$$

(b) $A\left[b^n - Cc^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{z(z+1)}{3z^2+2.1z+0.36} \quad , \ |z| > 0.4$

$$(7/3)\left[(-0.3)^n - (6/7)(-0.4)^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{3}\frac{z(z+1)}{(z+0.4)(z+0.3)} = \frac{z}{3}\left[-\frac{6}{z+0.4} + \frac{7}{z+0.3}\right] \quad , \ |z| > 0.4$$

or

$$-2\left[(-0.4)^n - (7/6)(-0.3)^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{3}\frac{z(z+1)}{(z+0.4)(z+0.3)} = \frac{z}{3}\left[-\frac{6}{z+0.4} + \frac{7}{z+0.3}\right] \quad , \ |z| > 0.4$$

(c) $(0.3)^n \mathrm{u}[-n+8] \xleftrightarrow{\mathcal{Z}} A\dfrac{z^b}{z+a} \quad , \ |z| < |c|$

$$0.3^n \mathrm{u}[-n-1] = 0.3^n \mathrm{u}[-(n+1)] \xleftrightarrow{\mathcal{Z}} -\frac{z}{z-0.3} \quad , \ |z| < 0.3$$

$$(0.3)^9\, 0.3^{n-9} \mathrm{u}[-(n-8)] = (0.3)^9\, 0.3^{n-9} \mathrm{u}[-n+8] \xleftrightarrow{\mathcal{Z}} -(0.3)^9 z^{-9}\frac{z}{z-0.3} = -\frac{1.97\times 10^{-5} z^{-8}}{z-0.3} \quad , \ |z| < 0.3$$

(d) $Aa^{n-c}\left[\cos(2\pi b(n-c)) + B\sin(2\pi b(n-c))\right]\mathrm{u}[n-c] \xleftrightarrow{\mathcal{Z}} z^{-1}\dfrac{1.5625z-1}{z^2+0.64} \quad , \ |z| > 0.8$

2. Find the numerical region of convergence of the z transform of

$$\mathrm{x}[n] = 12(0.85)^n \cos(2\pi n/10)\mathrm{u}[-n-1] + 3(0.4)^{n+2}\mathrm{u}[n+2]$$

or, if it does not exist, state that fact and explain why.

ROC is $0.4 < |z| < 0.85$

3. A periodic discrete-time signal with fundamental period $N = 3$ has the values $x[1] = 7$, $x[2] = -3$, $x[3] = 1$. If $x[n] \xleftrightarrow[3]{\mathcal{DFT}} X[k]$, find the numerical magnitude and angle (in radians) of $X[1]$.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} = \sum_{n=0}^{2} x[n] e^{-j2\pi kn/3}$$

$$X[1] = \sum_{n=0}^{2} x[n] e^{-j2\pi n/3} = x[0] + x[1] e^{-j2\pi/3} + x[2] e^{-j4\pi/3}$$

$$X[1] = 1 + 7e^{-j2\pi/3} - 3e^{-j4\pi/3} = -1 - j8.66 = 8.7178\angle -1.6858$$

4. Given that $x[n]$ is periodic and real-valued with fundamental period 8, $x[n] \xleftrightarrow[8]{\mathcal{DFT}} X_8[k]$, $X_8[5] = -2 + j5$ and $x[n] \xleftrightarrow[16]{\mathcal{DFT}} X_{16}[k]$,

(a) What is the numerical value of $X_{16}[10]$?

The change of period property for the DFT is

$$\text{If } x[n] \xleftrightarrow[N]{\mathcal{DFT}} X[k] \text{ then } x[n] \xleftrightarrow[mN]{\mathcal{DFT}} \begin{cases} m X[k/m] & , \ k/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

$$X_{16}[10] = 2\, X_8[5] = -4 + j10$$

(b) What is the numerical value of $X_8[3]$?

For real-valued $x[n]$ $X[k] = X^*[-k]$. Also, $X_8[k]$ is periodic with period 8 so that $X_8[3] = X_8[-5] = X_8^*[5] = -2 - j5$.

5. Let $x = \begin{bmatrix} 1 \\ e^{-j\pi/3} \\ e^{-j2\pi/3} \\ -1 \\ e^{-j4\pi/3} \\ e^{-j5\pi/3} \end{bmatrix}$ and $y = \begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix}$. Are they orthogonal? Explain how you know.

Explanation:

$$x^H y = \begin{bmatrix} 1 & e^{j\pi/3} & e^{j2\pi/3} & -1 & e^{j4\pi/3} & e^{j5\pi/3} \end{bmatrix} \begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix} = e^{-j4\pi/3} + e^{-j7\pi/3} + e^{j2\pi/3} - e^{-j16\pi/3} + e^{-j16\pi/3} + e^{j5\pi/3}$$

$x^H y = 0$. The inner product is zero. Therefore they are orthogonal.

Solution of ECE 315 Final Examination Su09

1. Find the numerical values of the constants in the z-transform pairs below.

(a) $(-1.5)^n \mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} A\dfrac{z}{z+a} \quad , \quad |z|>|b|$

$$(-1.5)^n \mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{z+1.5} \quad , \quad |z|>1.5$$

(b) $A\left[b^n + Cc^n\right]\mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \dfrac{z(z+1)}{4z^2+2.8z+0.4} \quad , \quad |z|>0.5$

$$(2/3)\left[(-0.2)^n - (5/8)(-0.5)^n\right]\mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{4}\frac{z(z+1)}{(z+0.5)(z+0.2)} = \frac{z}{4}\left[-\frac{5/3}{z+0.5}+\frac{8/3}{z+0.2}\right] \quad , \quad |z|>0.5$$

or

$$-(5/12)\left[(-0.5)^n - (8/5)(-0.2)^n\right]\mathrm{u}[n] \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z}{4}\left[-\frac{5/3}{z+0.5}+\frac{8/3}{z+0.2}\right] \; , \; |z|>0.5$$

(c) $(0.6)^n \mathrm{u}[-n+4] \overset{\mathcal{Z}}{\longleftrightarrow} A\dfrac{z^b}{z+a} \quad , \quad |z|<|c|$

$$0.6^n \mathrm{u}[-n-1] = 0.6^n \mathrm{u}[-(n+1)] \overset{\mathcal{Z}}{\longleftrightarrow} -\frac{z}{z-0.6} \quad , \quad |z|<0.6$$

$$(0.6)^5\, 0.6^{n-5} \mathrm{u}[-(n-4)] = (0.6)^5\, 0.6^{n-5} \mathrm{u}[-n+4] \overset{\mathcal{Z}}{\longleftrightarrow} -(0.6)^5 z^{-5}\frac{z}{z-0.6} = -\frac{0.778z^{-4}}{z-0.6} \quad , \quad |z|<0.6$$

(d) $Aa^{n-c}\left[\cos(2\pi b(n-c)) + B\sin(2\pi b(n-c))\right]\mathrm{u}[n-c] \overset{\mathcal{Z}}{\longleftrightarrow} z^{-1}\dfrac{1.2346z-1}{z^2+0.81} \quad , \quad |z|>0.9$

2. Find the numerical region of convergence of the z transform of

$$\mathrm{x}[n] = 9(0.7)^n \cos(2\pi n/10)\mathrm{u}[-n-1] + 3(0.6)^{n+2}\mathrm{u}[n+2]$$

or, if it does not exist, state that fact and explain why.

ROC is $0.6<|z|<0.7$

3. A periodic discrete-time signal with fundamental period $N = 3$ has the values $\mathrm{x}[1]=5$, $\mathrm{x}[2]=2$, $\mathrm{x}[3]=-1$. If $\mathrm{x}[n] \xleftrightarrow[3]{\mathcal{DFT}} \mathrm{X}[k]$, find the numerical magnitude and angle (in radians) of $\mathrm{X}[1]$.

$$\mathrm{X}[k]=\sum_{n=0}^{N-1}\mathrm{x}[n]e^{-j2\pi kn/N}=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j2\pi kn/3}$$

$$\mathrm{X}[1]=\sum_{n=0}^{2}\mathrm{x}[n]e^{-j2\pi n/3}=\mathrm{x}[0]+\mathrm{x}[1]e^{-j2\pi/3}+\mathrm{x}[2]e^{-j4\pi/3}$$

$$\mathrm{X}[1]=-1+5e^{-j2\pi/3}+2e^{-j4\pi/3}=-4.5-j2.5981=5.1962\angle-2.618$$

4. Given that $\mathrm{x}[n]$ is periodic and real-valued with fundamental period 8, $\mathrm{x}[n] \xleftrightarrow[8]{\mathcal{DFT}} \mathrm{X}_8[k]$, $\mathrm{X}_8[5]=-1+j2$ and $\mathrm{x}[n] \xleftrightarrow[16]{\mathcal{DFT}} \mathrm{X}_{16}[k]$,

(a) What is the numerical value of $\mathrm{X}_{16}[10]$?

The change of period property for the DFT is

$$\text{If } \mathrm{x}[n] \xleftrightarrow[N]{\mathcal{DFT}} \mathrm{X}[k] \text{ then } \mathrm{x}[n] \xleftrightarrow[mN]{\mathcal{DFT}} \begin{cases} m\mathrm{X}[k/m] & , \ k/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

$$\mathrm{X}_{16}[10]=2\,\mathrm{X}_8[5]=-2+j4$$

(b) What is the numerical value of $\mathrm{X}_8[3]$?

For real-valued $\mathrm{x}[n]$ $\mathrm{X}[k]=\mathrm{X}^*[-k]$. Also, $\mathrm{X}_8[k]$ is periodic with period 8 so that $\mathrm{X}_8[3]=\mathrm{X}_8[-5]=\mathrm{X}_8^*[5]=-1-j2$.

5. Let $\mathrm{x}=\begin{bmatrix} 1 \\ e^{j\pi/3} \\ e^{j2\pi/3} \\ -1 \\ e^{j4\pi/3} \\ e^{j5\pi/3} \end{bmatrix}$ and $\mathrm{y}=\begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix}$. Are they orthogonal? Explain how you know.

Orthogonal Not Orthogonal

Explanation:

$$\mathrm{x}^H\mathrm{y}=\begin{bmatrix} 1 & e^{-j\pi/3} & e^{-j2\pi/3} & -1 & e^{-j4\pi/3} & e^{-j5\pi/3} \end{bmatrix}\begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix}=e^{-j4\pi/3}+e^{-j9\pi/3}+e^{-j2\pi/3}-e^{-j16\pi/3}+e^{-j24\pi/3}+e^{-j5\pi/3}$$

$\mathrm{x}^H\mathrm{y}=0$ The inner product is zero. Therefore they are orthogonal.

Solution of ECE 315 Final Examination Su09

1. Find the numerical values of the constants in the z-transform pairs below.

(a) $(-2.5)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} A\dfrac{z}{z+a} \quad , \ |z|>|b|$

$$(-2.5)^n \mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{z}{z+2.5} \quad , \ |z|>|2.5|$$

(b) $A\left[b^n + Cc^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{z(z+1)}{5z^2+3.5z+0.3} \quad , \ |z|>0.6$

$$(9/25)\left[(-0.1)^n - (4/9)(-0.6)^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{5}\frac{z(z+1)}{(z+0.6)(z+0.1)} = \frac{z}{5}\left[-\frac{4/5}{z+0.6}+\frac{9/5}{z+0.1}\right] \quad , \ |z|>0.6$$

or

$$-(4/25)\left[(-0.6)^n - (9/4)(-0.1)^n\right]\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \frac{1}{5}\frac{z(z+1)}{(z+0.6)(z+0.1)} = \frac{z}{5}\left[-\frac{4/5}{z+0.6}+\frac{9/5}{z+0.1}\right] \quad , \ |z|>0.6$$

(c) $(0.1)^n \mathrm{u}[-n+6] \xleftrightarrow{\mathcal{Z}} A\dfrac{z^b}{z+a} \quad , \ |z|<|c|$

$$(0.1)^n \mathrm{u}[-n-1] = (0.1)^n \mathrm{u}[-(n+1)] \xleftrightarrow{\mathcal{Z}} -\frac{z}{z-0.1} \quad , \ |z|<0.1$$

$$(0.1)^7(0.1)^{n-7}\mathrm{u}[-(n-6)] = (0.1)^7(0.1)^{n-7}\mathrm{u}[-n+6] \xleftrightarrow{\mathcal{Z}} -(0.1)^7 z^{-7}\frac{z}{z-0.1} = -\frac{10^{-7}z^{-6}}{z-0.1} \quad , \ |z|<0.1$$

(d) $Aa^{n-c}\left[\cos(2\pi b(n-c)) + B\sin(2\pi b(n-c))\right]\mathrm{u}[n-c] \xleftrightarrow{\mathcal{Z}} z^{-1}\dfrac{2.0408z-1}{z^2+0.49} \quad , \ |z|>0.7$

$$z^{-1}\frac{2.0408z-1}{z^2+0.49} = 2.0408z^{-2}\frac{z^2-0.49z}{z^2+0.49} = 2.0408z^{-2}\left(\frac{z^2}{z^2+0.49}-\frac{0.49z}{z^2+0.49}\right) \quad , \ |z|>0.7$$

In the damped sinusoid forms $\alpha = 0.7$ and $\Omega_0 = \pi/2$.

$$2.0408(0.7)^{n-2}\left[\cos(\pi(n-2)/2) - 0.7\sin(\pi(n-2)/2)\right]\mathrm{u}[n-2] \xleftrightarrow{\mathcal{Z}} 2.0408z^{-2}\left(\frac{z^2}{z^2+0.49}-\frac{0.49z}{z^2+0.49}\right) \quad , \ |z|>0.7$$

2. Find the numerical region of convergence of the z transform of

$$\mathrm{x}[n] = 12(1.2)^n\cos(2\pi n/10)\mathrm{u}[-n-1] + 3(0.9)^{n+2}\mathrm{u}[n+2]$$

or, if it does not exist, state that fact and explain why.

ROC is $0.9 < |z| < 1.2$

3. A periodic discrete-time signal with fundamental period $N = 3$ has the values $\mathrm{x}[1] = 7$, $\mathrm{x}[2] = 9$, $\mathrm{x}[3] = 1$. If $\mathrm{x}[n] \xleftrightarrow[3]{\mathcal{DFT}} \mathrm{X}[k]$, find the numerical magnitude and angle (in radians) of $\mathrm{X}[1]$.

$$\mathrm{X}[k] = \sum_{n=0}^{N-1} \mathrm{x}[n] e^{-j2\pi kn/N} = \sum_{n=0}^{2} \mathrm{x}[n] e^{-j2\pi kn/3}$$

$$\mathrm{X}[1] = \sum_{n=0}^{2} \mathrm{x}[n] e^{-j2\pi n/3} = \mathrm{x}[0] + \mathrm{x}[1] e^{-j2\pi/3} + \mathrm{x}[2] e^{-j4\pi/3}$$

$$\mathrm{X}[1] = 1 + 7e^{-j2\pi/3} + 9e^{-j4\pi/3} = -7 + j1.732 = 7.2111\angle 2.899$$

4. Given that $\mathrm{x}[n]$ is periodic and real-valued with fundamental period 8, $\mathrm{x}[n] \xleftrightarrow[8]{\mathcal{DFT}} \mathrm{X}_8[k]$, $\mathrm{X}_8[5] = 5 + j2$ and $\mathrm{x}[n] \xleftrightarrow[16]{\mathcal{DFT}} \mathrm{X}_{16}[k]$,

(a) What is the numerical value of $\mathrm{X}_{16}[10]$?

The change of period property for the DFT is

$$\text{If } \mathrm{x}[n] \xleftrightarrow[N]{\mathcal{DFT}} \mathrm{X}[k] \text{ then } \mathrm{x}[n] \xleftrightarrow[mN]{\mathcal{DFT}} \begin{cases} m\mathrm{X}[k/m] & , \ k/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

$$\mathrm{X}_{16}[10] = 2\,\mathrm{X}_8[5] = 10 + j4$$

(b) What is the numerical value of $\mathrm{X}_8[3]$?

For real-valued $\mathrm{x}[n]$ $\mathrm{X}[k] = \mathrm{X}^*[-k]$. Also, $\mathrm{X}_8[k]$ is periodic with period 8 so that $\mathrm{X}_8[3] = \mathrm{X}_8[-5] = \mathrm{X}_8^*[5] = 5 - j2$.

5. Let $\mathrm{x} = \begin{bmatrix} 1 \\ e^{-j\pi/3} \\ e^{-j2\pi/3} \\ -1 \\ e^{-j4\pi/3} \\ e^{-j5\pi/3} \end{bmatrix}$ and $\mathrm{y} = \begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix}$. Are they orthogonal? Explain how you know.

Orthogonal Not Orthogonal

Explanation:

$$\mathrm{x}^H \mathrm{y} = \begin{bmatrix} 1 & e^{j\pi/3} & e^{j2\pi/3} & -1 & e^{j4\pi/3} & e^{j5\pi/3} \end{bmatrix} \begin{bmatrix} e^{-j4\pi/3} \\ e^{-j8\pi/3} \\ 1 \\ e^{-j16\pi/3} \\ e^{-j20\pi/3} \\ 1 \end{bmatrix} = e^{-j4\pi/3} + e^{-j7\pi/3} + e^{j2\pi/3} - e^{-j16\pi/3} + e^{-j16\pi/3} + e^{j5\pi/3}$$

$\mathrm{x}^H \mathrm{y} = 0$. The inner product is zero. Therefore they are orthogonal.