Solution of EECS 315 Final Examination F09

1. Find the numerical value of $\int_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(-2t)\,dt$.

$$\int_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(-2t)\,dt = \mathrm{ramp}(-2(-4)) = \mathrm{ramp}(8) = 8$$

2. Find the numerical signal energy of $\mathrm{x}[n] = (\delta_3[n] - 3\delta_6[n])(\mathrm{u}[n+1] - \mathrm{u}[n-12])$.

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} \left|(\delta_3[n] - 3\delta_6[n])(\mathrm{u}[n+1] - \mathrm{u}[n-12])\right|^2$$

$$E_{\mathrm{x}} = \sum_{n=-1}^{11} \left|(\delta_3[n] - 3\delta_6[n])\right|^2 = \underbrace{(1-3)^2}_{n=0} + \underbrace{1^2}_{n=3} + \underbrace{(1-3)^2}_{n=6} + \underbrace{1^2}_{n=9} = 10$$

3. Find the numerical strength of the impulse $-3\delta(-4t)$.

$$-3\delta(-4t) = -3 \times \frac{1}{|-4|}\delta(t) \Rightarrow \text{Strength is } -3/4$$

4. Find the numerical fundamental period of $\mathrm{x}(t) = 3\cos(2000\pi t) - 8\sin(2500\pi t)$.

The fundamental frequency is the greatest common divisor of 1000 and 1250 which is 250. Therefore the fundamental period is 1/250 or 4 ms.

5. Let $\mathrm{x}(t) = 3\,\mathrm{rect}(t/5) - 7\,\mathrm{rect}(t/2)$. If $\mathrm{y}(t) = \mathrm{x}(2(t-1))$ find the numerical value of $\mathrm{y}(2)$.

$$\mathrm{y}(t) = 3\,\mathrm{rect}(2(t-1)/5) - 7\,\mathrm{rect}(2(t-1)/2) = 3\,\mathrm{rect}(0.4(t-1)) - 7\,\mathrm{rect}(t-1)$$

$$\mathrm{y}(2) = 3\,\mathrm{rect}(0.4(2-1)) - 7\,\mathrm{rect}(2-1) = 3\,\mathrm{rect}(0.4) - 7\,\mathrm{rect}(1) = 3$$

6. A continuous-time system is described by $3\mathrm{y}'(t) + A\,\mathrm{y}(t) = 2\,\mathrm{x}(t)$.

(a) If its impulse response is $\mathrm{h}(t) = Ke^{st}\,\mathrm{u}(t)$, find the value of *s* in terms of *A* (all numbers except *A*).

(b) Find the numerical value of *K*.

(c) For what numerical range of values of *A* is the system stable?

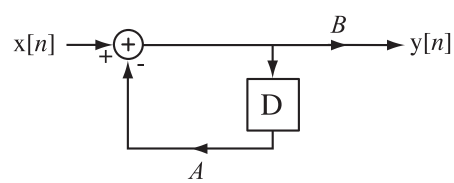
$$3s + A = 0 \Rightarrow s = -A/3 \Rightarrow \mathrm{h}(t) = Ke^{-At/3}\,\mathrm{u}(t)$$

$$3\left[\underbrace{\mathrm{h}(0^+)}_{K} - \underbrace{\mathrm{h}(0^-)}_{=0}\right] + A\underbrace{\int_{0^-}^{0^+} \mathrm{h}(t)\,dt}_{=0} = 2\left[\underbrace{\mathrm{u}(0^+)}_{1} - \underbrace{\mathrm{u}(0^-)}_{=0}\right] \Rightarrow 3K = 2 \Rightarrow K = 2/3$$

$$\mathrm{h}(t) = (2/3)e^{-At/3}\,\mathrm{u}(t)$$

System is stable for $A > 0$.

7. What numerical ranges of values of *A* and *B* make this system stable?



The output of the summing junction is $\mathrm{y}[n]/B$. Therefore

$$\mathrm{y}[n] = B(\mathrm{x}[n] - A\,\mathrm{y}[n-1]/B)$$

$$\mathrm{y}[n] = B(\mathrm{x}[n] - A\,\mathrm{y}[n-1]/B) \Rightarrow \mathrm{y}[n] + A\,\mathrm{y}[n-1] = B\,\mathrm{x}[n]$$

The eigenvalue is -*A* and the impulse response is

$$\mathrm{h}[n] = B(-A)^n\,\mathrm{u}[n]$$

System is stable for any $|A| < 1$ and for any *B*.

8. (a) If $\mathrm{x}(t) = \mathrm{tri}(t/3) * \delta(t-2)$ and $\mathrm{y}(t) = \mathrm{x}(2t)$, what is the numerical range of values of t for which $\mathrm{y}(t)$ is not zero?

$$-1/2 < t < 5/2$$

(b) If $\mathrm{x}(t) = \mathrm{tri}(t/w) * \delta(t+t_0)$ and $\mathrm{y}(t) = \mathrm{x}(at)$, what is the range of values of t (in terms of w, t_0 and a) for which $\mathrm{y}(t)$ is not zero?

$$\frac{-w-t_0}{a} < t < \frac{w-t_0}{a}$$

9. If $\mathrm{x}[n] = (\mathrm{u}[n+4] - \mathrm{u}[n-3]) * \delta[n+3]$ and $\mathrm{y}[n] = \mathrm{x}[n-4]$, what is the range of values for which $\mathrm{y}[n]$ is not zero?

x is non-zero for $-7 \le n < 0$. Therefore y is non-zero for $-3 \le n < 4$ or $-3 \le n \le 3$.

10. Find the numerical fundamental periods of

(a) $4\cos(6\pi n/7)$

$$4\cos(6\pi n/7) = 4\cos(2\pi n(3/7)) \Rightarrow N_0 = 7$$

(b) $2\sin(15\pi n/12)$

$$2\sin(15\pi n/12) = 2\sin(2\pi n(15/24)) = 2\sin(2\pi n(8/8)) \Rightarrow N_0 = 8$$

(c) What is the smallest positive value of n_0 that makes the signal

$$[4\cos(6\pi n/7) - 2\sin(15\pi n/12)] * (\mathrm{u}[n] - \mathrm{u}[n-n_0])$$

zero for all n?

This occurs when n_0 is the length of the common period between the two sinusoids because then the sum of the points is the sum over an integer number of periods of each sinusoid and that must be zero. In this case that value is $\mathrm{LCM}(7,8) = 56$.

11. Classify these systems according to stability, linearity and time invariance.

(a) $y[n] = |x[n]|$

Stable — If x is bounded then so is y.

Non-Linear — If x is A, $A > 0$, then y is also A. Then if we multiply x by -1, x is $-A$ but y is still A. Not homogeneous, therefore not linear.

Time Invariant — This is a static system. y is the magnitude of x, no matter when x is applied (for any n.)

(b) $y(t) = \sin(100\pi t)x(t)$

Stable — The sinusoid is bounded between -1 and 1. If x is bounded, so is y.

Linear — Multiplying x by any constant multiplies y by the same constant and if x is the sum of two signals, y is the sum of the responses. Therefore the system is homogeneous and additive and therefore linear.

Time Variant — The response y at time t depends on x at time t but also on the value of the sinusoid at time t. Therefore the response if x is delayed is not, in general, simply a delayed version of the response when x is not delayed.

12. Find the regions of convergence of the Laplace transforms of the following functions.

(a) $3e^{2t}\,u(t)$ — ROC is $\sigma > 2$

(b) $-10e^{t/2}\,u(-t)$ — ROC is $\sigma < 1/2$

13. A continuous-time system is described by the differential equation

$$4\,\mathrm{y}'''(t) - 2\,\mathrm{y}''(t) + 3\mathrm{y}'(t) - \mathrm{y}(t) = 8\,\mathrm{x}''(t) + \mathrm{x}'(t) - 4\,\mathrm{x}(t)\,.$$

Its transfer function can be written in the standard form

$$\mathrm{H}(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{b_M s^M + b_{M-1} s^{M-1} + \cdots + b_2 s^2 + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Find the numerical values of M, N and all the a and b coefficients $\left(a_N \rightarrow a_0 \text{ and } b_M \rightarrow b_0\right)$.

$M =$ ____________ , $N =$ ____________

$M = 2$, $N = 3$

a coefficients are ____

$$a_3 = 4\ ,\ a_2 = -2\ ,\ a_1 = 3\ ,\ a_0 = -1$$

b coefficients are ____

$$b_2 = 8\ ,\ b_1 = 1\ ,\ b_0 = -4$$

(Your identification of the a and b coefficients must be in the form $a_2 = 18$ or $b_3 = -7$ for example. That is, identify them individually, not just as a set of numbers.)

14. A continuous-time system has a transfer function $\mathrm{H}(s) = \dfrac{s}{(s + 0.1 + j10)(s + 0.1 - j10)}$.

(a) At what numerical <u>cyclic</u> frequency (in Hz) will the magnitude frequency response of this system be a maximum?

The maximum will occur at the closest approach to a pole which occurs at $\omega = \pm 10 \Rightarrow f = \pm 1.59$

(b) What will that numerical maximum magnitude be?

At $\omega = \pm 10$, the magnitude frequency response will be (substituting $j10$ for s), 4.9999.

15. Find the numerical values of the constants in $3(\mathrm{u}[n-1]-\mathrm{u}[n+1]) \xleftrightarrow{\mathcal{Z}} Az^a + Bz^b$.

$$3(\mathrm{u}[n-1]-\mathrm{u}[n+1]) = -3(\delta[n+1]+\delta[n]) \xleftrightarrow{\mathcal{Z}} -3z^1 - 3z^0$$

Alternate Solution:

$$\mathrm{x}[n] = 3(\mathrm{u}[n-1]-\mathrm{u}[n+1]) \xleftrightarrow{\mathcal{Z}} 3\left(\frac{z^{-1}z}{z-1} - \frac{z^2}{z-1}\right) = 3\frac{1-z^2}{z-1} = -3\frac{z^2-1}{z-1} = -3z^0 - 3z^1$$

16. A digital filter has a transfer function $\mathrm{H}(z) = 0.9525\dfrac{z^2-z+1}{z^2-0.95z+0.9025}$.

(a) What are the numerical locations of its poles and zeros?

Poles at $z = 0.95e^{\pm j\pi/3}$ or $0.475 \pm j0.8227$

Zeros at $z = e^{\pm j\pi/3}$ or $0.5 \pm j0.866$

(b) Find the numerical frequency response magnitude at these radian frequencies.

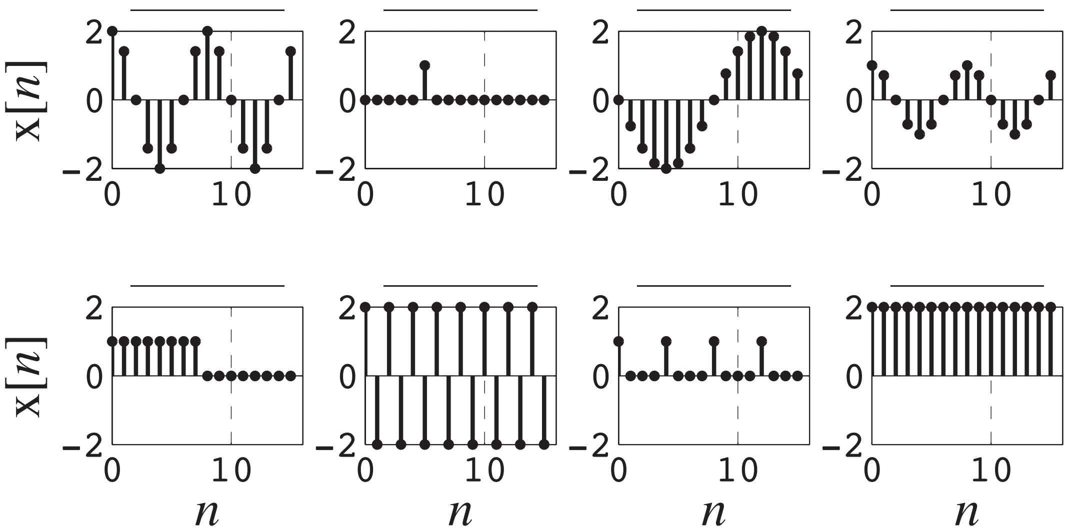
$\Omega = 0$

$$\left|\mathrm{H}(e^{j0})\right| = \left|\mathrm{H}(1)\right| = 0.9525\frac{1-1+1}{1-0.951+0.9025} = 0.9525\frac{1}{1-0.951+0.9025} = 1.05$$

$\Omega = \pi/3$

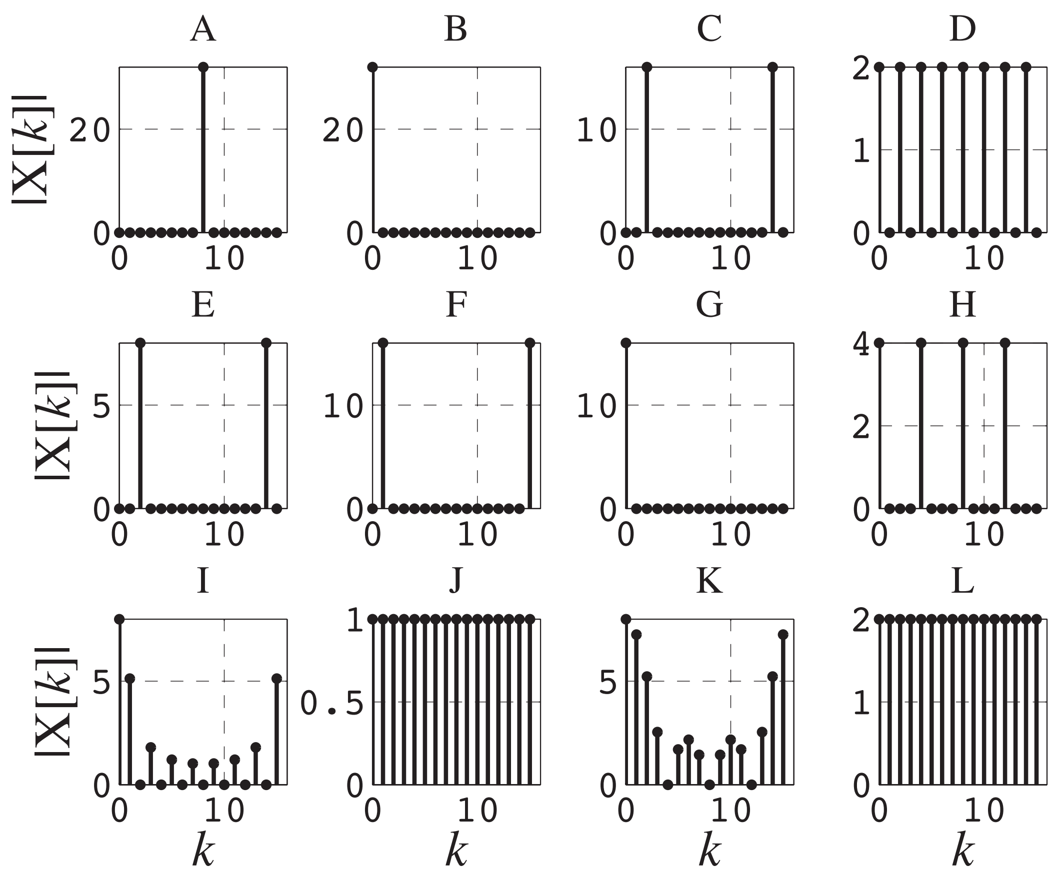
$$\left|\mathrm{H}(e^{j\pi/3})\right| = 0.9525\frac{e^{j2\pi/3} - e^{j\pi/3} + 1}{e^{j2\pi/3} - 0.951 \times e^{j\pi/3} + 0.9025} = 0$$

17. Below are some graphs of 8 discrete-time signals and below them some graphs of the magnitudes of 12 DFT's, all based on 16 points. Match the discrete-time signals to the DFT magnitudes by writing in the letter designation of the DFT magnitude corresponding to each discrete-time signal in the space provided at the top of its graph.



C J F E

I A H B



Solution of EECS 315 Final Examination F09

1. Find the numerical value of $\int_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(-3t)\,dt$.

$$\int_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(-3t)\,dt = \mathrm{ramp}(-3(-4)) = \mathrm{ramp}(12) = 12$$

2. Find the numerical signal energy of $\mathrm{x}[n] = (\delta_3[n] - 2\delta_6[n])(\mathrm{u}[n+4] - \mathrm{u}[n-9])$.

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} \left|(\delta_3[n] - 2\delta_6[n])(\mathrm{u}[n+4] - \mathrm{u}[n-9])\right|^2$$

$$E_{\mathrm{x}} = \sum_{n=-4}^{8} \left|(\delta_3[n] - 2\delta_6[n])\right|^2 = \underbrace{1^2}_{n=-3} + \underbrace{(1-2)^2}_{n=0} + \underbrace{1^2}_{n=3} + \underbrace{(1-2)^2}_{n=6} = 4$$

3. Find the numerical strength of the impulse $-5\delta(-4t)$.

$$-5\delta(-4t) = -5 \times \frac{1}{|-4|}\delta(t) \Rightarrow \text{Strength is } -5/4$$

4. Find the numerical fundamental period of $\mathrm{x}(t) = 3\cos(3000\pi t) - 8\sin(4000\pi t)$.

The fundamental frequency is the greatest common divisor of 1500 and 2000 which is 500. Therefore the fundamental period is 1/500 or 2 ms.

5. Let $\mathrm{x}(t) = 6\,\mathrm{rect}(t/5) - 7\,\mathrm{rect}(t/2)$. If $\mathrm{y}(t) = \mathrm{x}(2(t-1))$ find the numerical value of $\mathrm{y}(2)$.

$\mathrm{y}(t) = 6\,\mathrm{rect}(2(t-1)/5) - 7\,\mathrm{rect}(2(t-1)/2) = 6\,\mathrm{rect}(0.4(t-1)) - 7\,\mathrm{rect}(t-1)$

$\mathrm{y}(2) = 6\,\mathrm{rect}(0.4(2-1)) - 7\,\mathrm{rect}(2-1) = 6\,\mathrm{rect}(0.4) - 7\,\mathrm{rect}(1) = 6$

6. A continuous-time system is described by $5\,\mathrm{y}'(t) + A\,\mathrm{y}(t) = 2\,\mathrm{x}(t)$.

(a) If its impulse response is $\mathrm{h}(t) = Ke^{st}\,\mathrm{u}(t)$, find the value of s in terms of A (all numbers except A).

(b) Find the numerical value of K.

(b) For what numerical range of values of A is the system stable?

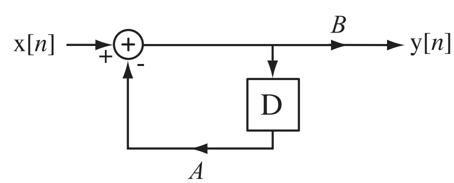
$$5s + A = 0 \Rightarrow s = -A/5 \Rightarrow \mathrm{h}(t) = Ke^{-At/5}\,\mathrm{u}(t)$$

$$5\left[\underbrace{\mathrm{h}(0^+)}_{K} - \underbrace{\mathrm{h}(0^-)}_{=0}\right] + A\underbrace{\int_{0^-}^{0^+}\mathrm{h}(t)\,dt}_{=0} = 2\left[\underbrace{\mathrm{u}(0^+)}_{1} - \underbrace{\mathrm{u}(0^-)}_{=0}\right] \Rightarrow 5K = 2 \Rightarrow K = 2/5$$

$$\mathrm{h}(t) = (2/5)e^{-At/5}\,\mathrm{u}(t)$$

System is stable for $A > 0$.

7. What numerical ranges of values of A and B make this system stable?



The output of the summing junction is $\mathrm{y}[n]/B$. Therefore

$$\mathrm{y}[n] = B(\mathrm{x}[n] - A\,\mathrm{y}[n-1]/B)$$

$$\mathrm{y}[n] = B(\mathrm{x}[n] - A\,\mathrm{y}[n-1]/B) \Rightarrow \mathrm{y}[n] + A\,\mathrm{y}[n-1] = B\,\mathrm{x}[n]$$

The eigenvalue is -A and the impulse response is

$$\mathrm{h}[n] = B(-A)^n\,\mathrm{u}[n]$$

System is stable for any $|A| < 1$ and for any B.

8. (a) If $x(t) = \text{tri}(t/5) * \delta(t-2)$ and $y(t) = x(2t)$, what is the numerical range of values of t for which $y(t)$ is not zero?

$$-3/2 < t < 7/2$$

(b) If $x(t) = \text{tri}(t/w) * \delta(t+t_0)$ and $y(t) = x(at)$, what is the range of values of t (in terms of w, t_0 and a) for which $y(t)$ is not zero?

$$\frac{-w-t_0}{a} < t < \frac{w-t_0}{a}$$

9. If $x[n] = (u[n+6] - u[n-3]) * \delta[n+3]$ and $y[n] = x[n-4]$, what is the range of values for which $y[n]$ is not zero?

x is non-zero for $-9 \le n < 0$. Therefore y is non-zero for $-5 \le n < 4$ or $-5 \le n \le 3$.

10. Find the numerical fundamental periods of

(a) $4\cos(12\pi n/11)$

$$4\cos(12\pi n/11) = 4\cos(2\pi n(6/11)) \Rightarrow N_0 = 11$$

(b) $2\sin(35\pi n/14)$

$$2\sin(35\pi n/14) = 2\sin(2\pi n(35/28)) = 2\sin(2\pi n(5/4)) \Rightarrow N_0 = 4$$

(c) What is the smallest positive value of n_0 that makes the signal

$$[4\cos(6\pi n/7) - 2\sin(15\pi n/12)] * (u[n] - u[n-n_0])$$

zero for all n?

This occurs when n_0 is the length of the common period between the two sinusoids because then the sum of the points is the sum over an integer number of periods of each sinusoid and that must be zero. In this case that value is $\text{LCM}(7,8) = 56$.

11. Classify these systems according to stability, linearity and time invariance.

(a) $y[n] = x^2[n]$

Stable: If x is bounded then so is y.

Non-Linear: If x is A, $A > 0$, then y is A^2. Then if we multiply x by -1, x is -A but y is still A^2. Not homogeneous, therefore not linear.

Time Invariant: This is a static system. y is x squared, no matter when x is applied (for any n.)

(b) $y(t) = 5\,x(t/3)$

Stable: If x is bounded, so is y.

Linear: Multiplying x by any constant multiplies y by the same constant and if x is the sum of two signals, y is the sum of the responses. Therefore the system is homogeneous and additive and therefore linear.

Time Variant: If $x_1(t) = g(t)$, then $y_1(t) = g(t/3)$. If $x_2(t) = g(t - t_0)$ then $y_2(t) = g(t/3 - t_0)$. $y_1(t - t_0) = g((t - t_0)/3)$. Therefore y_2 is not (in general) simply a delayed version of y_1.

12. Find the regions of convergence of the Laplace transforms of the following functions.

(a) $3e^{5t}\,u(t)$ ROC is $\sigma > 5$

(b) $-10e^{t/4}\,u(-t)$ ROC is $\sigma < 1/4$

13. A continuous-time system is described by the differential equation

$$3\,\mathrm{y}'''(t) - 8\,\mathrm{y}''(t) + 7\,\mathrm{y}'(t) - 5\,\mathrm{y}(t) = 4\,\mathrm{x}''(t) - 6\,\mathrm{x}(t)\,.$$

Its transfer function can be written in the standard form

$$\mathrm{H}(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{b_M s^M + b_{M-1} s^{M-1} + \cdots + b_2 s^2 + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Find the numerical values of M, N and all the a and b coefficients $\left(a_N \to a_0 \text{ and } b_M \to b_0\right)$.

$M = 2$, $N = 3$

a coefficients are ______________________________

$$a_3 = 3\ ,\ a_2 = -8\ ,\ a_1 = 7\ ,\ a_0 = -5$$

b coefficients are ______________________________

$$b_2 = 4\ ,\ b_1 = 0\ ,\ b_0 = -6$$

(Your identification of the a and b coefficients must be in the form $a_2 = 18$ or $b_3 = -7$ for example. That is, identify them individually, not just as a set of numbers.)

14. A continuous-time system has a transfer function $\mathrm{H}(s) = \dfrac{s}{(s + 0.1 + j12)(s + 0.1 - j12)}$.

(a) At what numerical cyclic frequency (in Hz) will the magnitude frequency response of this system be a maximum?

Frequency for maximum is ____________________Hz

The maximum will occur at the closest approach to a pole which occurs at $\omega = \pm 12 \Rightarrow f = \pm 1.91$

(b) What will that numerical maximum magnitude be?

Maximum frequency response magnitude is ______________________________

At $\omega = \pm 12$, the magnitude frequency response will be (substituting $j12$ for s), 4.9999.

15. Find the numerical values of the constants in $2(\mathrm{u}[n]-\mathrm{u}[n+2]) \xleftrightarrow{\mathcal{Z}} Az^a + Bz^b$.

$$2(\mathrm{u}[n]-\mathrm{u}[n+2]) = -2(\delta[n+2]+\delta[n+1]) \xleftrightarrow{\mathcal{Z}} -2z^2 - 2z^1$$

Alternate Solution:

$$\mathrm{x}[n] = 2(\mathrm{u}[n]-\mathrm{u}[n+2]) \xleftrightarrow{\mathcal{Z}} 2\left(\frac{z}{z-1} - \frac{z^3}{z-1}\right) = 2z\frac{1-z^2}{z-1} = -2z\frac{z^2-1}{z-1} = -2z^1 - 2z^2$$

16. A digital filter has a transfer function $\mathrm{H}(z) = 1.9025\dfrac{z^2+1}{z^2+0.9025}$.

(a) What are the numerical locations of its poles and zeros?

$$\text{Poles at } z = 0.95e^{\pm j\pi/2} \text{ or } \pm j0.95$$

$$\text{Zeros at } z = e^{\pm j\pi/2} \text{ or } \pm j$$

(b) Find the numerical frequency response magnitude at these radian frequencies.

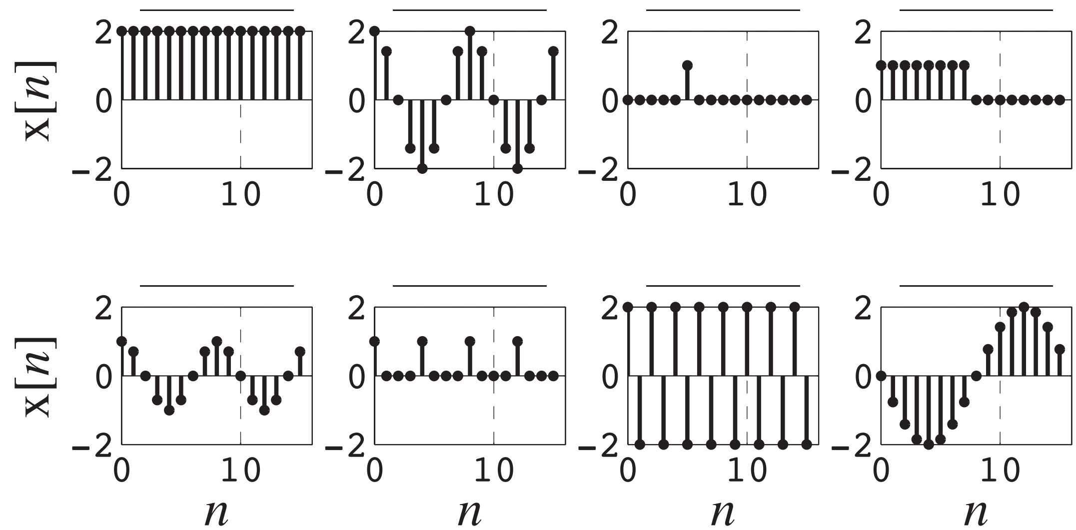
$\Omega = 0$

$$\left|\mathrm{H}(e^{j0})\right| = \left|\mathrm{H}(1)\right| = 1.9025\frac{1+1}{1+0.9025} = 2$$

$\Omega = \pi/2$

$$\left|\mathrm{H}(e^{j\pi/2})\right| = \left|\mathrm{H}(j)\right| = 1.9025\left|\frac{-1+1}{-1+0.9025}\right| = 0$$

17. Below are some graphs of 8 discrete-time signals and below them some graphs of the magnitudes of 12 DFT's, all based on 16 points. Match the discrete-time signals to the DFT magnitudes by writing in the letter designation of the DFT magnitude corresponding to each discrete-time signal in the space provided at the top of its graph.



G C E A

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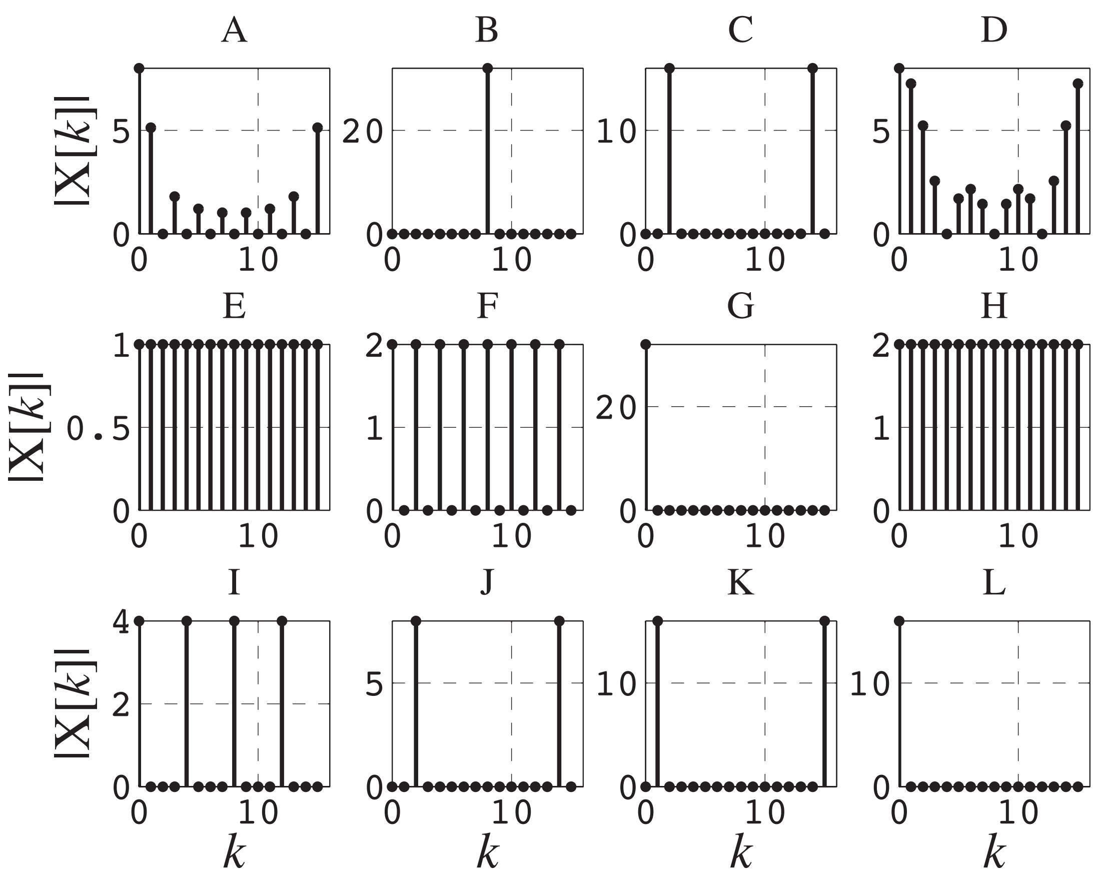


Solution of EECS 315 Final Examination F09

1. Find the numerical value of $\int_{-\infty}^{\infty} \delta(t+3)\,\mathrm{ramp}(-2t)\,dt$.

$$\int_{-\infty}^{\infty} \delta(t+3)\,\mathrm{ramp}(-2t)\,dt = \mathrm{ramp}(-2(-3)) = \mathrm{ramp}(6) = 6$$

2. Find the numerical signal energy of $\mathrm{x}[n] = (4\delta_3[n] - 3\delta_6[n])(\mathrm{u}[n+1] - \mathrm{u}[n-12])$.

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} \left|(4\delta_3[n] - 3\delta_6[n])(\mathrm{u}[n+1] - \mathrm{u}[n-12])\right|^2$$

$$E_{\mathrm{x}} = \sum_{n=-1}^{11} \left|(4\delta_3[n] - 3\delta_6[n])\right|^2 = \underbrace{(4-3)^2}_{n=0} + \underbrace{4^2}_{n=3} + \underbrace{(4-3)^2}_{n=6} + \underbrace{4^2}_{n=9} = 34$$

3. Find the numerical strength of the impulse $-3\delta(-7t)$.

$$-3\delta(-7t) = -3 \times \frac{1}{|-7|}\delta(t) \Rightarrow \text{Strength is } -3/7$$

4. Find the numerical fundamental period of $\mathrm{x}(t) = 3\cos(600\pi t) - 8\sin(480\pi t)$.

The fundamental frequency is the greatest common divisor of 300 and 240 which is 60. Therefore the fundamental period is 1/60 or 16.667 ms.

5. Let $x(t) = 4\,\text{rect}(t/5) - 9\,\text{rect}(t/2)$. If $y(t) = x(2(t-1))$ find the numerical value of $y(2)$.

$y(t) = 4\,\text{rect}(2(t-1)/5) - 9\,\text{rect}(2(t-1)/2) = 4\,\text{rect}(0.4(t-1)) - 9\,\text{rect}(t-1)$

$y(2) = 4\,\text{rect}(0.4(2-1)) - 4\,\text{rect}(2-1) = 4\,\text{rect}(0.4) - 9\,\text{rect}(1) = 4$

6. A continuous-time system is described by $8\,y'(t) + A\,y(t) = 3\,x(t)$.

(a) If its impulse response is $h(t) = Ke^{st}\,u(t)$, find the value of s in terms of A (all numbers except A).

(b) Find the numerical value of K.

(b) For what numerical range of values of A is the system stable?

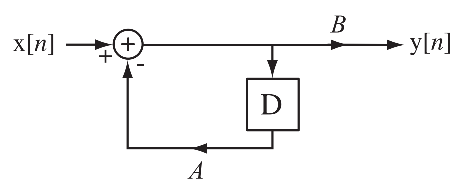
$$8s + A = 0 \Rightarrow s = -A/8 \Rightarrow h(t) = Ke^{-At/8}\,u(t)$$

$$8\left[\underbrace{h(0^+)}_{K} - \underbrace{h(0^-)}_{=0}\right] + A\underbrace{\int_{0^-}^{0^+} h(t)\,dt}_{=0} = 3\left[\underbrace{u(0^+)}_{1} - \underbrace{u(0^-)}_{=0}\right] \Rightarrow 8K = 3 \Rightarrow K = 3/8$$

$$h(t) = (3/8)e^{-At/8}\,u(t)$$

System is stable for $A > 0$.

7. What numerical ranges of values of A and B make this system stable?



The output of the summing junction is $y[n]/B$. Therefore

$$y[n] = B(x[n] - A\,y[n-1]/B)$$

$$y[n] = B(x[n] - A\,y[n-1]/B) \Rightarrow y[n] + A\,y[n-1] = B\,x[n]$$

The eigenvalue is -A and the impulse response is

$$h[n] = B(-A)^n\,u[n]$$

System is stable for any $|A| < 1$ and for any B.

8. (a) If $\mathrm{x}(t)=\mathrm{tri}(t/4)*\delta(t-2)$ and $\mathrm{y}(t)=\mathrm{x}(2t)$, what is the numerical range of values of t for which $\mathrm{y}(t)$ is not zero?

$$-1<t<3$$

(b) If $\mathrm{x}(t)=\mathrm{tri}(t/w)*\delta(t+t_0)$ and $\mathrm{y}(t)=\mathrm{x}(at)$, what is the range of values of t (in terms of w, t_0 and a) for which $\mathrm{y}(t)$ is not zero?

$$\frac{-w-t_0}{a}<t<\frac{w-t_0}{a}$$

9. If $\mathrm{x}[n]=(\mathrm{u}[n+2]-\mathrm{u}[n-5])*\delta[n+3]$ and $\mathrm{y}[n]=\mathrm{x}[n-4]$, what is the range of values for which $\mathrm{y}[n]$ is not zero?

x is non-zero for $-5\le n<2$. Therefore y is non-zero for $-1\le n<6$ or $-1\le n\le 5$.

10. Find the numerical fundamental periods of

(a) $4\cos(6\pi n/13)$

$4\cos(6\pi n/13)=4\cos(2\pi n(3/13))\Rightarrow N_0=13$

(b) $2\sin(12\pi n/9)$

$2\sin(12\pi n/9)=2\sin(2\pi n(12/18))=2\sin(2\pi n(2/3))\Rightarrow N_0=3$

(c) What is the smallest positive value of n_0 that makes the signal

$$[4\cos(6\pi n/7)-2\sin(15\pi n/12)]*(\mathrm{u}[n]-\mathrm{u}[n-n_0])$$

zero for all n?

This occurs when n_0 is the length of the common period between the two sinusoids because then the sum of the points is the sum over an integer number of periods of each sinusoid and that must be zero. In this case that value is $\mathrm{LCM}(7,8)=56$.

11. Classify these systems according to stability, linearity and time invariance.

(a) $y[n] = x[n]u[n]$

Stable — If x is bounded then so is y.

Linear — If x is multiplied by a constant y is multiplied by the same constant. If two excitations are added, the response is the sum of the individual responses. Therefore it is linear.

Time Variant — If $x_1[n] = g[n]$, then $y_1[n] = g[n]u[n]$. If $x_2[n] = g[n - n_0]$ then $y_2[n] = g[n - n_0]u[n]$. $y_1[n - n_0] = g[n - n_0]u[n - n_0]$. Therefore y_2 is not (in general) simply a delayed version of y_1.

(b) $y(t) = 3t\,x(t)$

Unstable — x is multiplied by $3t$ to get y. There is no upper bound on *t*. Therefore a bounded x can produce an unbounded y.

Linear — Multiplying x by any constant multiplies y by the same constant and if x is the sum of two signals, y is the sum of the responses. Therefore the system is homogeneous and additive and therefore linear.

Time Variant — If $x_1(t) = g(t)$, then $y_1(t) = 3t\,g(t)$. If $x_2(t) = g(t - t_0)$ then $y_2(t) = 3t\,g(t - t_0)$. $y_1(t - t_0) = 3(t - t_0)g(t - t_0)$. Therefore y_2 is not (in general) simply a delayed version of y_1.

12. Find the regions of convergence of the Laplace transforms of the following functions.

(a) $3e^{12t}u(t)$ — ROC is $\sigma > 12$

(b) $-10e^{t/7}u(-t)$ — ROC is $\sigma < 1/7$

13. A continuous-time system is described by the differential equation

$$2\,y'''(t) - 5\,y''(t) - 4\,y(t) = 9\,x''(t) + 2\,x'(t) - 7\,x(t).$$

Its transfer function can be written in the standard form

$$H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{b_M s^M + b_{M-1} s^{M-1} + \cdots + b_2 s^2 + b_1 s + b_0}{a_N s^N + a_{N-1} s^{N-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Find the numerical values of M, N and all the a and b coefficients.

$M = 2$, $N = 3$

$$a_3 = 2 \;,\; a_2 = -5 \;,\; a_1 = 0 \;,\; a_0 = -4$$
$$b_2 = 9 \;,\; b_1 = 2 \;,\; b_0 = -7$$

(Your identification of the a and b coefficients must be in the form $a_2 = 18$ or $b_3 = -7$ for example. That is, identify them individually, not just as a set of numbers.)

14. A continuous-time system has a transfer function $H(s) = \dfrac{s}{(s + 0.1 + j15)(s + 0.1 - j15)}$.

(a) At what numerical cyclic frequency (in Hz) will the magnitude frequency response of this system be a maximum?

The maximum will occur at the closest approach to a pole which occurs at $\omega = \pm 15 \Rightarrow f = \pm 2.39$

(b) What will that numerical maximum magnitude be?

At $\omega = \pm 15$, the magnitude frequency response will be (substituting $j15$ for s), 4.9999.

15. Find the numerical values of the constants in $11\left(\mathrm{u}[n-2]-\mathrm{u}[n]\right) \xleftrightarrow{\mathcal{Z}} Az^a + Bz^b$.

$$1\left(\mathrm{u}[n-2]-\mathrm{u}[n]\right) = -11\left(\delta[n]+\delta[n-1]\right) \xleftrightarrow{\mathcal{Z}} -11z^0 - 11z^{-1}$$

Alternate Solution:

$$\mathrm{x}[n] = 11\left(\mathrm{u}[n-2]-\mathrm{u}[n]\right) \xleftrightarrow{\mathcal{Z}} 11\left(\frac{z^{-2}z}{z-1} - \frac{z}{z-1}\right) = 11z^{-1}\frac{1-z^2}{z-1} = -11z^{-1}\frac{z^2-1}{z-1} = -11z^0 - 11z^{-1}$$

16. A digital filter has a transfer function $\mathrm{H}(z) = 0.9525\dfrac{z^2+z+1}{z^2+0.95z+0.9025}$.

(a) What are the numerical locations of its poles and zeros?

$$\text{Poles at } z = 0.95e^{\pm j2\pi/3} = -0.475 \pm j0.8227$$

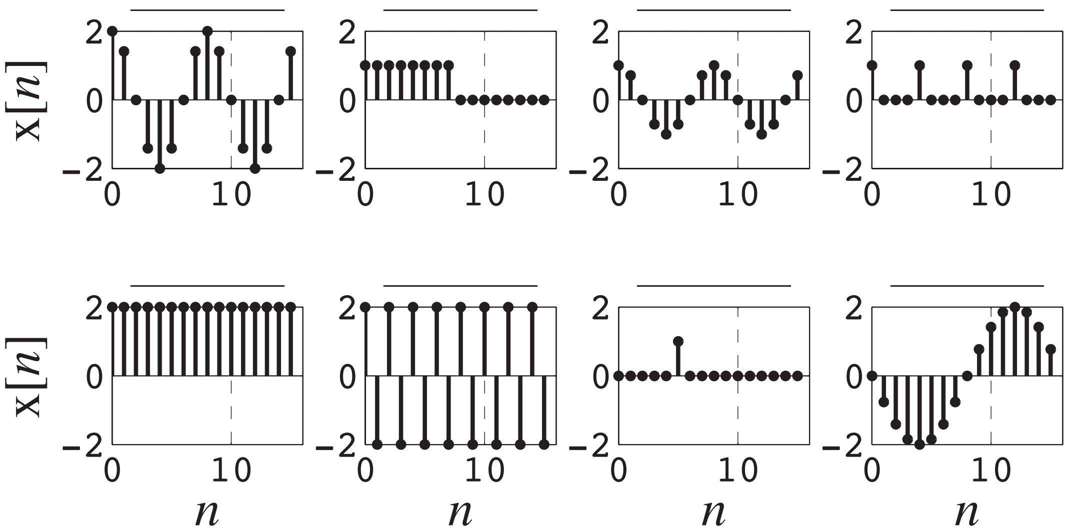
$$\text{Zeros at } z = e^{\pm j2\pi/3} = -0.5 \pm j0.866$$

(b) Find the numerical frequency response magnitude at these radian frequencies.

$\Omega = 0$ $\qquad \left|\mathrm{H}\left(e^{j0}\right)\right| = \left|\mathrm{H}(1)\right| = 0.9525\dfrac{1+1+1}{1+0.95+0.9025} = 0.9525\dfrac{3}{2.8525} = 1.0018$

$\Omega = 2\pi/3$ $\qquad \left|\mathrm{H}\left(e^{j2\pi/3}\right)\right| = 0.9525\left(\dfrac{e^{j4\pi/3}+e^{j2\pi/3}+1}{e^{j4\pi/3}+0.95e^{j2\pi/3}+0.9025}\right) = 0$

17. Below are some graphs of 8 discrete-time signals and below them some graphs of the magnitudes of 12 DFT's, all based on 16 points. Match the discrete-time signals to the DFT magnitudes by writing in the letter designation of the DFT magnitude corresponding to each discrete-time signal in the space provided at the top of its graph.



H E C K

G D I B