Solution of ECE 315 Final Examination F10

1. Four samples are taken from a continuous-time signal. They are

$$x(0) = 3 \text{ , } x(0.025) = -1 \text{ , } x(0.05) = -5 \text{ , } x(0.075) = 4$$

If these four samples cover exactly one period of a periodic continuous-time signal $x(t)$ and $x(t) \xleftrightarrow{\mathcal{F}} X(f)$, what is the best estimate, based solely on this set of data, of the numerical value of the strength of the impulse in $X(f)$ at $f = 0$?

It is the sum of the values divided by the number of values, $X(0) \cong \dfrac{3-1-5+4}{4} = 1/4$

2. Four values of a discrete-time signal are $x[0] = -2$, $x[1] = 7$, $x[2] = 11$, $x[3] = 3$.

(a) If these four values are exactly one period of a periodic discrete-time signal $x[n]$ and $x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\Omega})$, what is the numerical value of the strength of the impulse in $X(e^{j\Omega})$ at $\Omega = 0$?

It is the sum of the values divided by the number of values and multiplied by 2π ,

$$X(e^{j0}) = 2\pi \frac{-2+7+11+3}{4} = 38\pi / 4 = 29.85$$

(b) If these four values are all the non-zero values of a discrete-time signal $x[n]$ and $x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\Omega})$, what is the numerical value of $X(e^{j\Omega})$ at $\Omega = 0$?

It is the sum of the values,

$$X(e^{j0}) = -2 + 7 + 11 + 3 = 19$$

3. Find the numerical values of the constants.

(a) $4(0.7)^{n-2}\,\mathrm{u}[n-2] \xleftrightarrow{\mathcal{F}} \dfrac{Ae^{a\Omega}}{1+be^{c\Omega}}$

$$4(0.7)^{n-2}\,\mathrm{u}[n-2] \xleftrightarrow{\mathcal{F}} \frac{4e^{-j2\Omega}}{1-0.7e^{-j\Omega}}$$

(b) $-2(\mathrm{u}[n+1]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} Ae^{aF}\,\mathrm{drcl}(bF,B)$

$$-2(\mathrm{u}[n+1]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} -8e^{-j\pi F}\,\mathrm{drcl}(F,4)$$

(c) $Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{2+e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$

This problem is flawed and has no solution. It was not graded. The error is in the numerator. It should have been $2+0.5e^{-j\Omega}$ instead of $2+e^{-j\Omega}$. With that change the solution is

$$Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.25e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

Using $\alpha^n\cos(\Omega_0 n)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{1-\alpha\cos(\Omega_0)e^{-j\Omega}}{1-2\alpha\cos(\Omega_0)e^{-j\Omega}+\alpha^2e^{-j2\Omega}}$, $|\alpha|<1$

$\alpha^2 = 0.25$ and $\alpha = -0.5$

If $\alpha = -0.5 \Rightarrow \cos(\Omega_0) = 0.5 \Rightarrow \Omega_0 = \cos^{-1}(-0.5) = \pm\pi/3$

$$2(-0.5)^n\cos(\pm\pi n/3)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.5e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

4. Find the numerical values of the constants.

(a) $3\cos(2\pi n/18) \xleftrightarrow[18]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$3\cos(2\pi n/18) \xleftrightarrow[18]{\mathcal{DFT}} 27(\delta_{18}[k-1]+\delta_{18}[k+1])$$

(b) $3\cos(2\pi n/18) \xleftrightarrow[72]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$3\cos(2\pi n/18) \xleftrightarrow[72]{\mathcal{DFT}} 108(\delta_{72}[k-4]+\delta_{72}[k+4])$$

(c) $6(\mathrm{u}[n+4]-\mathrm{u}[n-1]) * \delta_{15}[n] \xleftrightarrow[15]{\mathcal{DFT}} Ae^{bk}\,\mathrm{drcl}(ck,D)$

Using $(\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1]) * \delta_N[n] \xleftrightarrow[mN]{\mathcal{DFT}} \dfrac{e^{-j\pi k(n_1+n_0)/mN}}{e^{-j\pi k/mN}}(n_1-n_0)m\,\mathrm{drcl}(k/mN, n_1-n_0)\delta_m[k]$

$$6(\mathrm{u}[n+4]-\mathrm{u}[n-1]) * \delta_{15}[n] \xleftrightarrow[15]{\mathcal{DFT}} 30e^{j4\pi k/15}\,\mathrm{drcl}(k/15,5)$$

5. A signal $\mathrm{x}(t)$ has the following description:

1. It is zero for all time $t < -4$.
2. It is a straight line from the point $t = -4, \mathrm{x} = 0$ to the point $t = -4, \mathrm{x} = 4$.
3. It is a straight line from the point $t = -4, \mathrm{x} = 4$ to the point $t = 3, \mathrm{x} = 0$.
4. It is zero for all time $t > 3$.

(a) What is the numerical value of $\mathrm{x}(-2)$?

$\mathrm{x}(-1) = 4 - (4/7)(2) = 20/7 = 2.86$

(b) If $\mathrm{y}(t) = \mathrm{x}(t/2)$ what is the numerical value of $\mathrm{y}(3)$?

$\mathrm{y}(3) = \mathrm{x}(3/2) = 4 - (4/7)(11/2) = 0.857$

(c) What is the numerical signal energy of $\mathrm{x}(t)$?
(You can time-shift a signal without changing its energy.)

The only non-zero values of $\mathrm{x}(t)$ lie on a straight line between $t = -4$ and $t = 3$. The signal energy of this signal is therefore the same as this signal shifted to the left so that it starts its non-zero values at $t = -7$ and goes to zero at $t = 0$. Such a signal would be described in its non-zero range by

$$\mathrm{x}(t) = -(4/7)t \quad , \ -7 \le t < 0$$

Its signal energy is $E_{\mathrm{x}} = \int_{-7}^{0} |(-4/7)t|^2 \, dt = (16/49) \int_{-7}^{0} (t^3/3) dt = (1/3)(16/49) \times 343 = 37.333$

6. A signal $\mathrm{x}[n]$ has the following description:

n	-2	-1	0	1	2	3	4	5	6	7
$\mathrm{x}[n]$	4	-1	7	2	-3	8	2	0	-1	5

Also $\mathrm{x}[n]$ is zero for $n < -2$ and for $n > 7$.

(a) If $\mathrm{y}[n] = \mathrm{x}[2n]$ what is the numerical value of $\mathrm{y}[2]$?

$\mathrm{y}[2] = \mathrm{x}[4] = 2$

(b) Some of the values of $\mathrm{x}[n]$ are not needed to form $\mathrm{y}[n]$. At which values of *n* from -2 through 7 do these unused values of $\mathrm{x}[n]$ occur?

The odd values are not used. Those are at -1,1,3,5,7.

(b) What is the numerical signal energy of $\mathrm{x}[n]$?

The signal energy is simply the sum of the squares of the values

$$E_{\mathrm{x}} = 16 + 1 + 49 + 4 + 9 + 64 + 4 + 0 + 1 + 25 = 173$$

7. If $x(t) = -2\,\mathrm{rect}(t/6)$, $h(t) = 2\delta(t+1) - 5\delta(t-2)$ and $y(t) = x(t) * h(t)$,

(a) Fill in the blanks with numbers.

t	-3.5	0	1.5	3.5
$x(t)$	________	________	________	________
$y(t)$	________	________	________	________

$$y(t) = x(t) * h(t) = -2\,\mathrm{rect}(t/6) * [2\delta(t+1) - 5\delta(t-2)] = -4\,\mathrm{rect}((t+1)/6) + 10\,\mathrm{rect}((t-2)/6)$$

t	-3.5	0	1.5	3.5
$x(t)$	0	-2	-2	0
$y(t)$	-4	6	6	10

(b) Find the numerical signal energy of $y(t)$.

Signal Energy of $y(t)$ is ____.

The numerical signal energy is the area under the square of the magnitude of $y(t)$. In this case that consists of three rectangles.

Signal Energy of $y(t)$ is $3 \times (-4)^2 + 3 \times (6)^2 + 3 \times (10)^2 = 3(16 + 36 + 100) = 456$

8. Circle the correct property for the systems described below.

(a) $\mathrm{y}(t)=3\mathrm{x}(\sin(t))$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant
Stable Unstable Static Dynamic Causal Non-Causal

Linear, Invertible, Time Invariant, Stable, Dynamic and Non-Causal

The most common errors on this problem were in believing that the system is Non-Invertible, Static, Time Variant and Causal. It is invertible because $\mathrm{x}(\sin(t))=\mathrm{y}(t)/3$. It is dynamic because y at any time depends on x at time $\sin(t)$ and that is, in general a different time. It is time invariant because

$\mathrm{x}_1(t)=\mathrm{g}(t)\Rightarrow \mathrm{y}_1(t)=\mathrm{g}(\sin(t))$ and $\mathrm{x}_2(t)=\mathrm{g}(t-t_0)\Rightarrow \mathrm{y}_2(t)=\mathrm{g}(\sin(t-t_0))=\mathrm{y}_1(t-t_0)$. And it is non-causal because, for example, $\mathrm{y}(-\pi)=3\mathrm{x}(\sin(-\pi))=3\mathrm{x}(0)$ and $\mathrm{x}(0)$ occurs after time $-\pi$.

(b) $\mathrm{y}[n]=\sqrt{\mathrm{x}[n+1]}$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant

Stable Unstable Static Dynamic Causal Non-Causal

Non-Linear, Invertible, Time Invariant, Stable, Dynamic and Non-Causal

(c) $2\mathrm{y}[n]-\mathrm{y}[n-1]=\mathrm{x}[n]$

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Solution of ECE 315 Final Examination F10

1. Four samples are taken from a continuous-time signal. They are

$$\mathrm{x}(0)=3\ ,\ \mathrm{x}(0.025)=-2\ ,\ \mathrm{x}(0.05)=-5\ ,\ \mathrm{x}(0.075)=-4$$

If these four samples cover exactly one period of a periodic continuous-time signal $\mathrm{x}(t)$ and $\mathrm{x}(t)\xleftrightarrow{\mathcal{F}}\mathrm{X}(f)$, what is the best estimate, based solely on this set of data, of the numerical value of the strength of the impulse in $\mathrm{X}(f)$ at $f=0$?

It is the sum of the values divided by the number of values, $\mathrm{X}(0)\cong\dfrac{3-2-5-4}{4}=-2$

2. Four values of a discrete-time signal are $\mathrm{x}[0]=-2$, $\mathrm{x}[1]=-7$, $\mathrm{x}[2]=11$, $\mathrm{x}[3]=3$.

(a) If these four values are exactly one period of a periodic discrete-time signal $\mathrm{x}[n]$ and $\mathrm{x}[n]\xleftrightarrow{\mathcal{F}}\mathrm{X}(e^{j\Omega})$, what is the numerical value of the strength of the impulse in $\mathrm{X}(e^{j\Omega})$ at $\Omega=0$?

It is the sum of the values divided by the number of values and multiplied by 2π ,

$$\mathrm{X}(e^{j0})=2\pi\frac{-2-7+11+3}{4}=10\pi/4=7.85$$

(b) If these four values are all the non-zero values of a discrete-time signal $\mathrm{x}[n]$ and $\mathrm{x}[n]\xleftrightarrow{\mathcal{F}}\mathrm{X}(e^{j\Omega})$, what is the numerical value of $\mathrm{X}(e^{j\Omega})$ at $\Omega=0$?

It is the sum of the values,

$$\mathrm{X}(e^{j0})=-2-7+11+3=5$$

3. Find the numerical values of the constants.

(a) $$4(0.5)^{n-3}\mathrm{u}[n-3] \xleftrightarrow{\mathcal{F}} \frac{Ae^{a\Omega}}{1+be^{c\Omega}}$$

$$4(0.5)^{n-3}\mathrm{u}[n-3] \xleftrightarrow{\mathcal{F}} \frac{4e^{-j3\Omega}}{1-0.5e^{-j\Omega}}$$

(b) $$2(\mathrm{u}[n+3]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} Ae^{aF}\,\mathrm{drcl}(bF,B)$$

$$2(\mathrm{u}[n+3]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} 12e^{j\pi F}\,\mathrm{drcl}(F,6)$$

(c) $$Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \frac{2+e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

This problem is flawed and has no solution. It was not graded. The error is in the numerator. It should have been $2+0.5e^{-j\Omega}$ instead of $2+e^{-j\Omega}$. With that change the solution is

$$Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.25e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

Using $\alpha^n\cos(\Omega_0 n)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{1-\alpha\cos(\Omega_0)e^{-j\Omega}}{1-2\alpha\cos(\Omega_0)e^{-j\Omega}+\alpha^2e^{-j2\Omega}}$, $|\alpha|<1$

$\alpha^2=0.25$ and $\alpha=-0.5$

If $\alpha=-0.5 \Rightarrow \cos(\Omega_0)=0.5 \Rightarrow \Omega_0=\cos^{-1}(-0.5)=\pm\pi/3$

$$2(-0.5)^n\cos(\pm\pi n/3)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.5e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

4. Find the numerical values of the constants.

(a) $$5\cos(2\pi n/14) \xleftrightarrow[18]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$$

There was a typo in this problem. The "18" should have been a "14". If a student used the formulas consistent with "18" I did not count the answer wrong even though the result was absurd.

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$5\cos(2\pi n/14) \xleftrightarrow[14]{\mathcal{DFT}} 35(\delta_{14}[k-1]+\delta_{14}[k+1])$$

(b) $$5\cos(2\pi n/14) \xleftrightarrow[42]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$$

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$5\cos(2\pi n/14) \xleftrightarrow[42]{\mathcal{DFT}} 105(\delta_{42}[k-3]+\delta_{42}[k+3])$$

(c) $$4(\mathrm{u}[n+1]-\mathrm{u}[n-1])*\delta_9[n] \xleftrightarrow[9]{\mathcal{DFT}} Ae^{bk}\,\mathrm{drcl}(ck,D)$$

Using $(\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1])*\delta_N[n] \xleftrightarrow[mN]{\mathcal{DFT}} \dfrac{e^{-j\pi k(n_1+n_0)/mN}}{e^{-j\pi k/mN}}(n_1-n_0)m\,\mathrm{drcl}(k/mN,n_1-n_0)\delta_m[k]$

$$4(\mathrm{u}[n+1]-\mathrm{u}[n-1])*\delta_9[n] \xleftrightarrow[9]{\mathcal{DFT}} 8e^{j\pi k/9}\,\mathrm{drcl}(k/9,2)$$

5. A signal $x(t)$ has the following description:

1. It is zero for all time $t < -4$.
2. It is a straight line from the point $t = -4, x = 0$ to the point $t = -4, x = 6$.
3. It is a straight line from the point $t = -4, x = 6$ to the point $t = 3, x = 0$.
4. It is zero for all time $t > 3$.

(a) What is the numerical value of $x(-2)$?

$x(-1) = 6 - (6/7)(2) = 30/7 = 4.286$

(b) If $y(t) = x(t/2)$ what is the numerical value of $y(3)$? $y(3) =$ ____________

$y(3) = x(3/2) = 6 - (6/7)(11/2) = 1.286$

(c) What is the numerical signal energy of $x(t)$?
(You can time-shift a signal without changing its energy.)

The only non-zero values of $x(t)$ lie on a straight line between $t = -4$ and $t = 3$. The signal energy of this signal is therefore the same as this signal shifted to the left so that it starts its non-zero values at $t = -7$ and goes to zero at $t = 0$. Such a signal would be described in its non-zero range by

$$x(t) = -(6/7)t \quad , \ -7 \le t < 0$$

Its signal energy is $E_x = \int_{-7}^{0} |(-6/7)t|^2 \, dt = (36/49)\int_{-7}^{0} (t^3/3) dt = (1/3)(36/49) \times 343 = 84$

6. A signal $x[n]$ has the following description:

n	-2	-1	0	1	2	3	4	5	6	7
$x[n]$	4	-1	7	2	-3	8	-4	0	-1	5

Also $x[n]$ is zero for $n < -2$ and for $n > 7$.

(a) If $y[n] = x[2n]$ what is the numerical value of $y[2]$?

$y[2] = x[4] = -4$

(b) Some of the values of $x[n]$ are not needed to form $y[n]$. At which values of n from -2 through 7 do these unused values of $x[n]$ occur?

The odd values are not used. Those are at -1,1,3,5,7.

(b) What is the numerical signal energy of $x[n]$?

The signal energy is simply the sum of the squares of the values

$$E_x = 16 + 1 + 49 + 4 + 9 + 64 + 16 + 0 + 1 + 25 = 185$$

7. If $x(t) = -3\text{rect}(t/6)$, $h(t) = 2\delta(t+1) - 5\delta(t-2)$ and $y(t) = x(t) * h(t)$,

(a) Fill in the blanks with numbers.

t	-3.5	0	1.5	3.5
$x(t)$	________	________	________	________
$y(t)$	________	________	________	________

$$y(t) = x(t) * h(t) = -3\text{rect}(t/6) * [2\delta(t+1) - 5\delta(t-2)] = -6\,\text{rect}((t+1)/6) + 15\,\text{rect}((t-2)/6)$$

t	-3.5	0	1.5	3.5
$x(t)$	0	-3	-3	0
$y(t)$	-6	9	9	15

(b) Find the numerical signal energy of $y(t)$.

Signal Energy of $y(t)$ is ____________________________________.

The numerical signal energy is the area under the square of the magnitude of $y(t)$. In this case that consists of three rectangles.

Signal Energy of $y(t)$ is $3 \times (-6)^2 + 3 \times (9)^2 + 3 \times (15)^2 = 3(36 + 81 + 225) = 1026$

8. Circle the correct property for the systems described below.

(a) $y(t) = 3x(\sin(t))$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant
Stable Unstable Static Dynamic Causal Non-Causal

Linear, Invertible, Time Invariant, Stable, Dynamic and Non-Causal

The most common errors on this problem were in believing that the system is Non-Invertible, Static, Time Variant and Causal. It is invertible because $x(\sin(t)) = y(t)/3$. It is dynamic because y at any time depends on x at time $\sin(t)$ and that is, in general a different time. It is time invariant because

$x_1(t) = g(t) \Rightarrow y_1(t) = g(\sin(t))$ and $x_2(t) = g(t - t_0) \Rightarrow y_2(t) = g(\sin(t - t_0)) = y_1(t - t_0)$. And it is non-causal because, for example, $y(-\pi) = 3x(\sin(-\pi)) = 3x(0)$ and $x(0)$ occurs after time $-\pi$.

(b) $y[n] = \sqrt{x[n+1]}$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant

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(c) $2y[n] - y[n-1] = x[n]$

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Solution of ECE 315 Final Examination F10

1. Four samples are taken from a continuous-time signal. They are

$$x(0) = 3 \;,\; x(0.025) = -2 \;,\; x(0.05) = -8 \;,\; x(0.075) = -4$$

If these four samples cover exactly one period of a periodic continuous-time signal $x(t)$ and $x(t) \xleftrightarrow{\mathcal{F}} X(f)$, what is the best estimate, based solely on this set of data, of the numerical value of the strength of the impulse in $X(f)$ at $f = 0$?

It is the sum of the values divided by the number of values, $X(0) \cong \frac{3-2-8-4}{4} = -11/4$

2. Four values of a discrete-time signal are $x[0] = -2$, $x[1] = 7$, $x[2] = -11$, $x[3] = 3$.

(a) If these four values are exactly one period of a periodic discrete-time signal $x[n]$ and $x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\Omega})$, what is the numerical value of the strength of the impulse in $X(e^{j\Omega})$ at $\Omega = 0$?

It is the sum of the values divided by the number of values and multiplied by 2π ,

$$X(e^{j0}) = 2\pi \frac{-2+7-11+3}{4} = -6\pi / 4 = -4.71$$

(b) If these four values are all the non-zero values of a discrete-time signal $x[n]$ and $x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\Omega})$, what is the numerical value of $X(e^{j\Omega})$ at $\Omega = 0$?

It is the sum of the values,

$$X(e^{j0}) = -2+7-11+3 = -3$$

3. Find the numerical values of the constants.

(a) $5(0.7)^{n-4}\,\mathrm{u}[n-4] \xleftrightarrow{\mathcal{F}} \dfrac{Ae^{a\Omega}}{1+be^{c\Omega}}$

$$5(0.7)^{n-4}\,\mathrm{u}[n-4] \xleftrightarrow{\mathcal{F}} \frac{5e^{-j4\Omega}}{1-0.7e^{-j\Omega}}$$

(b) $-6(\mathrm{u}[n+5]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} Ae^{aF}\,\mathrm{drcl}(bF,B)$

$$-6(\mathrm{u}[n+5]-\mathrm{u}[n-3]) \xleftrightarrow{\mathcal{F}} -48e^{j3\pi F}\,\mathrm{drcl}(F,8)$$

(c) $Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{2+e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$

This problem is flawed and has no solution. It was not graded. The error is in the numerator. It should have been $2+0.5e^{-j\Omega}$ instead of $2+e^{-j\Omega}$. With that change the solution is

$$Aa^n\cos(bn)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.25e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

Using $\alpha^n\cos(\Omega_0 n)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} \dfrac{1-\alpha\cos(\Omega_0)e^{-j\Omega}}{1-2\alpha\cos(\Omega_0)e^{-j\Omega}+\alpha^2e^{-j2\Omega}}$, $|\alpha|<1$

$\alpha^2 = 0.25$ and $\alpha = -0.5$

If $\alpha=-0.5 \Rightarrow \cos(\Omega_0)=0.5 \Rightarrow \Omega_0=\cos^{-1}(-0.5)=\pm\pi/3$

$$2(-0.5)^n\cos(\pm\pi n/3)\mathrm{u}[n] \xleftrightarrow{\mathcal{F}} 2\frac{1+0.5e^{-j\Omega}}{1+0.5e^{-j\Omega}+0.25e^{-j2\Omega}}$$

4. Find the numerical values of the constants.

(a) $-12\cos(2\pi n/7) \xleftrightarrow[7]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$-12\cos(2\pi n/7) \xleftrightarrow[7]{\mathcal{DFT}} -42(\delta_7[k-1]+\delta_7[k+1])$$

(b) $-12\cos(2\pi n/7) \xleftrightarrow[35]{\mathcal{DFT}} A(\delta_a[k-b]+\delta_a[k+b])$

$$\cos(2\pi qn/N) \xleftrightarrow[mN]{\mathcal{DFT}} (mN/2)(\delta_{mN}[k-mq]+\delta_{mN}[k+mq])$$
$$-12\cos(2\pi n/7) \xleftrightarrow[35]{\mathcal{DFT}} -210(\delta_{35}[k-5]+\delta_{35}[k+5])$$

(c) $3(\mathrm{u}[n+5]-\mathrm{u}[n-1]) * \delta_{15}[n] \xleftrightarrow[15]{\mathcal{DFT}} Ae^{bk}\,\mathrm{drcl}(ck,D)$

$$(\mathrm{u}[n-n_0]-\mathrm{u}[n-n_1]) * \delta_N[n] \xleftrightarrow[mN]{\mathcal{DFT}} \frac{e^{-j\pi k(n_1+n_0)/mN}}{e^{-j\pi k/mN}}(n_1-n_0)m\,\mathrm{drcl}(k/mN, n_1-n_0)\delta_m[k]$$
$$3(\mathrm{u}[n+5]-\mathrm{u}[n-1]) * \delta_{15}[n] \xleftrightarrow[15]{\mathcal{DFT}} 18e^{j\pi k/3}\,\mathrm{drcl}(k/15,6)$$

5. A signal $x(t)$ has the following description:

1. It is zero for all time $t < -4$.
2. It is a straight line from the point $t = -4, x = 0$ to the point $t = -4, x = 8$.
3. It is a straight line from the point $t = -4, x = 8$ to the point $t = 3, x = 0$.
4. It is zero for all time $t > 3$.

(a) What is the numerical value of $x(-2)$?

$x(-2) = 8 - (8/7)(2) = 40/7 = 5.71$

(b) If $y(t) = x(t/2)$ what is the numerical value of $y(3)$? $y(3) =$ ____________

$y(3) = x(3/2) = 8 - (8/7)(11/2) = 1.714$

(c) What is the numerical signal energy of $x(t)$?
(You can time-shift a signal without changing its energy.)

The only non-zero values of $x(t)$ lie on a straight line between $t = -4$ and $t = 3$. The signal energy of this signal is therefore the same as this signal shifted to the left so that it starts its non-zero values at $t = -7$ and goes to zero at $t = 0$. Such a signal would be described in its non-zero range by

$$x(t) = -(8/7)t \quad , \ -7 \le t < 0$$

Its signal energy is $E_x = \int_{-7}^{0} |(-8/7)t|^2 \, dt = (64/49) \int_{-7}^{0} t^2 \, dt = (1/3)(64/49) \times 343 = 149.333$

6. A signal $x[n]$ has the following description:

n	-2	-1	0	1	2	3	4	5	6	7
$x[n]$	4	-1	7	2	-3	8	-5	0	-1	5

Also $x[n]$ is zero for $n < -2$ and for $n > 7$.

(a) If $y[n] = x[2n]$ what is the numerical value of $y[2]$?

$y[2] = x[4] = -5$

(b) Some of the values of $x[n]$ are not needed to form $y[n]$. At which values of *n* from -2 through 7 do these unused values of $x[n]$ occur?

The odd values are not used. Those are at -1,1,3,5,7.

(b) What is the numerical signal energy of $x[n]$?

The signal energy is simply the sum of the squares of the values

$$E_x = 16 + 1 + 49 + 4 + 9 + 64 + 25 + 0 + 1 + 25 = 194$$

7. If $\mathrm{x}(t) = -4\,\mathrm{rect}(t/6)$, $\mathrm{h}(t) = 2\delta(t+1) - 5\delta(t-2)$ and $\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t)$,

(a) Fill in the blanks with numbers.

t	-3.5	0	1.5	3.5
$\mathrm{x}(t)$	________	________	________	________
$\mathrm{y}(t)$	________	________	________	________

$$\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) = -4\,\mathrm{rect}(t/6) * \left[2\delta(t+1) - 5\delta(t-2)\right] = -8\,\mathrm{rect}((t+1)/6) + 20\,\mathrm{rect}((t-2)/6)$$

t	-3.5	0	1.5	3.5
$\mathrm{x}(t)$	0	-4	-4	0
$\mathrm{y}(t)$	-8	12	12	20

(b) Find the numerical signal energy of $\mathrm{y}(t)$.

Signal Energy of $\mathrm{y}(t)$ is ______________________________.

The numerical signal energy is the area under the square of the magnitude of $\mathrm{y}(t)$. In this case that consists of three rectangles.

Signal Energy of $\mathrm{y}(t)$ is $3\times(-8)^2 + 3\times(12)^2 + 3\times(20)^2 = 3(64 + 144 + 400) = 1824$

8. Circle the correct property for the systems described below.

(a) $y(t) = 3x(\sin(t))$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant
Stable Unstable Static Dynamic Causal Non-Causal

Linear, Invertible, Time Invariant, Stable, Dynamic and Non-Causal

The most common errors on this problem were in believing that the system is Non-Invertible, Static, Time Variant and Causal. It is invertible because $x(\sin(t)) = y(t)/3$. It is dynamic because y at any time depends on x at time $\sin(t)$ and that is, in general a different time. It is time invariant because

$x_1(t) = g(t) \Rightarrow y_1(t) = g(\sin(t))$ and $x_2(t) = g(t - t_0) \Rightarrow y_2(t) = g(\sin(t - t_0)) = y_1(t - t_0)$. And it is non-causal because, for example, $y(-\pi) = 3x(\sin(-\pi)) = 3x(0)$ and $x(0)$ occurs after time $-\pi$.

(b) $y[n] = \sqrt{x[n+1]}$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant

Stable Unstable Static Dynamic Causal Non-Causal

Non-Linear, Invertible, Time Invariant, Stable, Dynamic and Non-Causal

(c) $2y[n] - y[n-1] = x[n]$

Linear Non-Linear Invertible Non-Invertible Time Invariant Time Variant

Stable Unstable Static Dynamic Causal Non-Causal

Linear, Invertible, Time Invariant, Stable, Dynamic and Causal