

Solution of EECS Final Examination F13

1. Given the $g[n]$ graphed below, if $h[n] = g[2n-3]$ and $x[n] = h[n] - h[n-1]$ and $y[n] = \sum_{m=-\infty}^n h[m]$ find the numerical values of

(a) $h[-2] = \underline{\hspace{2cm}}$

$$h[-2] = g[2(-2) - 3] = g[-7] = 4$$

(b) The signal energy of $g[n] = \underline{\hspace{2cm}}$

$$E_g = \sum_{n=-\infty}^{\infty} |g[n]|^2 = 36 + 16 + 4 + 0 + 4 + 16 + 4 + 0 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4$$

$$E_g = \sum_{n=-\infty}^{\infty} |g[n]|^2 = 36 + 16 + 16 + 48 = 116$$

(c) $x[-1] = \underline{\hspace{2cm}}$

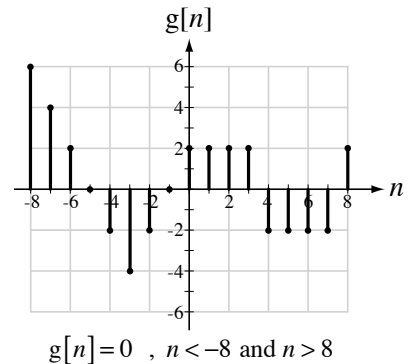
$$x[-1] = h[-1] - h[-2] = g[2(-1) - 3] - g[2(-2) - 3] = g[-5] - g[-7] = 0 - 4 = -4$$

(d) $y[2] = \underline{\hspace{2cm}}$

Since $h[n] = g[2n-3]$ and $g[n]$ is zero for $n < -8$,

$$h[n] = 0 \text{ for } 2n - 3 < -8 \text{ or } 2n < -5 \text{ or } n < -2.5 \Rightarrow n < -2 \text{ (} n \text{ must be an integer)}$$

$$y[2] = \sum_{m=-\infty}^2 h[m] = \sum_{m=-2}^2 h[m] = \sum_{m=-2}^2 g[2m-3] = [4 + 0 - 4 + 0 + 2] = 2$$



2. Refer to the system whose block diagram description is below with $a = 5$ and $b = 2$.

- (a) Fill in the blanks in the differential equation with correct numbers. (Be sure to observe the signs on the summers.)

$$(\text{_____})y''(t) + (\text{_____})y'(t) + (\text{_____})y(t) = (\text{_____})x(t)$$

$$y''(t) + 5y'(t) + 2y(t) = x(t)$$

- (b) The impulse response of this system can be written in the form, $h(t) = (K_1 e^{s_1 t} + K_2 e^{s_2 t})u(t)$. Find the numerical values of K_1 , K_2 , s_1 and s_2 .

$$K_1 = \text{_____}, \quad K_2 = \text{_____}$$

$$s_1 = \text{_____}, \quad s_2 = \text{_____}$$

Eigenvalues are the solutions of $s^2 + 5s + 2 = 0$, which are $s_{1,2} = -4.5616$ and -0.4384 .

$$h''(t) + 5h'(t) + 2h(t) = \delta(t)$$

Integrating once from 0^- to 0^+ ,

$$\underbrace{h'(0^+) - h'(0^-)}_{=s_1 K_1 + s_2 K_2} + 5 \underbrace{[h(0^+) - h(0^-)]}_{=0} + 2 \underbrace{\int_{0^-}^{0^+} h(t) dt}_{=0} = \underbrace{u(0^+) - u(0^-)}_1$$

$$-4.5616K_1 - 0.4384K_2 + 5K_1 + 5K_2 = 1$$

Integrating twice from 0^- to 0^+ ,

$$\underbrace{h(0^+) - h(0^-)}_{=K_1 + K_2} + 5 \underbrace{[0] + 2[0]}_{=0} = \underbrace{\text{ramp}(0^+) - \text{ramp}(0^-)}_0$$

$$K_1 + K_2 = 0$$

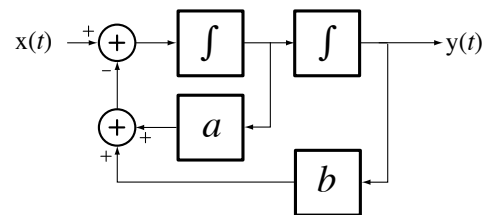
$$\begin{bmatrix} 0.4384 & 4.5616 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -0.2425 \\ 0.2425 \end{bmatrix}$$

- (c) Is this system stable? Yes No

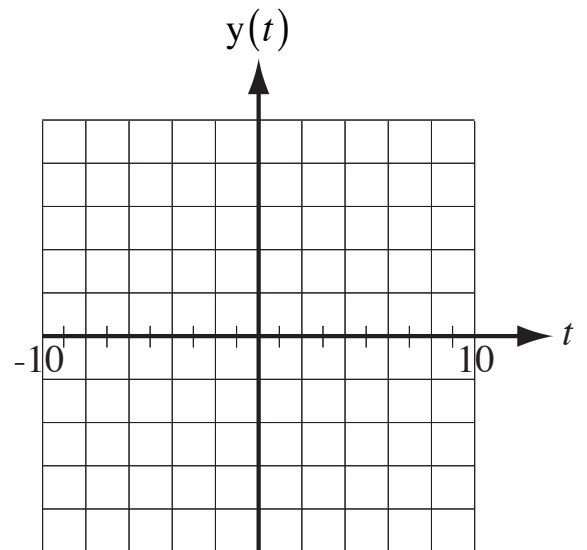
Yes

Explain how you know.

Both eigenvalues have negative real parts.

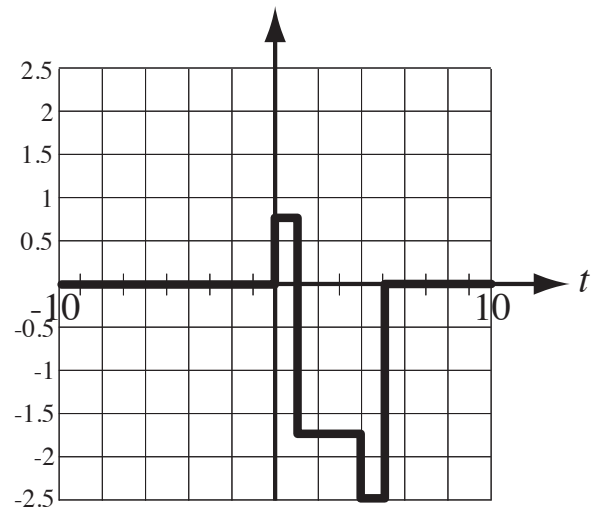


3. Let $x(t) = 0.5 \operatorname{rect}\left(\frac{t-2}{4}\right)$ and $h(t) = 3\delta(2t) - 5\delta(t-1)$ and let $y(t) = h(t) * x(t)$. Graph $y(t)$ in the space provided below. The graph should clearly show all points where $y(t)$ is discontinuous. Provide a numerical scale for the vertical axis.



$$y(t) = \left[1.5\delta(2t) * \operatorname{rect}\left(\frac{t-2}{4}\right) - 2.5\delta(t-1) * \operatorname{rect}\left(\frac{t-2}{4}\right) \right]$$

$$y(t) = \left[0.75 \operatorname{rect}\left(\frac{t-2}{4}\right) - 2.5 \operatorname{rect}\left(\frac{t-3}{4}\right) \right]$$



4. A continuous-time system has a frequency response $H(f) = 4 \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/16}$. A signal $x(t) = \operatorname{rect}(16t) * \delta_{1/8}(t)$ excites it.

- (a) Find the numerical CTFT of the excitation written as a product of a sinc function and a periodic impulse. As far as possible reduce all expressions to numerical values.

$$X(f) = \underline{\hspace{15cm}}$$

$$X(f) = (1/16) \operatorname{sinc}(f/16) \times 8 \delta_8(f) = (1/2) \operatorname{sinc}(f/16) \delta_8(f)$$

- (b) Find the numerical CTFT of the response written as the sum of three impulses. As far as possible reduce all expressions to numerical values.

$$Y(f) = \underline{\hspace{15cm}}$$

$$Y(f) = (1/2) \operatorname{sinc}(f/16) \delta_8(f) \times 4 \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/16}$$

$$Y(f) = 2 \operatorname{sinc}(f/16) \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/16} \sum_{k=-\infty}^{\infty} \delta(f - 8k)$$

$$Y(f) = 2 \sum_{k=-\infty}^{\infty} \operatorname{sinc}(k/2) \operatorname{rect}\left(\frac{8k}{20}\right) e^{-j\pi k/2} \delta(f - 8k)$$

$$Y(f) = 2 \left[\delta(f) + \operatorname{sinc}(1/2) e^{-j\pi/2} \delta(f - 8) + \operatorname{sinc}(-1/2) e^{j\pi/2} \delta(f + 8) \right]$$

$$Y(f) = 2\delta(f) + 1.2732 \left[-j\delta(f - 8) + j\delta(f + 8) \right]$$

- (c) The response is a single sinusoid, plus a constant, of the form $y(t) = A \cos(2\pi f_0 t + \theta) + B$. Find the numerical values of A , f_0 , θ and B .

$$A = \underline{\hspace{2cm}}, \quad f_0 = \underline{\hspace{2cm}} \text{ Hz}$$

$$\theta = \underline{\hspace{2cm}} \text{ radians}, \quad B = \underline{\hspace{2cm}}$$

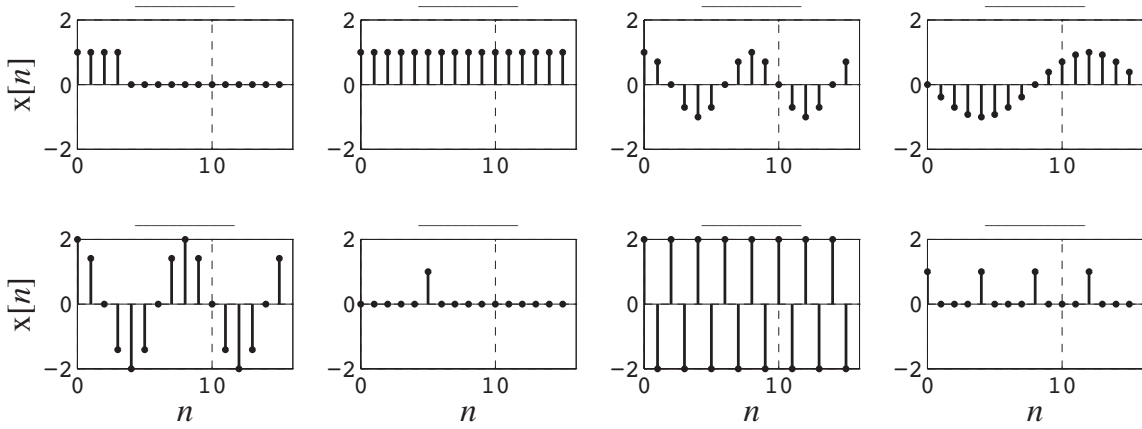
$$y(t) = 2 \left[1 + \operatorname{sinc}(1/2) e^{j16\pi t} e^{-j\pi/2} + \operatorname{sinc}(-1/2) e^{-j16\pi t} e^{j\pi/2} \right]$$

$$y(t) = 2 \left[1 + 2 \operatorname{sinc}(1/2) \cos(16\pi t - \pi/2) \right]$$

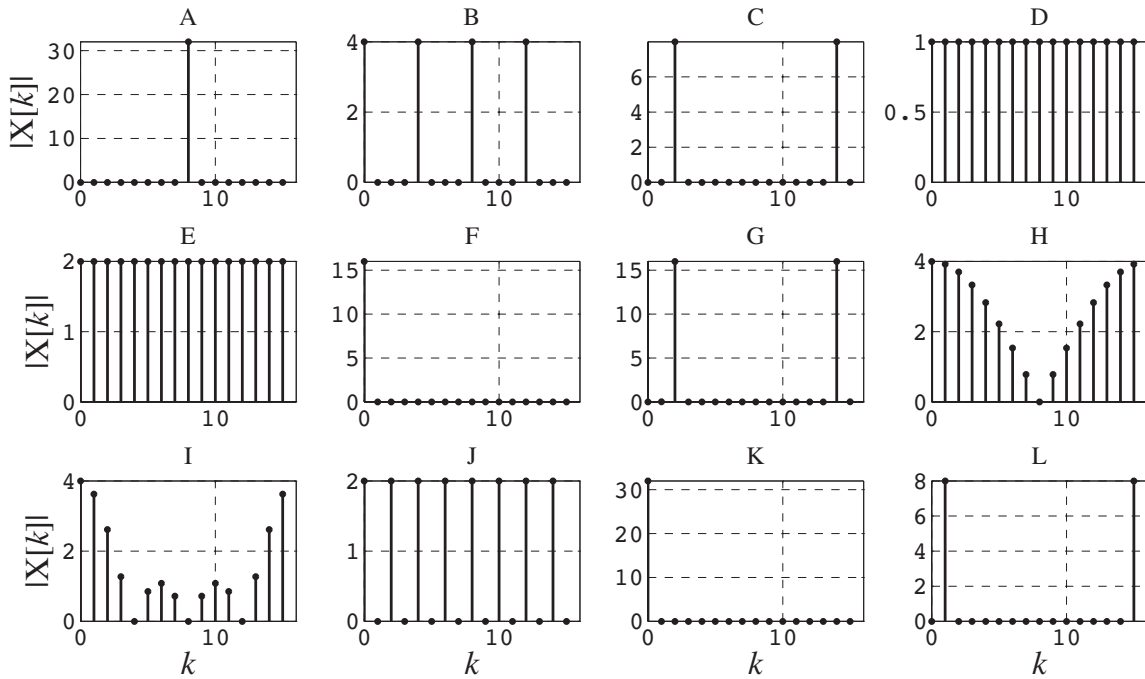
$$y(t) = 2 + 2.5465 \cos(16\pi t - \pi/2)$$

$$A = 2.5465, \quad f_0 = 8 \text{ Hz}, \quad \theta = -\pi/2, \quad B = 2$$

5. Below are graphs of some discrete-time signals and below them are some graphs of magnitudes of DFT's of discrete-time signals. Match the DFT magnitudes to the discrete-time signals by writing in each space provided above a discrete-time graph the letter designation of the corresponding DFT magnitude graph. These DFT magnitudes are the direct result of the FFT function in MATLAB. They have not been shifted or scaled in any way. If there is no match for a discrete-time signal just write in "None".



I F C L
G D A B



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1. Given the $g[n]$ graphed below, if $h[n] = g[2n-3]$ and $x[n] = h[n] - h[n-1]$ and $y[n] = \sum_{m=-\infty}^n h[m]$ find the numerical values of

(a) $h[-2] = \underline{\hspace{2cm}}$

$$h[-2] = g[2(-2)-3] = g[-7] = 6$$

(b) The signal energy of $g[n] = \underline{\hspace{2cm}}$

$$E_g = \sum_{n=-\infty}^{\infty} |g[n]|^2 = 4 + 36 + 16 + 0 + 16 + 36 + 16 + 0 + 4 + 16 + 36 + 36 + 4 + 4 + 16 + 16 + 36 = 292$$

(c) $x[-1] = \underline{\hspace{2cm}}$

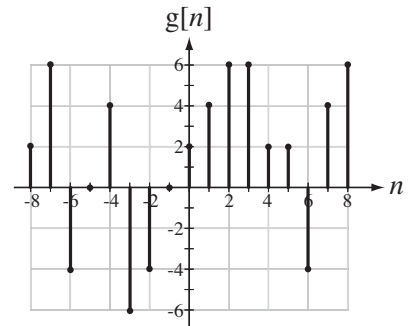
$$x[-1] = h[-1] - h[-2] = g[2(-1)-3] - g[2(-2)-3] = g[-5] - g[-7] = 0 - 6 = -6$$

(d) $y[2] = \underline{\hspace{2cm}}$

Since $h[n] = g[2n-3]$ and $g[n]$ is zero for $n < -8$,

$$h[n] = 0 \text{ for } 2n-3 < -8 \text{ or } 2n < -5 \text{ or } n < -2.5 \Rightarrow n < -2 \text{ (} n \text{ must be an integer)}$$

$$y[2] = \sum_{m=-\infty}^2 h[m] = \sum_{m=-2}^2 h[m] = \sum_{m=-2}^2 g[2m-3] = [6+0-6+0+4] = 4$$



$$g[n] = 0, \quad n < -8 \text{ and } n > 8$$

2. Refer to the system whose block diagram description is below with $a = 7$ and $b = 3$.

- (a) Fill in the blanks in the differential equation with correct numbers. (Be sure to observe the signs on the summers.)

$$(\text{_____})y''(t) + (\text{_____})y'(t) + (\text{_____})y(t) = (\text{_____})x(t)$$

$$y''(t) + 7y'(t) + 3y(t) = x(t)$$

- (b) The impulse response of this system can be written in the form, $h(t) = (K_1 e^{s_1 t} + K_2 e^{s_2 t})u(t)$. Find the numerical values of K_1 , K_2 , s_1 and s_2 .

$$K_1 = \text{_____}, \quad K_2 = \text{_____}$$

$$s_1 = \text{_____}, \quad s_2 = \text{_____}$$

Eigenvalues are the solutions of $s^2 + 7s + 3 = 0$, which are $s_{1,2} = -6.5414$ and -0.4586 .

$$h''(t) + 7h'(t) + 3h(t) = \delta(t)$$

Integrating once from 0^- to 0^+ ,

$$\underbrace{h'(0^+) - h'(0^-)}_{=s_1 K_1 + s_2 K_2} + 7 \underbrace{[h(0^+) - h(0^-)]}_{=0} + 3 \underbrace{\int_{0^-}^{0^+} h(t) dt}_{=0} = \underbrace{u(0^+) - u(0^-)}_1$$

$$-6.5414K_1 - 0.4586K_2 + 7K_1 + 7K_2 = 1$$

Integrating twice from 0^- to 0^+ ,

$$\underbrace{h(0^+) - h(0^-)}_{=K_1 + K_2} + 7 \underbrace{[0]}_{=0} + 3 \underbrace{[0]}_0 = \underbrace{\text{ramp}(0^+) - \text{ramp}(0^-)}_0$$

$$K_1 + K_2 = 0$$

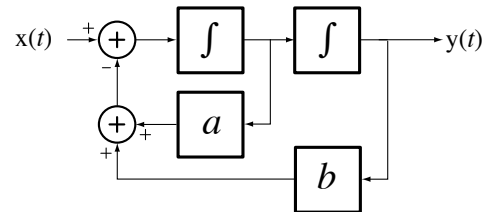
$$\begin{bmatrix} 0.4586 & 6.5414 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -0.1644 \\ 0.1644 \end{bmatrix}$$

- (c) Is this system stable? Yes No

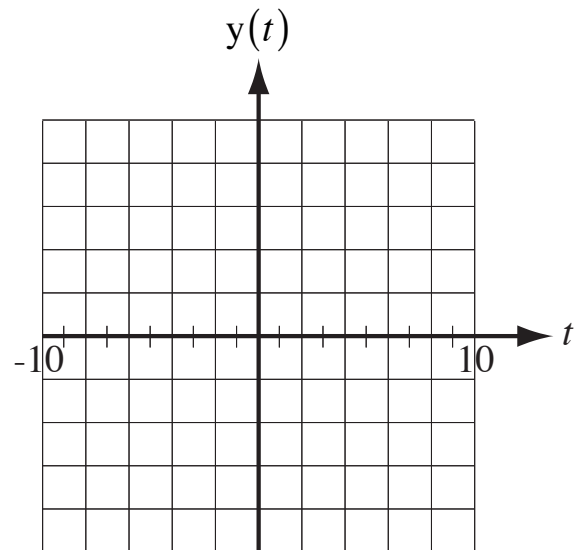
Yes

Explain how you know.

Both eigenvalues have negative real parts.

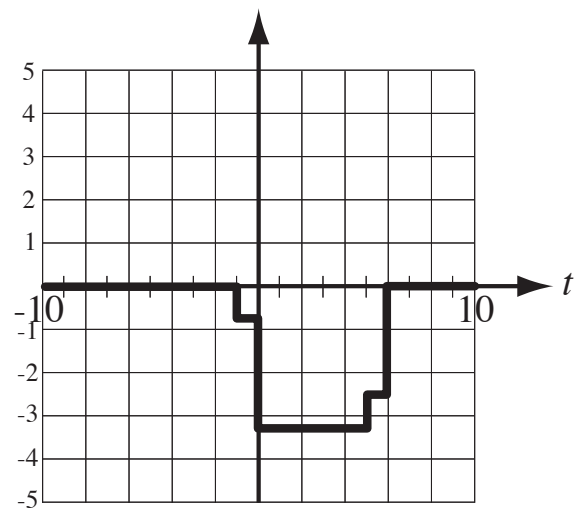


3. Let $x(t) = 0.5 \operatorname{rect}\left(\frac{t-2}{6}\right)$ and $h(t) = -3\delta(2t) - 5\delta(t-1)$ and let $y(t) = h(t) * x(t)$. Graph $y(t)$ in the space provided below. The graph should clearly show all points where $y(t)$ is discontinuous. Provide a numerical scale for the vertical axis.



$$y(t) = \left[-1.5\delta(2t) * \operatorname{rect}\left(\frac{t-2}{6}\right) - 2.5\delta(t-1) * \operatorname{rect}\left(\frac{t-2}{6}\right) \right]$$

$$y(t) = \left[-0.75 \operatorname{rect}\left(\frac{t-2}{6}\right) - 2.5 \operatorname{rect}\left(\frac{t-3}{6}\right) \right]$$



4. A continuous-time system has a frequency response $H(f) = 6 \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/14}$. A signal $x(t) = \operatorname{rect}(14t) * \delta_{1/7}(t)$ excites it.

- (a) Find the numerical CTFT of the excitation written as a product of a sinc function and a periodic impulse. As far as possible reduce all expressions to numerical values.

$$X(f) = \underline{\hspace{15cm}}$$

$$X(f) = (1/14) \operatorname{sinc}(f/14) \times 7 \delta_7(f) = (1/2) \operatorname{sinc}(f/14) \delta_7(f)$$

- (b) Find the numerical CTFT of the response written as the sum of three impulses. As far as possible reduce all expressions to numerical values.

$$Y(f) = \underline{\hspace{15cm}}$$

$$Y(f) = (1/2) \operatorname{sinc}(f/14) \delta_7(f) \times 6 \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/14}$$

$$Y(f) = 3 \operatorname{sinc}(f/14) \operatorname{rect}\left(\frac{f}{20}\right) e^{-j\pi f/14} \sum_{k=-\infty}^{\infty} \delta(f - 7k)$$

$$Y(f) = 3 \sum_{k=-\infty}^{\infty} \operatorname{sinc}(k/2) \operatorname{rect}\left(\frac{7k}{20}\right) e^{-j\pi k/2} \delta(f - 7k)$$

$$Y(f) = 3 \left[\delta(f) + \operatorname{sinc}(1/2) e^{-j\pi/2} \delta(f - 7) + \operatorname{sinc}(-1/2) e^{j\pi/2} \delta(f + 7) \right]$$

$$Y(f) = 3\delta(f) + 1.9099 \left[-j\delta(f - 7) + j\delta(f + 7) \right]$$

- (c) The response is a single sinusoid, plus a constant, of the form $y(t) = A \cos(2\pi f_0 t + \theta) + B$. Find the numerical values of A , f_0 , θ and B .

$$A = \underline{\hspace{2cm}}, \quad f_0 = \underline{\hspace{2cm}} \text{ Hz}$$

$$\theta = \underline{\hspace{2cm}} \text{ radians}, \quad B = \underline{\hspace{2cm}}$$

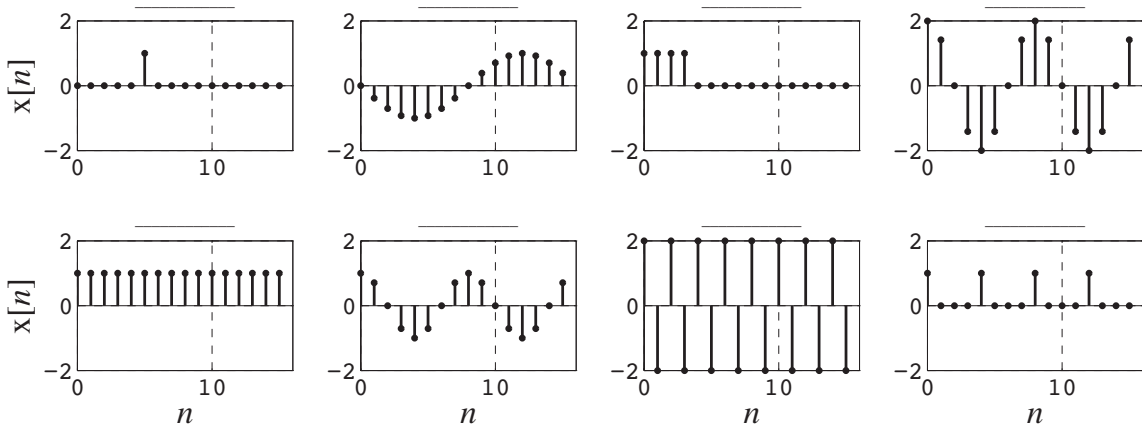
$$y(t) = 3 \left[1 + \operatorname{sinc}(1/2) e^{j14\pi t} e^{-j\pi/2} + \operatorname{sinc}(-1/2) e^{-j14\pi t} e^{j\pi/2} \right]$$

$$y(t) = 3 \left[1 + 2 \operatorname{sinc}(1/2) \cos(14\pi t - \pi/2) \right]$$

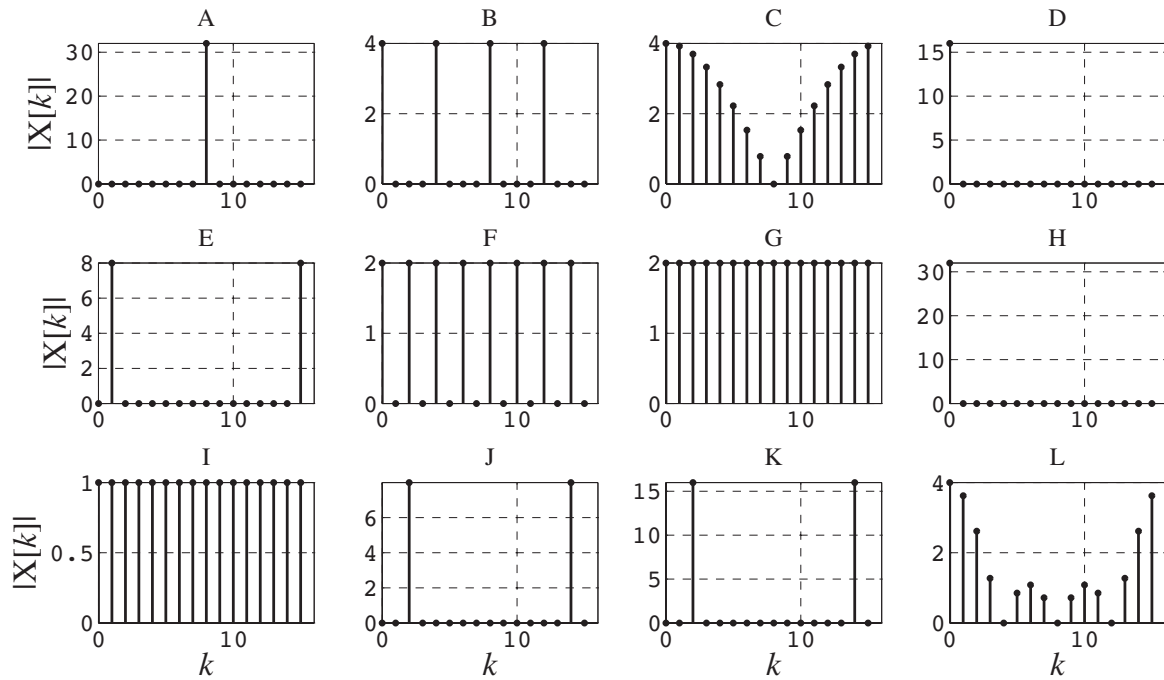
$$y(t) = 3 + 3.8197 \cos(14\pi t - \pi/2)$$

$$A = 3.8197, \quad f_0 = 7 \text{ Hz}, \quad \theta = -\pi/2, \quad B = 3$$

5. Below are graphs of some discrete-time signals and below them are some graphs of magnitudes of DFT's of discrete-time signals. Match the DFT magnitudes to the discrete-time signals by writing in each space provided above a discrete-time graph the letter designation of the corresponding DFT magnitude graph. These DFT magnitudes are the direct result of the FFT function in MATLAB. They have not been shifted or scaled in any way. If there is no match for a discrete-time signal just write in "None".



IELK
DJAB



Solution of EECS Final Examination F13

1. Given the $g[n]$ graphed below, if $h[n] = g[2n-3]$ and $x[n] = h[n] - h[n-1]$ and $y[n] = \sum_{m=-\infty}^n h[m]$ find the numerical values of

(a) $h[-2] = \underline{\hspace{2cm}}$

$$h[-2] = g[2(-2) - 3] = g[-7] = 4$$

(b) The signal energy of $g[n] = \underline{\hspace{2cm}}$

$$E_g = \sum_{n=-\infty}^{\infty} |g[n]|^2 = 16 + 16 + 36 + 0 + 36 + 4 + 4 + 0 + 4 + 4 + 36 + 16 + 4 + 36 + 36 + 4 + 36 + 16 = 300$$

(c) $x[-1] = \underline{\hspace{2cm}}$

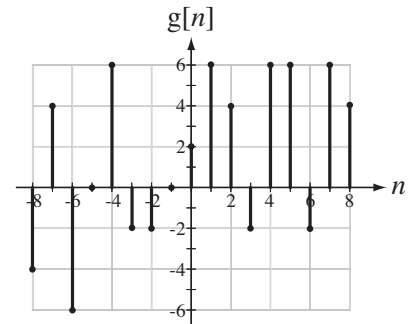
$$x[-1] = h[-1] - h[-2] = g[2(-1) - 3] - g[2(-2) - 3] = g[-5] - g[-7] = 0 - 4 = -4$$

(d) $y[2] = \underline{\hspace{2cm}}$

Since $h[n] = g[2n-3]$ and $g[n]$ is zero for $n < -8$,

$$h[n] = 0 \text{ for } 2n-3 < -8 \text{ or } 2n < -5 \text{ or } n < -2.5 \Rightarrow n < -2 \text{ (} n \text{ must be an integer)}$$

$$y[2] = \sum_{m=-\infty}^2 h[m] = \sum_{m=-2}^2 h[m] = \sum_{m=-2}^2 g[2m-3] = [4 + 0 - 2 + 0 + 6] = 8$$



$$g[n] = 0, \quad n < -8 \text{ and } n > 8$$

2. Refer to the system whose block diagram description is below with $a = 6$ and $b = 4$.

- (a) Fill in the blanks in the differential equation with correct numbers. (Be sure to observe the signs on the summers.)

$$(\text{_____})y''(t) + (\text{_____})y'(t) + (\text{_____})y(t) = (\text{_____})x(t)$$

$$y''(t) + 6y'(t) + 4y(t) = x(t)$$

- (b) The impulse response of this system can be written in the form, $h(t) = (K_1 e^{s_1 t} + K_2 e^{s_2 t})u(t)$. Find the numerical values of K_1 , K_2 , s_1 and s_2 .

$$K_1 = \text{_____}, \quad K_2 = \text{_____}$$

$$s_1 = \text{_____}, \quad s_2 = \text{_____}$$

Eigenvalues are the solutions of $s^2 + 6s + 4 = 0$, which are $s_{1,2} = -5.2361$ and -0.7639 .

$$h''(t) + 6h'(t) + 4h(t) = \delta(t)$$

Integrating once from 0^- to 0^+ ,

$$\underbrace{h'(0^+) - h'(0^-)}_{=s_1 K_1 + s_2 K_2} + \underbrace{6[h(0^+) - h(0^-)]}_{=0} + 4 \underbrace{\int_{0^-}^{0^+} h(t) dt}_{=0} = \underbrace{u(0^+) - u(0^-)}_1 - \underbrace{u(0^-)}_0$$

$$-5.2361K_1 - 0.7639K_2 + 6K_1 + 6K_2 = 1$$

Integrating twice from 0^- to 0^+ ,

$$\underbrace{h(0^+) - h(0^-)}_{=K_1 + K_2} + 6[0] + 4[0] = \underbrace{\text{ramp}(0^+) - \text{ramp}(0^-)}_0 - \underbrace{\text{ramp}(0^-)}_0$$

$$K_1 + K_2 = 0$$

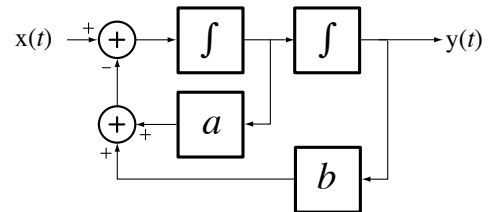
$$\begin{bmatrix} 0.7639 & 5.2361 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} -0.2236 \\ 0.2236 \end{bmatrix}$$

- (c) Is this system stable? Yes No

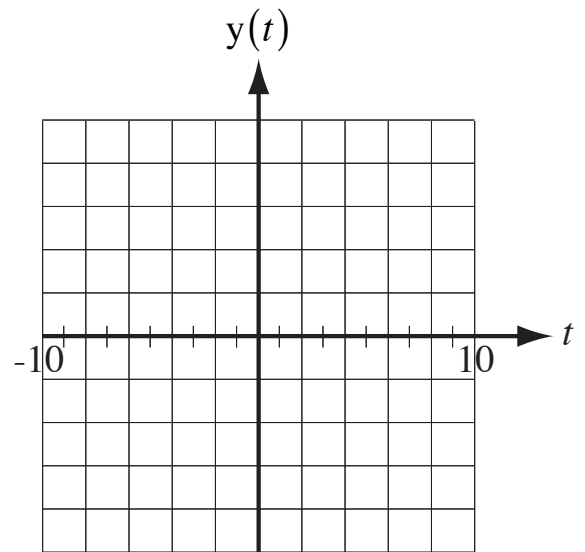
Yes

Explain how you know.

Both eigenvalues have negative real parts.

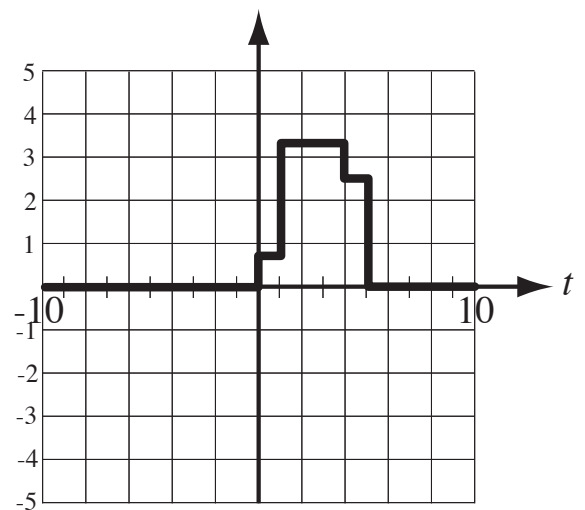


3. Let $x(t) = 0.5 \text{rect}\left(\frac{t-2}{4}\right)$ and $h(t) = 3\delta(2t) + 5\delta(t-1)$ and let $y(t) = h(t) * x(t)$. Graph $y(t)$ in the space provided below. The graph should clearly show all points where $y(t)$ is discontinuous. Provide a numerical scale for the vertical axis.



$$y(t) = \left[1.5\delta(2t) * \text{rect}\left(\frac{t-2}{4}\right) + 2.5\delta(t-1) * \text{rect}\left(\frac{t-2}{4}\right) \right]$$

$$y(t) = \left[0.75 \text{rect}\left(\frac{t-2}{4}\right) + 2.5 \text{rect}\left(\frac{t-3}{4}\right) \right]$$



4. A continuous-time system has a frequency response $H(f) = 3\text{rect}\left(\frac{f}{20}\right)e^{-j\pi f/12}$. A signal $x(t) = \text{rect}(12t) * \delta_{1/6}(t)$ excites it.

- (a) Find the numerical CTFT of the excitation written as a product of a sinc function and a periodic impulse. As far as possible reduce all expressions to numerical values.

$$X(f) = \underline{\hspace{15cm}}$$

$$X(f) = (1/12)\text{sinc}(f/12) \times 6\delta_6(f) = (1/2)\text{sinc}(f/12)\delta_6(f)$$

- (b) Find the numerical CTFT of the response written as the sum of three impulses. As far as possible reduce all expressions to numerical values.

$$Y(f) = \underline{\hspace{15cm}}$$

$$Y(f) = (1/2)\text{sinc}(f/12)\delta_6(f) \times 3\text{rect}\left(\frac{f}{20}\right)e^{-j\pi f/12}$$

$$Y(f) = 1.5\text{sinc}(f/12)\text{rect}\left(\frac{f}{20}\right)e^{-j\pi f/12} \sum_{k=-\infty}^{\infty} \delta(f-6k)$$

$$Y(f) = 1.5 \sum_{k=-\infty}^{\infty} \text{sinc}(k/2)\text{rect}\left(\frac{6k}{20}\right)e^{-j\pi k/2} \delta(f-6k)$$

$$Y(f) = 1.5[\delta(f) + \text{sinc}(1/2)e^{-j\pi/2}\delta(f-6) + \text{sinc}(-1/2)e^{j\pi/2}\delta(f+6)]$$

$$Y(f) = 1.5\delta(f) + 0.955[-j\delta(f-6) + j\delta(f+6)]$$

- (c) The response is a single sinusoid, plus a constant, of the form $y(t) = A\cos(2\pi f_0 t + \theta) + B$. Find the numerical values of A , f_0 , θ and B .

$$A = \underline{\hspace{2cm}}, \quad f_0 = \underline{\hspace{2cm}} \text{ Hz}$$

$$\theta = \underline{\hspace{2cm}} \text{ radians}, \quad B = \underline{\hspace{2cm}}$$

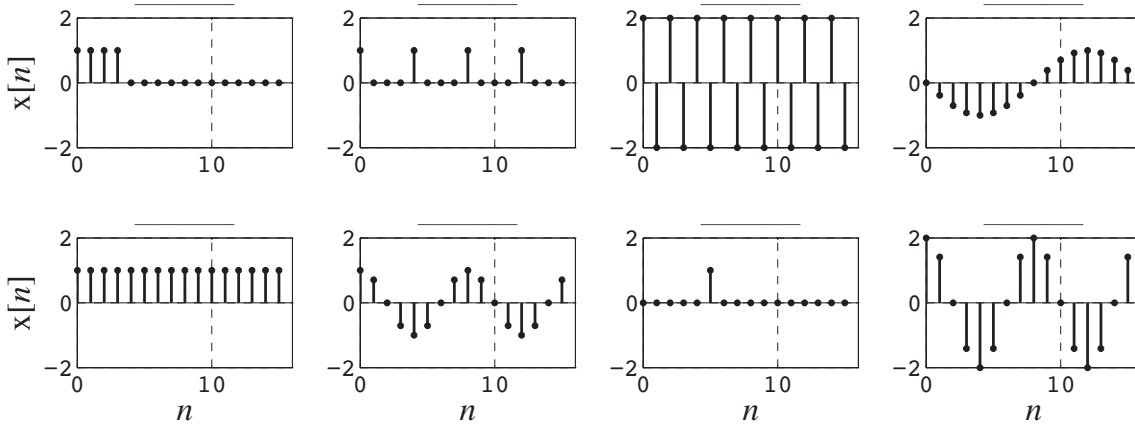
$$y(t) = 1.5[1 + \text{sinc}(1/2)e^{j12\pi t}e^{-j\pi/2} + \text{sinc}(-1/2)e^{-j12\pi t}e^{j\pi/2}]$$

$$y(t) = 1.5[1 + 2\text{sinc}(1/2)\cos(12\pi t - \pi/2)]$$

$$y(t) = 1.5 + 1.9099\cos(16\pi t - \pi/2)$$

$$A = 1.9099, \quad f_0 = 6 \text{ Hz}, \quad \theta = -\pi/2, \quad B = 1.5$$

5. Below are graphs of some discrete-time signals and below them are some graphs of magnitudes of DFT's of discrete-time signals. Match the DFT magnitudes to the discrete-time signals by writing in each space provided above a discrete-time graph the letter designation of the corresponding DFT magnitude graph. These DFT magnitudes are the direct result of the FFT function in MATLAB. They have not been shifted or scaled in any way. If there is no match for a discrete-time signal just write in "None".



F J L K
H E C D

