Solution to ECE Test #11 F03

1. A CT signal, $\mathrm{x}(t)$, has a CTFT, $\mathrm{X}(f) = 4\,\mathrm{rect}\left(\frac{f}{2}\right)$. A new signal, $\mathrm{x}_p(t)$, is formed by periodically repeating $\mathrm{x}(t)$ with a period of 8. The CTFS harmonic function of $\mathrm{x}_p(t)$ is $\mathrm{X}_p[k]$. Find $\mathrm{X}_p[k]$.

$$\mathrm{X}_p[k] = f_p\,\mathrm{X}(kf_p) \text{ where } f_p = \frac{1}{T_p} = \frac{1}{8}$$

$$\mathrm{X}_p[k] = \frac{1}{8}\mathrm{X}\left(\frac{k}{8}\right) = \frac{1}{2}\mathrm{rect}\left(\frac{k}{16}\right)$$

Find the numerical value of $\mathrm{X}_p[3]$

$$\mathrm{X}_p[3] = \frac{1}{2}\mathrm{rect}\left(\frac{3}{16}\right) = \frac{1}{2}$$

2. A DT signal, $\mathrm{x}[n]$, is formed by sampling a CT signal, $\mathrm{x}(t) = 12\,\mathrm{sinc}(5t)$, with a time between samples, $T_s = 0.1$. The DTFT of $\mathrm{x}[n]$ is $\mathrm{X}_{DTFT}(F)$.
Find the numerical value of $\mathrm{X}_{DTFT}(0.2)$.

$$\mathrm{X}_{DTFT}(F) = f_s \sum_{k=-\infty}^{\infty} \mathrm{X}_{CTFT}(f_s(F-k)) \quad , \quad \mathrm{X}_{CTFT}(f) = \frac{12}{5}\mathrm{rect}\left(\frac{f}{5}\right) \quad , \quad f_s = \frac{1}{T_s} = 10$$

$$\mathrm{X}_{DTFT}(F) = \frac{12 \times 10}{5} \sum_{k=-\infty}^{\infty} \mathrm{rect}\left(\frac{10(F-k)}{5}\right) = 24 \sum_{k=-\infty}^{\infty} \mathrm{rect}(2(F-k))$$

$$\mathrm{X}_{DTFT}(0.2) = 24 \sum_{k=-\infty}^{\infty} \mathrm{rect}(2(0.2-k)) = 24\,\mathrm{rect}(0.4) = 24$$

Alternate Solution:

$$\mathrm{x}[n] = \mathrm{x}(nT_s) = 12\,\mathrm{sinc}(5nT_s) = 12\,\mathrm{sinc}\left(\frac{n}{2}\right)$$

Using $\mathrm{sinc}\left(\frac{n}{w}\right) = w\,\mathrm{rect}(wF) * \mathrm{comb}(F)$,

$$\mathrm{X}(F) = 12 \times 2\,\mathrm{rect}(2F) * \mathrm{comb}(F) = 24\,\mathrm{rect}(2F) * \sum_{k=-\infty}^{\infty} \delta(F-k)$$

$$\mathrm{X}(F) = 24 \sum_{k=-\infty}^{\infty} \mathrm{rect}(2(F-k))$$

$$\mathrm{X}_{DTFT}(0.2) = 24 \sum_{k=-\infty}^{\infty} \mathrm{rect}(2(0.2-k)) = 24\,\mathrm{rect}(0.4) = 24$$