Solution to ECE Test #2 Su04

1. For any even periodic function of time, its complex Fourier series harmonic function using the fundamental period as the representation time is always <u>purely real.</u>

2. For any real-valued signal, the CTFS harmonic function has a magnitude that is an <u>even</u> function of harmonic number, *k*.

3. The signal, $\mathrm{x}[n]=1$, has a fundamental period, $N_0=1$.

(a) Find its DTFS harmonic function using that fundamental period as the representation time.

$$\mathrm{X}[k]=\mathrm{comb}_1[k]=1$$

(b) Now let $\mathrm{z}[n]=\begin{cases}\mathrm{x}\left[\frac{n}{4}\right] & , \ \frac{n}{4} \text{ an integer} \\ 0 & , \ \text{otherwise}\end{cases}$. Find the DTFS harmonic function for $\mathrm{z}[n]$ using its fundamental period as the representation time.

$$\mathrm{Z}[k]=\frac{1}{4}\mathrm{X}[k]=\frac{1}{4}$$

(c) Verify that $\mathrm{z}[0]=1$ and that $\mathrm{z}[1]=0$ by using the DTFS representation of z,

$$\mathrm{z}[n]=\sum_{k=\langle N_F\rangle}\mathrm{Z}[k]e^{j2\pi(kF_0)n} \quad .$$

$$\mathrm{z}[0]=\sum_{k=\langle 4\rangle}\frac{1}{4}e^0=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1 \quad \text{Check}$$

$$\mathrm{z}[1]=\sum_{k=\langle 4\rangle}\frac{1}{4}e^{j\frac{2\pi k}{4}}=\frac{1}{4}\left(e^0+e^{j\frac{\pi}{2}}+e^{j\pi}+e^{j\frac{3\pi}{2}}\right)=\frac{1}{4}(1+j-1-j)=0 \quad \text{Check}$$

4. A signal, $\mathrm{x}(t)=4\cos(30\pi t)-3$ can be represented for all time by a CTFS but its integral, $\int_{-\infty}^{t}\mathrm{x}(\lambda)d\lambda$, cannot. Why? Because the integral is not periodic.

5. Using Parseval's theorem, find the numerical value of the signal power of the signal, $x(t) = 3\cos(10\pi t) - 2\sin(10\pi t)$ from its CTFS harmonic function.

$$X[k] = \frac{3}{2}(\delta[k-1] + \delta[k+1]) - j(\delta[k+1] - \delta[k-1]) = \delta[k-1]\left(\frac{3}{2} + j\right) + \delta[k+1]\left(\frac{3}{2} - j\right)$$

$$P_x = \sum_{k=-\infty}^{\infty} |X[k]|^2 = \sum_{k=-\infty}^{\infty} \left|\delta[k-1]\left(\frac{3}{2} + j\right) + \delta[k+1]\left(\frac{3}{2} - j\right)\right|^2$$

$$P_x = \sum_{k=-\infty}^{\infty} \left(\frac{3}{2}\delta[k-1] + \frac{3}{2}\delta[k+1]\right)^2 + (\delta[k-1] - \delta[k+1])^2$$

$$P_x = \sum_{k=-\infty}^{\infty} \frac{9}{4}\left(\underbrace{\delta^2[k-1]}_{=\delta[k-1]} + \underbrace{\delta^2[k+1]}_{-\delta[k+1]} + \underbrace{2\delta[k-1]\delta[k+1]}_{=0}\right) + \left(\underbrace{\delta^2[k-1]}_{=\delta[k-1]} + \underbrace{\delta^2[k+1]}_{-\delta[k+1]} - \underbrace{2\delta[k-1]\delta[k+1]}_{=0}\right)$$

$$P_x = \frac{9}{4} + \frac{9}{4} + 1 + 1 = \frac{13}{2} = 6.5$$

6. A DT signal, $x[n]$ with fundamental period, $N_0 = 6$, has a DTFS harmonic function, $X[k]$, with the following numerical values at the given harmonic numbers, *k*.

k	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$X[k]$	1	$-j2$	-1	4	-1	$j2$	1	$-j2$	-1	4	-1	$j2$	1

(a) Find the numerical value of $x[0]$.

$$x[n] = \sum_{k=\langle 6\rangle} X[k]e^{j2\pi\frac{kn}{N_0}} \Rightarrow x[0] = \sum_{k=\langle 6\rangle} X[k]e^{0} = \sum_{k=0}^{5} X[k] = 4 - 1 + j2 + 1 - j2 - 1 = 3$$

(b) Find the numerical average value of $x[n]$.

The average value of $x[n]$ is $X[0]$ which is 4.

(c) Find the numerical value of the average signal power of $x[n]$.

From Parseval's theorem,

$$P_x = \sum_{k=\langle N_F\rangle} |X[k]|^2 = \sum_{k=0}^{5} |X[k]|^2 = 16 + 1 + 4 + 1 + 4 + 1 = 27$$