Solution of ECE 315 Test 6 F07

1. If $x(t) = 3\text{rect}(t/2) * \delta_5(t) \xleftarrow{\text{FS}} X[k] = A\,\text{sinc}(bk)$ and $T_F = T_0$, find the numerical values of A and b.

   $A = $ _____ , $b = $ _____

$$(1/w)\text{rect}(t/w) * \delta_{T_0}(t) \xleftarrow{\text{FS}} f_0\,\text{sinc}(wkf_0) \ , \ T_F = T_0$$

$$(1/2)\text{rect}(t/2) * \delta_5(t) \xleftarrow{\text{FS}} (1/5)\text{sinc}(2k/5) \ , \ T_F = T_0 = 5$$

$$3\text{rect}(t/2) * \delta_5(t) \xleftarrow{\text{FS}} (6/5)\text{sinc}(2k/5) \ , \ T_F = T_0 = 5$$

$$A = 6/5 \ , \ b = 2/5$$

2. If $x(t) = A\,\text{tri}(bt) * \delta_c(t) \xleftarrow{\text{FS}} X[k] = 10\,\text{sinc}^2(k/3)$ and $T_F = T_0 = 8$, find the numerical values of A, b and c. $A = $ _____ , $b = $ _____ , $c = $ _____

$$(1/w)\text{tri}(t/w) * \delta_{T_0}(t) \xleftarrow{\text{FS}} f_0\,\text{sinc}^2(wkf_0) \ , \ T_F = T_0$$

$$(1/w)\text{tri}(t/w) * \delta_8(t) \xleftarrow{\text{FS}} (1/8)\text{sinc}^2(wk/8) \ , \ T_F = T_0 = 8$$

$$(1/(8/3))\text{tri}(t/(8/3)) * \delta_8(t) \xleftarrow{\text{FS}} (1/8)\text{sinc}^2((8/3)k/8) \ , \ T_F = T_0 = 8$$

$$(3/8)\text{tri}(3t/8) * \delta_8(t) \xleftarrow{\text{FS}} (1/8)\text{sinc}^2(k/3) \ , \ T_F = T_0 = 8$$

$$30\,\text{tri}(3t/8) * \delta_8(t) \xleftarrow{\text{FS}} 10\,\text{sinc}^2(k/3) \ , \ T_F = T_0 = 8$$

$$A = 30 \ , \ b = 3/8 \ , \ c = 8$$

3. Given that $x(t) \xleftarrow{\text{FS}} \delta[k] + 5\delta[k-3] + 5\delta[k+3]$ using a representation time $T_F = mT_{0x}$, and given that $z(t) = x(2t) \xleftarrow{\text{FS}} Z[k]$ using the same representation time, find these specific numerical values of $Z[k]$.

k	0	1	6	−3	−12
$Z[k]$	___	___	___	___	___

$$Z[k] = \begin{cases} X[k/2] & , \ k/2 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

$$Z[k] = \begin{cases} \delta[k/2] + 5\delta[k/2-3] + 5\delta[k/2+3] & , \ k/2 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

k	0	1	6	−3	−12
$Z[k]$	1	0	5	0	0

Solution of ECE 315 Test 6 F07

1. If $x(t) = 5\,\text{rect}(t/3) * \delta_4(t) \xleftarrow{\text{FS}} X[k] = A\,\text{sinc}(bk)$ and $T_F = T_0$, find the numerical values of A and b.

    $A = \underline{\hspace{3cm}}$, $b = \underline{\hspace{3cm}}$

    $$(1/w)\,\text{rect}(t/w) * \delta_{T_0}(t) \xleftarrow{\text{FS}} f_0\,\text{sinc}(wkf_0) \ , \ T_F = T_0$$

    $$(1/3)\,\text{rect}(t/3) * \delta_4(t) \xleftarrow{\text{FS}} (1/4)\,\text{sinc}(3k/4) \ , \ T_F = T_0 = 4$$

    $$5\,\text{rect}(t/3) * \delta_4(t) \xleftarrow{\text{FS}} (15/4)\,\text{sinc}(3k/4) \ , \ T_F = T_0 = 4$$

    $$A = 15/4 \ , \ b = 3/4$$

2. If $x(t) = A\,\text{tri}(bt) * \delta_c(t) \xleftarrow{\text{FS}} X[k] = 8\,\text{sinc}^2(k/5)$ and $T_F = T_0 = 12$, find the numerical values of A, b and c. $\quad A = \underline{\hspace{2.5cm}}$, $b = \underline{\hspace{2.5cm}}$, $c = \underline{\hspace{2.5cm}}$

    $$(1/w)\,\text{tri}(t/w) * \delta_{T_0}(t) \xleftarrow{\text{FS}} f_0\,\text{sinc}^2(wkf_0) \ , \ T_F = T_0$$

    $$(1/w)\,\text{tri}(t/w) * \delta_{12}(t) \xleftarrow{\text{FS}} (1/12)\,\text{sinc}^2(wk/12) \ , \ T_F = T_0 = 12$$

    $$(1/(12/5))\,\text{tri}(t/(12/5)) * \delta_8(t) \xleftarrow{\text{FS}} (1/12)\,\text{sinc}^2((12/5)k/12) \ , \ T_F = T_0 = 12$$

    $$(5/12)\,\text{tri}(5t/12) * \delta_8(t) \xleftarrow{\text{FS}} (1/12)\,\text{sinc}^2(k/5) \ , \ T_F = T_0 = 12$$

    $$40\,\text{tri}(5t/12) * \delta_8(t) \xleftarrow{\text{FS}} 8\,\text{sinc}^2(k/5) \ , \ T_F = T_0 = 12$$

    $$A = 40 \ , \ b = 5/12 \ , \ c = 12$$

3. Given that $x(t) \xleftarrow{\text{FS}} 2\delta[k] + 3\delta[k-2] + 3\delta[k+2]$ using a representation time $T_F = mT_{0x}$, and given that $z(t) = x(2t) \xleftarrow{\text{FS}} Z[k]$ using the same representation time, find these specific numerical values of $Z[k]$.

k	0	4	1	-4	-8
$Z[k]$	___	___	___	___	___

    $$Z[k] = \begin{cases} X[k/2] & , \ k/2 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

    $$Z[k] = \begin{cases} 2\delta[k/2] + 3\delta[k/2-2] + 3\delta[k/2+2] & , \ k/2 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

k	0	4	1	-4	-8
$Z[k]$	2	3	0	3	0

Solution of ECE 315 Test 6 F07

1. If $x(t) = 7 \operatorname{rect}(t/6) * \delta_{10}(t) \xleftrightarrow{\text{FS}} X[k] = A \operatorname{sinc}(bk)$ and $T_F = T_0$, find the numerical values of A and b.

   $A = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$

$$(1/w)\operatorname{rect}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\text{FS}} f_0 \operatorname{sinc}(wkf_0) \ , \ T_F = T_0$$

$$(1/6)\operatorname{rect}(t/6) * \delta_{10}(t) \xleftrightarrow{\text{FS}} (1/10)\operatorname{sinc}(6k/10) \ , \ T_F = T_0 = 10$$

$$7\operatorname{rect}(t/6) * \delta_{10}(t) \xleftrightarrow{\text{FS}} (21/5)\operatorname{sinc}(3k/5) \ , \ T_F = T_0 = 10$$

$$A = 21/5 \ , \ b = 3/5$$

2. If $x(t) = A \operatorname{tri}(bt) * \delta_c(t) \xleftrightarrow{\text{FS}} X[k] = 4\operatorname{sinc}^2(k/7)$ and $T_F = T_0 = 9$, find the numerical values of A, b and c.
   $A = \underline{\hspace{2cm}}$, $b = \underline{\hspace{2cm}}$, $c = \underline{\hspace{2cm}}$

$$(1/w)\operatorname{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\text{FS}} f_0 \operatorname{sinc}^2(wkf_0) \ , \ T_F = T_0$$

$$(1/w)\operatorname{tri}(t/w) * \delta_9(t) \xleftrightarrow{\text{FS}} (1/9)\operatorname{sinc}^2(wk/9) \ , \ T_F = T_0 = 9$$

$$(1/(9/7))\operatorname{tri}(t/(9/7)) * \delta_9(t) \xleftrightarrow{\text{FS}} (1/9)\operatorname{sinc}^2((9/7)k/9) \ , \ T_F = T_0 = 9$$

$$(7/9)\operatorname{tri}(7t/9) * \delta_9(t) \xleftrightarrow{\text{FS}} (1/9)\operatorname{sinc}^2(k/7) \ , \ T_F = T_0 = 9$$

$$28\operatorname{tri}(7t/9) * \delta_9(t) \xleftrightarrow{\text{FS}} 4\operatorname{sinc}^2(k/3) \ , \ T_F = T_0 = 9$$

$$A = 28 \ , \ b = 7/9 \ , \ c = 9$$

3. Given that $x(t) \xleftrightarrow{\text{FS}} 4\delta[k] + 7\delta[k-3] + 7\delta[k+3]$ using a representation time $T_F = mT_{0x}$, and given that $z(t) = x(3t) \xleftrightarrow{\text{FS}} Z[k]$ using the same representation time, find these specific numerical values of $Z[k]$.

k	0	1	6	−3	−12
$Z[k]$	___	___	___	___	___

$$Z[k] = \begin{cases} X[k/3] & , \ k/3 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

$$Z[k] = \begin{cases} 4\delta[k/3] + 7\delta[k/3-3] + 7\delta[k/3+3] & , \ k/3 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

k	0	1	6	−3	−12
$Z[k]$	4	0	0	0	0