Solution of ECE 315 Test 7 F07

1. If $\mathrm{x}(t) = 4\,\mathrm{sinc}(t/2) * \delta_9(t)$ and $T_F = T_0$, use Parseval's theorem to find the numerical value of the signal power of $\mathrm{x}(t)$, P_x. $P_x =$ _______________

$$(1/w)\,\mathrm{sinc}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{rect}(wkf_0) \;\;,\;\; T_F = T_0$$

$$(1/2)\,\mathrm{sinc}(t/2) * \delta_9(t) \xleftrightarrow{\mathcal{FS}} (1/9)\,\mathrm{rect}(2k/9) \;\;,\;\; T_F = T_0 = 9$$

$$4\,\mathrm{sinc}(t/2) * \delta_9(t) \xleftrightarrow{\mathcal{FS}} (8/9)\,\mathrm{rect}(2k/9) \;\;,\;\; T_F = T_0 = 9$$

Using Parseval's theorem $\frac{1}{T_F}\int_{T_F} |\mathrm{x}(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2$,

$$P_x = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2 = \sum_{k=-\infty}^{\infty} |(8/9)\,\mathrm{rect}(2k/9)|^2 = (8/9)^2 \sum_{k=-2}^{2} \mathrm{rect}(2k/9) = 5 \times (8/9)^2 = \frac{320}{81} = 3.9506$$

2. If $\mathrm{x}(t) = 10\,\mathrm{tri}(3t) * \delta_2(t)$ with $T_F = T_0$ and $\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} Ak\,\mathrm{sinc}^2(bk) \;,\; T_F = T_0$ find the numerical values of A and b. $A =$ _______________ , $b =$ ________________

$$(1/w)\,\mathrm{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{sinc}^2(wkf_0) \;\;,\;\; T_F = T_0$$

$$3\,\mathrm{tri}(3t) * \delta_2(t) \xleftrightarrow{\mathcal{FS}} (1/2)\,\mathrm{sinc}^2(k/6) \;\;,\;\; T_F = T_0 = 2$$

$$10\,\mathrm{tri}(3t) * \delta_2(t) \xleftrightarrow{\mathcal{FS}} (5/3)\,\mathrm{sinc}^2(k/6) \;\;,\;\; T_F = T_0 = 2$$

$$\frac{d}{dt}\left(10\,\mathrm{tri}(3t) * \delta_2(t)\right) \xleftrightarrow{\mathcal{FS}} j2\pi kf_0\,(5/3)\,\mathrm{sinc}^2(k/6) \;\;,\;\; T_F = T_0 = 2$$

$$\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} (j5\pi k/3)\,\mathrm{sinc}^2(k/6) \;\;,\;\; T_F = T_0 = 2$$

$$A = j5\pi/3 = j5.236 \;\;,\;\; b = 1/6$$

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1. If $\mathrm{x}(t) = 7\,\mathrm{sinc}(t/4) * \delta_{10}(t)$ and $T_F = T_0$, use Parseval's theorem to find the numerical value of the signal power of $\mathrm{x}(t)$, P_x. $P_x =$ _______________

$$(1/w)\,\mathrm{sinc}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{rect}(wkf_0) \;\;,\;\; T_F = T_0$$

$$(1/4)\,\mathrm{sinc}(t/4) * \delta_{10}(t) \xleftrightarrow{\mathcal{FS}} (1/10)\,\mathrm{rect}(4k/10) \;\;,\;\; T_F = T_0 = 10$$

$$7\,\mathrm{sinc}(t/4) * \delta_{10}(t) \xleftrightarrow{\mathcal{FS}} (14/5)\,\mathrm{rect}(4k/10) \;\;,\;\; T_F = T_0 = 10$$

Using Parseval's theorem $\frac{1}{T_F}\int_{T_F} |\mathrm{x}(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2$,

$$P_x = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2 = \sum_{k=-\infty}^{\infty} |(14/5)\,\mathrm{rect}(2k/5)|^2 = (14/5)^2 \sum_{k=-1}^{1} \mathrm{rect}(2k/5) = 3 \times (14/5)^2 = \frac{588}{25} = 23.52$$

2. If $\mathrm{x}(t) = 6\,\mathrm{tri}(2t) * \delta_5(t)$ with $T_F = T_0$ and $\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} Ak\,\mathrm{sinc}^2(bk)$, $T_F = T_0$ find the numerical values of A and b. $A =$ _______________ , $b =$ ________________

$$(1/w)\,\mathrm{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{sinc}^2(wkf_0) \;\;,\;\; T_F = T_0$$

$$2\,\mathrm{tri}(2t) * \delta_5(t) \xleftrightarrow{\mathcal{FS}} (1/5)\,\mathrm{sinc}^2(k/10) \;\;,\;\; T_F = T_0 = 5$$

$$6\,\mathrm{tri}(2t) * \delta_5(t) \xleftrightarrow{\mathcal{FS}} (3/5)\,\mathrm{sinc}^2(k/10) \;\;,\;\; T_F = T_0 = 5$$

$$\frac{d}{dt}\left(6\,\mathrm{tri}(2t) * \delta_5(t)\right) \xleftrightarrow{\mathcal{FS}} j2\pi k f_0 (3/5)\,\mathrm{sinc}^2(k/10) \;\;,\;\; T_F = T_0 = 5$$

$$\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} (j6\pi k/25)\,\mathrm{sinc}^2(k/10) \;\;,\;\; T_F = T_0 = 5$$

$A = j6\pi/25 = j0.754$, $b = 1/10$

Solution of ECE 315 Test 7 F07

1. If $\mathrm{x}(t) = 9\,\mathrm{sinc}(t/4) * \delta_7(t)$ and $T_F = T_0$, use Parseval's theorem to find the numerical value of the signal power of $\mathrm{x}(t)$, P_x. $P_x =$ _______________

$$(1/w)\mathrm{sinc}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{rect}(wkf_0) \quad , \quad T_F = T_0$$

$$(1/4)\mathrm{sinc}(t/4) * \delta_7(t) \xleftrightarrow{\mathcal{FS}} (1/7)\mathrm{rect}(4k/7) \quad , \quad T_F = T_0 = 7$$

$$9\,\mathrm{sinc}(t/4) * \delta_7(t) \xleftrightarrow{\mathcal{FS}} (36/7)\mathrm{rect}(4k/7) \quad , \quad T_F = T_0 = 7$$

Using Parseval's theorem $\frac{1}{T_F}\int_{T_F} |\mathrm{x}(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2$,

$$P_x = \sum_{k=-\infty}^{\infty} |\mathrm{X}[k]|^2 = \sum_{k=-\infty}^{\infty} |(36/7)\mathrm{rect}(4k/7)|^2 = (36/7)^2 \sum_{k=-0}^{0} \mathrm{rect}(4k/7) = (36/7)^2 = \frac{1296}{49} = 26.449$$

2. If $\mathrm{x}(t) = 4\,\mathrm{tri}(8t) * \delta_3(t)$ with $T_F = T_0$ and $\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} Ak\,\mathrm{sinc}^2(bk) \quad , \quad T_F = T_0$ find the numerical values of A and b. $A =$ _______________ , $b =$ ________________

$$(1/w)\mathrm{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{sinc}^2(wkf_0) \quad , \quad T_F = T_0$$

$$8\,\mathrm{tri}(8t) * \delta_3(t) \xleftrightarrow{\mathcal{FS}} (1/3)\mathrm{sinc}^2(k/24) \quad , \quad T_F = T_0 = 3$$

$$4\,\mathrm{tri}(8t) * \delta_3(t) \xleftrightarrow{\mathcal{FS}} (1/6)\mathrm{sinc}^2(k/24) \quad , \quad T_F = T_0 = 3$$

$$\frac{d}{dt}(4\,\mathrm{tri}(8t) * \delta_3(t)) \xleftrightarrow{\mathcal{FS}} j2\pi k f_0 (1/6)\mathrm{sinc}^2(k/24) \quad , \quad T_F = T_0 = 3$$

$$\frac{d}{dt}(\mathrm{x}(t)) \xleftrightarrow{\mathcal{FS}} (j\pi k/9)\mathrm{sinc}^2(k/24) \quad , \quad T_F = T_0 = 3$$

$$A = j\pi/9 = j0.3491 \quad , \quad b = 1/24$$