Solution of ECE 315 Test 8 F07

1. A signal $\text{x}[n]$ is periodic with period 7 and $\text{x}[n] = n^2/3$, $0 \le n < 7$. Its DTFS harmonic function is $\text{X}[k]$. Let $N_F = N_0$. What is the numerical value of $\text{X}[0]$?

$$\text{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \text{x}[n] e^{-j2\pi kn/N_F}$$

$$\text{X}[0] = \frac{1}{7} \sum_{n=\langle 7 \rangle} \text{x}[n] = \frac{0+1+4+9+16+25+36}{3 \times 7} = \text{X}[0] = \frac{91}{21} = 4.333$$

2. Exactly one period of a periodic signal $\text{x}[n]$ has the following numerical values.

n	3	4	5	6
$\text{x}[n]$	–1	5	2	–2

Its DTFS harmonic function is $\text{X}[k]$. Let $N_F = N_0$.

(a) What is the numerical value of $\text{X}[1]$?

$$\text{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \text{x}[n] e^{-j2\pi kn/N_F}$$

$$\text{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \text{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=3}^{6} \text{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[j(-1) + 1(5) + (-j)2 + (-1)(-2) \right] = \frac{7-j3}{4} = 1.904 e^{-j0.4049}$$

or

$$\text{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \text{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=0}^{3} \text{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[1(5) + (-j)2 + (-1)(-2) + j(-1) \right] = \frac{7-j3}{4} = 1.904 e^{-j0.4049}$$

(b) What is the numerical value of $\text{X}[6]$?

$$\text{X}[6] = \text{X}[6-4] = \text{X}[2] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \text{x}[n] e^{-j\pi n} = \frac{1}{4} \sum_{n=3}^{6} \text{x}[n] (-1)^n = \frac{1}{4}\left[(-1)(-1) + 1(5) + (-1)2 + 1(-2) \right] = 1/2$$

(c) If $\text{y}[n] = \text{x}[n+1]$, $\text{y}[n] \overset{\mathcal{FS}}{\longleftrightarrow} \text{Y}[k]$ and $N_F = N_0$, what is the numerical value of $\text{Y}[1]$?

$$\text{Y}[1] = \text{X}[1] e^{-j2\pi(-1)/4} = \text{X}[1] e^{j\pi/2} = j\,\text{X}[1] = \frac{3+j7}{4} = 1.904 e^{j1.1659}$$

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1. A signal $\mathrm{x}[n]$ is periodic with period 7 and $\mathrm{x}[n] = n^2/2$, $0 \le n < 7$. Its DTFS harmonic function is $\mathrm{X}[k]$. Let $N_F = N_0$. What is the numerical value of $\mathrm{X}[0]$?

$$\mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

$$\mathrm{X}[0] = \frac{1}{7} \sum_{n=\langle 7 \rangle} \mathrm{x}[n] = \frac{0+1+4+9+16+25+36}{2 \times 7} = \mathrm{X}[0] = \frac{91}{14} = 6.5$$

2. Exactly one period of a periodic signal $\mathrm{x}[n]$ has the following numerical values.

n	3	4	5	6
$\mathrm{x}[n]$	1	4	3	–5

Its DTFS harmonic function is $\mathrm{X}[k]$. Let $N_F = N_0$.

(a) What is the numerical value of $\mathrm{X}[1]$?

$$\mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=3}^{6} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[j(1) + 1(4) + (-j)3 + (-1)(-5) \right] = \frac{9 - j2}{4} = 2.305 e^{-j0.2187}$$

or

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[1(4) + (-j)3 + (-1)(-5) + j(1) \right] = \frac{9 - j2}{4} = 2.305 e^{-j0.2187}$$

(b) What is the numerical value of $\mathrm{X}[7]$?

$$\mathrm{X}[7] = \mathrm{X}[7-4] = \mathrm{X}[3] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j3\pi n/2} = \frac{1}{4} \sum_{n=3}^{6} \mathrm{x}[n] (j)^n$$

$$= \frac{1}{4}\left[(-j)(1) + 1(4) + (j)3 + (-1)(-5) \right] = \frac{9 + j2}{4} = 2.305 e^{+j0.2187}$$

(c) If $\mathrm{y}[n] = \mathrm{x}[n+1]$, $\mathrm{y}[n] \xleftrightarrow{\mathcal{FS}} \mathrm{Y}[k]$ and $N_F = N_0$, what is the numerical value of $\mathrm{Y}[1]$?

$$\mathrm{Y}[1] = \mathrm{X}[1] e^{-j2\pi(-1)/4} = \mathrm{X}[1] e^{j\pi/2} = j\,\mathrm{X}[1] = \frac{2 + j9}{4} = 2.305 e^{j1.3521}$$

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1. A signal $\mathrm{x}[n]$ is periodic with period 7 and $\mathrm{x}[n] = n^2/4$, $0 \le n < 7$. Its DTFS harmonic function is $\mathrm{X}[k]$. Let $N_F = N_0$. What is the numerical value of $\mathrm{X}[0]$?

$$\mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

$$\mathrm{X}[0] = \frac{1}{7} \sum_{n=\langle 7 \rangle} \mathrm{x}[n] = \frac{0+1+4+9+16+25+36}{4 \times 7} = \mathrm{X}[0] = \frac{91}{28} = 3.25$$

2. Exactly one period of a periodic signal $\mathrm{x}[n]$ has the following numerical values.

n	3	4	5	6
$\mathrm{x}[n]$	4	1	–3	7

Its DTFS harmonic function is $\mathrm{X}[k]$. Let $N_F = N_0$.

(a) What is the numerical value of $\mathrm{X}[1]$?

$$\mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=3}^{6} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[j(4) + 1(1) + (-j)(-3) + (-1)(7) \right] = \frac{-6+j7}{4} = 2.305 e^{+j2.279}$$

or

$$\mathrm{X}[1] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n/2} = \frac{1}{4}\left[1(1) + (-j)(-3) + (-1)(7) + j(4) \right] = \frac{-6+j7}{4} = 2.305 e^{+j2.279}$$

(b) What is the numerical value of $\mathrm{X}[6]$?

$$\mathrm{X}[6] = \mathrm{X}[6-4] = \mathrm{X}[2] = \frac{1}{4} \sum_{n=\langle 4 \rangle} \mathrm{x}[n] e^{-j\pi n} = \frac{1}{4} \sum_{n=3}^{6} \mathrm{x}[n] (-1)^k = \frac{1}{4}\left[(-1)(4) + 1(1) + (-1)(-3) + 1(7) \right] = 7/4$$

(c) If $\mathrm{y}[n] = \mathrm{x}[n-1]$, $\mathrm{y}[n] \xleftrightarrow{\mathcal{FS}} \mathrm{Y}[k]$ and $N_F = N_0$, what is the numerical value of $\mathrm{Y}[1]$?

$$\mathrm{Y}[1] = \mathrm{X}[1] e^{-j2\pi(1)/4} = \mathrm{X}[1] e^{-j\pi/2} = -j\,\mathrm{X}[1] = -j\frac{-6+j7}{4} = \frac{7+j6}{4} = 2.305 e^{j0.7086}$$