Solution of ECE 315 Test 4 Su08

1. A signal $x(t) = [7\,\text{rect}(2t) - 5\,\text{rect}(4(t-1))] * \delta_8(t)$ has a CTFS harmonic function $X[k]$. What is the numerical value of $X[0]$? (It is not necessary to find a general expression for $X[k]$ to answer this question.)

   $X[0]$ is the average value of the periodic signal $x(t) = [7\,\text{rect}(2t) - 5\,\text{rect}(4(t-1))] * \delta_8(t)$ with fundamental period 8. In one period the area under the function is 7/2 from the first rectangle minus 5/4 from the second rectangle or 9/4. Therefore the average value of the function is 9/4 divided by 8 or 9/32 or 0.28125.

2. The DTFS harmonic function $X[k]$ of $x[n] = 5\cos(\pi n) \circledast 3\sin\left(\frac{2\pi n}{4}\right)$ using $N_F = 4$ can be written in the form $X[k] = A(\delta_4[k-a] + \delta_4[k+a])(\delta_4[k+b] - \delta_4[k-b])$. Find the numerical values of *A*, *a* and *b*.

From the tables,

$$\cos\left(\frac{2\pi n}{N_0}\right) \xleftrightarrow{\mathcal{FS}} \frac{1}{2}\left(\delta_{N_F}[k-m] + \delta_{N_F}[k+m]\right), \quad N_F = mN_0$$

Therefore

$$5\cos(\pi n) \xleftrightarrow{\mathcal{FS}} \frac{5}{2}\left(\delta_4[k-2] + \delta_4[k+2]\right), \quad N_F = 4$$

or, alternately,

$$5\cos(\pi n) \xleftrightarrow{\mathcal{FS}} \frac{5}{2}\left(\delta_2[k-1] + \delta_2[k+1]\right), \quad N_F = 2$$

$$5\cos(\pi n) \xleftrightarrow{\mathcal{FS}} \frac{5}{2}\begin{cases}\left(\delta_2[k/2-1] + \delta_2[k/2+1]\right) & , \ k/2 \text{ an integer} \\ 0 & , \ \text{otherwise}\end{cases}, \quad N_F = 4$$

$$5\cos(\pi n) \xleftrightarrow{\mathcal{FS}} \frac{5}{2}\left(\delta_4[k-2] + \delta_4[k+2]\right), \quad N_F = 4$$

From the tables,

$$\sin\left(\frac{2\pi n}{N_0}\right) \xleftrightarrow{\mathcal{FS}} \frac{j}{2}\left(\delta_{N_F}[k+m] - \delta_{N_F}[k-m]\right), \quad N_F = mN_0$$

Therefore

$$3\sin\left(\frac{2\pi n}{4}\right) \xleftrightarrow{\mathcal{FS}} j\frac{3}{2}\left(\delta_4[k+1] - \delta_4[k-1]\right), \quad N_F = 4$$

Then, using $x[n] \circledast y[n] = \sum_{p=\langle N_F \rangle} x[p]\,y[n-p] \xleftrightarrow{\mathcal{FS}} N_F\,Y[k]\,X[k]$

$$X[k] = 4 \times \frac{5}{2}\left(\delta_4[k-2] + \delta_4[k+2]\right) \times j\frac{3}{2}\left(\delta_4[k+1] - \delta_4[k-1]\right)$$

$$X[k] = j15\left(\delta_4[k-2] + \delta_4[k+2]\right)\left(\delta_4[k+1] - \delta_4[k-1]\right)$$

$A = \underline{j15}$, $a = \underline{2}$, $b = \underline{1}$

3. Using

$$\operatorname{sinc}(n/w) * \delta_{N_0}[n] \xleftrightarrow[N_F = N_0]{\mathcal{FS}} (w/N_0)\operatorname{rect}(wk/N_0) * \delta_{N_0}[k]$$

and Parseval's theorem, find the numerical average signal power P_x of $x[n] = 2\operatorname{sinc}(n/5) * \delta_{16}[n]$.

$$\operatorname{sinc}(n/5) * \delta_{16}[n] \xleftrightarrow[N_F = 16]{\mathcal{FS}} (5/16)\operatorname{rect}(5k/16) * \delta_{16}[k]$$

$$2\operatorname{sinc}(n/5) * \delta_{16}[n] \xleftrightarrow[N_F = 16]{\mathcal{FS}} (5/8)\operatorname{rect}(5k/16) * \delta_{16}[k]$$

Parseval's Theorem says

$$\frac{1}{N_F}\sum_{n=\langle N_F\rangle} |x[n]|^2 = \sum_{k=\langle N_F\rangle} |X[k]|^2$$

Therefore

$$P_x = \sum_{k=\langle 16\rangle} |X[k]|^2 = \sum_{k=\langle 16\rangle} |(5/8)\operatorname{rect}(5k/16) * \delta_{16}[k]|^2$$

$$P_x = (5/8)^2 \sum_{k=-8}^{7} |\operatorname{rect}(5k/16)|^2 = (5/8)^2 \sum_{k=-1}^{1} 1 = 75/64 = 1.172$$

4. Circle the correct answers.

(a) $x(t) = 8\cos(50\pi t) - 4\sin(22\pi t)$

$\operatorname{Re}(X[k]) = 0$, for all k — $\operatorname{Im}(X[k]) = 0$, for all k — **[Neither]**

(b) $x(t) = 32\cos(50\pi t)\sin(22\pi t)$

[$\operatorname{Re}(X[k]) = 0$, for all k] — $\operatorname{Im}(X[k]) = 0$, for all k — Neither

(c) $x(t) = \left[\operatorname{tri}\left(\frac{t-1}{4}\right) - \operatorname{tri}\left(\frac{t+1}{4}\right)\right]\sin(100\pi t)$

$\operatorname{Re}(X[k]) = 0$, for all k — **[$\operatorname{Im}(X[k]) = 0$, for all k]** — Neither

5. Using the definition of the DTFS,

$$\mathrm{x}[n] = \sum_{k=\langle N_F \rangle} \mathrm{X}[k] e^{j2\pi kn/N_F} \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

if $\mathrm{x}[n]$ is periodic with fundamental period $N_0 = 4$ and

$$\mathrm{x}[0] = 3 \ , \ \mathrm{x}[1] = 7 \ , \ \mathrm{x}[2] = -4 \ \text{and} \ \mathrm{x}[3] = 1,$$

find the numerical value of $\mathrm{X}[2]$ with $N_F = N_0 = 4$.

$$\mathrm{X}[2] = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j2\pi 2n/4} = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n} = \frac{1}{4} \sum_{n=0}^{3} \mathrm{x}[n] (-1)^n$$

$$\mathrm{X}[2] = \frac{3(1) + 7(-1) + (-4)(1) + 1(-1)}{4} = -9/4$$

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1. A signal $\mathrm{x}(t)=\left[9\,\mathrm{rect}(2t)-5\,\mathrm{rect}(4(t-1))\right]*\delta_8(t)$ has a CTFS harmonic function $\mathrm{X}[k]$. What is the numerical value of $\mathrm{X}[0]$? (It is not necessary to find a general expression for $\mathrm{X}[k]$ to answer this question.)

   $\mathrm{X}[0]$ is the average value of the periodic signal $\mathrm{x}(t)=\left[9\,\mathrm{rect}(2t)-5\,\mathrm{rect}(4(t-1))\right]*\delta_8(t)$ with fundamental period 8. In one period the area under the function is 9/2 from the first rectangle minus 5/4 from the second rectangle or 13/4. Therefore the average value of the function is 13/4 divided by 8 or 13/32 or 0.40625.

2. The DTFS harmonic function $\mathrm{X}[k]$ of $\mathrm{x}[n]=8\cos(\pi n)\circledast 3\sin\left(\frac{2\pi n}{4}\right)$ using $N_F=4$ can be written in the form $\mathrm{X}[k]=A\left(\delta_4[k-a]+\delta_4[k+a]\right)\left(\delta_4[k+b]-\delta_4[k-b]\right)$. Find the numerical values of *A*, *a* and *b*.

$$N_F=mN_0$$

From the tables,

$$\cos\left(\frac{2\pi n}{N_0}\right)\xleftrightarrow{\mathcal{FS}}\frac{1}{2}\left(\delta_{N_F}[k-m]+\delta_{N_F}[k+m]\right)$$

Therefore

$$8\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{8}{2}\left(\delta_4[k-2]+\delta_4[k+2]\right)\ ,\ N_F=4$$

or, alternately,

$$8\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{8}{2}\left(\delta_2[k-1]+\delta_2[k+1]\right)\ ,\ N_F=2$$

$$8\cos(\pi n)\xleftrightarrow{\mathcal{FS}}4\begin{cases}\left(\delta_2[k/2-1]+\delta_2[k/2+1]\right) & ,\ k/2 \text{ an integer}\\ 0 & ,\ \text{otherwise}\end{cases}\ ,\ N_F=4$$

$$8\cos(\pi n)\xleftrightarrow{\mathcal{FS}}4\left(\delta_4[k-2]+\delta_4[k+2]\right)\ ,\ N_F=4$$

$$N_F=mN_0$$

From the tables,

$$\sin\left(\frac{2\pi n}{N_0}\right)\xleftrightarrow{\mathcal{FS}}\frac{j}{2}\left(\delta_{N_F}[k+m]-\delta_{N_F}[k-m]\right)$$

Therefore

$$3\sin\left(\frac{2\pi n}{4}\right)\xleftrightarrow{\mathcal{FS}}j\frac{3}{2}\left(\delta_4[k+1]-\delta_4[k-1]\right)\ ,\ N_F=4$$

Then, using $\mathrm{x}[n]\circledast\mathrm{y}[n]=\sum_{p=\langle N_F\rangle}\mathrm{x}[p]\mathrm{y}[n-p]\xleftrightarrow{\mathcal{FS}}N_F\,\mathrm{Y}[k]\mathrm{X}[k]$

$$\mathrm{X}[k]=4\times4\left(\delta_4[k-2]+\delta_4[k+2]\right)\times j\frac{3}{2}\left(\delta_4[k+1]-\delta_4[k-1]\right)$$

$$\mathrm{X}[k]=j24\left(\delta_4[k-2]+\delta_4[k+2]\right)\left(\delta_4[k+1]-\delta_4[k-1]\right)$$

$A=\underline{j24}$, $a=\underline{2}$, $b=\underline{1}$

3. Using

$$\mathrm{sinc}(n/w) * \delta_{N_0}[n] \xleftrightarrow[N_F = N_0]{\mathcal{FS}} (w/N_0)\,\mathrm{rect}(wk/N_0) * \delta_{N_0}[k]$$

and Parseval's theorem, find the numerical average signal power P_x of $x[n] = 2\,\mathrm{sinc}(n/5) * \delta_{24}[n]$.

$$\mathrm{sinc}(n/5) * \delta_{24}[n] \xleftrightarrow[N_F = 24]{\mathcal{FS}} (5/24)\,\mathrm{rect}(5k/24) * \delta_{24}[k]$$

$$2\,\mathrm{sinc}(n/5) * \delta_{24}[n] \xleftrightarrow[N_F = 24]{\mathcal{FS}} (5/12)\,\mathrm{rect}(5k/24) * \delta_{24}[k]$$

Parseval's Theorem says

$$\frac{1}{N_F} \sum_{n=\langle N_F \rangle} |x[n]|^2 = \sum_{k=\langle N_F \rangle} |X[k]|^2$$

Therefore

$$P_x = \sum_{k=\langle 24 \rangle} |X[k]|^2 = \sum_{k=\langle 24 \rangle} |(5/12)\,\mathrm{rect}(5k/24) * \delta_{24}[k]|^2$$

$$P_x = (5/12)^2 \sum_{k=-12}^{11} |\mathrm{rect}(5k/24)|^2 = (5/12)^2 \sum_{k=-2}^{2} 1 = 125/144 = 0.8681$$

4. Circle the correct answers.

(a) $x(t) = \left[\mathrm{tri}\left(\frac{t-1}{4}\right) - \mathrm{tri}\left(\frac{t+1}{4}\right)\right]\sin(100\pi t)$

$\mathrm{Re}(X[k]) = 0$, for all k | **[$\mathrm{Im}(X[k]) = 0$, for all k]** | Neither

(b) $x(t) = 8\cos(50\pi t) - 4\sin(22\pi t)$

$\mathrm{Re}(X[k]) = 0$, for all k | $\mathrm{Im}(X[k]) = 0$, for all k | **[Neither]**

(c) $x(t) = 32\cos(50\pi t)\sin(22\pi t)$

[$\mathrm{Re}(X[k]) = 0$, for all k] | $\mathrm{Im}(X[k]) = 0$, for all k | Neither

5. Using the definition of the DTFS,

$$\mathrm{x}[n] = \sum_{k=\langle N_F \rangle} \mathrm{X}[k] e^{j2\pi kn/N_F} \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

if $\mathrm{x}[n]$ is periodic with fundamental period $N_0 = 4$ and

$$\mathrm{x}[0] = 3 \ , \ \mathrm{x}[1] = -7 \ , \ \mathrm{x}[2] = -4 \text{ and } \mathrm{x}[3] = 1,$$

find the numerical value of $\mathrm{X}[2]$ with $N_F = N_0 = 4$.

$$\mathrm{X}[2] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j2\pi 2n/4} = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n} = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n](-1)^n$$

$$\mathrm{X}[2] = \frac{3(1) + (-7)(-1) + (-4)(1) + 1(-1)}{4} = 5/4$$

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1. A signal $\mathrm{x}(t)=\left[7\,\mathrm{rect}(2t)-9\,\mathrm{rect}(4(t-1))\right]*\delta_8(t)$ has a CTFS harmonic function $\mathrm{X}[k]$. What is the numerical value of $\mathrm{X}[0]$? (It is not necessary to find a general expression for $\mathrm{X}[k]$ to answer this question.)

   $\mathrm{X}[0]$ is the average value of the periodic signal $\mathrm{x}(t)=\left[7\,\mathrm{rect}(2t)-9\,\mathrm{rect}(4(t-1))\right]*\delta_8(t)$ with fundamental period 8. In one period the area under the function is 7/2 from the first rectangle minus 9/4 from the second rectangle or 5/4. Therefore the average value of the function is 5/4 divided by 8 or 5/32 or 0.15625.

2. The DTFS harmonic function $\mathrm{X}[k]$ of $\mathrm{x}[n]=5\cos(\pi n)\circledast 7\sin\left(\frac{2\pi n}{4}\right)$ using $N_F=4$ can be written in the form $\mathrm{X}[k]=A\left(\delta_4[k-a]+\delta_4[k+a]\right)\left(\delta_4[k+b]-\delta_4[k-b]\right)$. Find the numerical values of *A*, *a* and *b*.

   From the tables,
   $$N_F=mN_0$$
   $$\cos\left(\frac{2\pi n}{N_0}\right)\xleftrightarrow{\mathcal{FS}}\frac{1}{2}\left(\delta_{N_F}[k-m]+\delta_{N_F}[k+m]\right)$$

   Therefore $$5\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{5}{2}\left(\delta_4[k-2]+\delta_4[k+2]\right)\ ,\ N_F=4$$

   or, alternately,
   $$5\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{5}{2}\left(\delta_2[k-1]+\delta_2[k+1]\right)\ ,\ N_F=2$$
   $$5\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{5}{2}\begin{cases}\left(\delta_2[k/2-1]+\delta_2[k/2+1]\right) & ,\ k/2\text{ an integer}\\ 0 & ,\ \text{otherwise}\end{cases}\ ,\ N_F=4$$
   $$5\cos(\pi n)\xleftrightarrow{\mathcal{FS}}\frac{5}{2}\left(\delta_4[k-2]+\delta_4[k+2]\right)\ ,\ N_F=4$$

   From the tables,
   $$N_F=mN_0$$
   $$\sin\left(\frac{2\pi n}{N_0}\right)\xleftrightarrow{\mathcal{FS}}\frac{j}{2}\left(\delta_{N_F}[k+m]-\delta_{N_F}[k-m]\right)$$

   Therefore $$7\sin\left(\frac{2\pi n}{4}\right)\xleftrightarrow{\mathcal{FS}}j\frac{7}{2}\left(\delta_4[k+1]-\delta_4[k-1]\right)\ ,\ N_F=4$$

   Then, using $\mathrm{x}[n]\circledast\mathrm{y}[n]=\sum_{p=\langle N_F\rangle}\mathrm{x}[p]\mathrm{y}[n-p]\xleftrightarrow{\mathcal{FS}}N_F\,\mathrm{Y}[k]\mathrm{X}[k]$
   $$\mathrm{X}[k]=4\times\frac{5}{2}\left(\delta_4[k-2]+\delta_4[k+2]\right)\times j\frac{7}{2}\left(\delta_4[k+1]-\delta_4[k-1]\right)$$
   $$\mathrm{X}[k]=j35\left(\delta_4[k-2]+\delta_4[k+2]\right)\left(\delta_4[k+1]-\delta_4[k-1]\right)$$

   $A=\underline{j35}$, $a=\underline{2}$, $b=\underline{1}$

3. Using

$$N_F = N_0$$

$$\text{sinc}(n/w) * \delta_{N_0}[n] \xleftrightarrow{FS} (w/N_0)\text{rect}(wk/N_0) * \delta_{N_0}[k]$$

and Parseval's theorem, find the numerical average signal power P_x of $x[n] = 2\,\text{sinc}(n/5) * \delta_{32}[n]$.

$$N_F = 32$$

$$\text{sinc}(n/5) * \delta_{32}[n] \xleftrightarrow{FS} (5/32)\text{rect}(5k/32) * \delta_{32}[k]$$

$$N_F = 32$$

$$2\,\text{sinc}(n/5) * \delta_{32}[n] \xleftrightarrow{FS} (5/16)\text{rect}(5k/32) * \delta_{32}[k]$$

Parseval's Theorem says

$$\frac{1}{N_F}\sum_{n=\langle N_F\rangle} |x[n]|^2 = \sum_{k=\langle N_F\rangle} |X[k]|^2$$

Therefore

$$P_x = \sum_{k=\langle 16\rangle} |X[k]|^2 = \sum_{k=\langle 16\rangle} |(5/16)\text{rect}(5k/32) * \delta_{32}[k]|^2$$

$$P_x = (5/16)^2 \sum_{k=-16}^{15} |\text{rect}(5k/32)|^2 = (5/16)^2 \sum_{k=-3}^{3} 1 = 175/256 = 0.6836$$

4. Circle the correct answers.

(a) $x(t) = 32\cos(50\pi t)\sin(22\pi t)$

[$\text{Re}(X[k]) = 0$, for all k] $\qquad$ $\text{Im}(X[k]) = 0$, for all k $\qquad$ Neither

(b) $x(t) = \left[\text{tri}\left(\frac{t-1}{4}\right) - \text{tri}\left(\frac{t+1}{4}\right)\right]\sin(100\pi t)$

$\text{Re}(X[k]) = 0$, for all k $\qquad$ **[$\text{Im}(X[k]) = 0$, for all k]** $\qquad$ Neither

(c) $x(t) = 8\cos(50\pi t) - 4\sin(22\pi t)$

$\text{Re}(X[k]) = 0$, for all k $\qquad$ $\text{Im}(X[k]) = 0$, for all k $\qquad$ **[Neither]**

5. Using the definition of the DTFS,

$$\mathrm{x}[n] = \sum_{k=\langle N_F \rangle} \mathrm{X}[k] e^{j2\pi kn/N_F} \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi kn/N_F}$$

if $\mathrm{x}[n]$ is periodic with fundamental period $N_0 = 4$ and

$$\mathrm{x}[0] = 3 \ , \ \mathrm{x}[1] = 7 \ , \ \mathrm{x}[2] = 4 \text{ and } \mathrm{x}[3] = 1,$$

find the numerical value of $\mathrm{X}[2]$ with $N_F = N_0 = 4$.

$$\mathrm{X}[2] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j2\pi 2n/4} = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi n} = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] (-1)^n$$

$$\mathrm{X}[2] = \frac{3(1) + 7(-1) + (4)(1) + 1(-1)}{4} = -1/4$$