Solution of ECE 315 Test 6 F08

1. Given $\mathrm{x}(t) \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k] = \begin{cases} j4k & , \ |k| < 4 \\ 0 & , \ \text{otherwise} \end{cases}$ and that the fundamental period T_0 of $\mathrm{x}(t)$ is 8 seconds and that $T_F = T_0$, if $\mathrm{y}(t) = \int_{-\infty}^{t} \mathrm{x}(\tau) d\tau$ and $\mathrm{y}(t) \xleftrightarrow{\mathcal{FS}} \mathrm{Y}[k]$, also with $T_F = T_0$,

   (a) What is the average value of $\mathrm{x}(t)$?

   The average value of $\mathrm{x}(t)$ is $\mathrm{X}[0] = 0$.

   (b) What is the numerical value of $\mathrm{Y}[1]$?

   By the integration property, $\mathrm{Y}[k] = \dfrac{\mathrm{X}[k]}{j2\pi k f_0} = \dfrac{\mathrm{X}[k]}{j\pi k / 4} \ , \ k \neq 0 \Rightarrow \mathrm{Y}[1] = 4\dfrac{\mathrm{X}[1]}{j\pi} = 4\dfrac{j4}{j\pi} = 16/\pi$

   (c) Is $\mathrm{x}(t)$ even, odd or neither?

   Odd because the harmonic function is purely imaginary.

2. If $\mathrm{X}[k] = 3(\delta[k-1] + \delta[k+1])$ and $\mathrm{Y}[k] = j2(\delta[k+2] - \delta[k-2])$ and both are based on the same T_F and $\mathrm{z}(t) = \mathrm{x}(t)\mathrm{y}(t)$, $Z[k]$ can be written in the form

$$\mathrm{Z}[k] = A(\delta[k-a] - \delta[k-b] + \delta[k-c] - \delta[k-d])$$

   Find the numerical values of A , a, b, c and d.

$$\mathrm{Z}[k] = \mathrm{X}[k] * \mathrm{Y}[k] = 3(\delta[k-1] + \delta[k+1]) * j2(\delta[k+2] - \delta[k-2])$$

$$\mathrm{Z}[k] = j6(\delta[k+1] - \delta[k-3] + \delta[k+3] - \delta[k-1])$$

Solution of ECE 315 Test 6 F08

1. Given $\text{x}(t) \xleftrightarrow{\text{FS}} \text{X}[k] = \begin{cases} j8k & , \quad |k| < 4 \\ 0 & , \quad \text{otherwise} \end{cases}$ and that the fundamental period T_0 of $\text{x}(t)$ is 8 seconds and that $T_F = T_0$, if $\text{y}(t) = \int_{-\infty}^{t} \text{x}(\tau)\,d\tau$ and $\text{y}(t) \xleftrightarrow{\text{FS}} \text{Y}[k]$, also with $T_F = T_0$,

(a) What is the average value of $\text{x}(t)$?

The average value of $\text{x}(t)$ is $\text{X}[0] = 0$.

(b) What is the numerical value of $\text{Y}[1]$?

By the integration property, $\text{Y}[k] = \dfrac{\text{X}[k]}{j2\pi k f_0} = \dfrac{\text{X}[k]}{j\pi k / 4}$, $k \neq 0 \Rightarrow \text{Y}[1] = 4\dfrac{\text{X}[1]}{j\pi} = 4\dfrac{j8}{j\pi} = 32/\pi$

(c) Is $\text{x}(t)$ even, odd or neither?

Odd because the harmonic function is purely imaginary.

2. If $\text{X}[k] = 5(\delta[k-1] + \delta[k+1])$ and $\text{Y}[k] = j2(\delta[k+3] - \delta[k-3])$ and both are based on the same T_F and $\text{z}(t) = \text{x}(t)\text{y}(t)$, $Z[k]$ can be written in the form

$$\text{Z}[k] = A(\delta[k-a] - \delta[k-b] + \delta[k-c] - \delta[k-d])$$

Find the numerical values of *A* , *a*, *b*, *c* and *d*.

$$\text{Z}[k] = \text{X}[k] * \text{Y}[k] = 5(\delta[k-1] + \delta[k+1]) * j2(\delta[k+3] - \delta[k-3])$$

$$\text{Z}[k] = j10(\delta[k+2] - \delta[k-4] + \delta[k+4] - \delta[k-2])$$

Solution of ECE 315 Test 6 F08

1. Given $\mathrm{x}(t) \xleftrightarrow{FS} \mathrm{X}[k] = \begin{cases} j5k & , \ |k| < 4 \\ 0 & , \ \text{otherwise} \end{cases}$ and that the fundamental period T_0 of $\mathrm{x}(t)$ is 8 seconds and that $T_F = T_0$, if $\mathrm{y}(t) = \int_{-\infty}^{t} \mathrm{x}(\tau)\,d\tau$ and $\mathrm{y}(t) \xleftrightarrow{FS} \mathrm{Y}[k]$, also with $T_F = T_0$,

(a) What is the average value of $\mathrm{x}(t)$?

The average value of $\mathrm{x}(t)$ is $\mathrm{X}[0] = 0$.

(b) What is the numerical value of $\mathrm{Y}[1]$?

By the integration property, $\mathrm{Y}[k] = \dfrac{\mathrm{X}[k]}{j2\pi k f_0} = \dfrac{\mathrm{X}[k]}{j\pi k / 4}$, $k \neq 0 \Rightarrow \mathrm{Y}[1] = 4\dfrac{\mathrm{X}[1]}{j\pi} = 4\dfrac{j5}{j\pi} = 20/\pi$

(c) Is $\mathrm{x}(t)$ even, odd or neither?

Odd because the harmonic function is purely imaginary.

2. If $\mathrm{X}[k] = 7(\delta[k-3] + \delta[k+3])$ and $\mathrm{Y}[k] = j3(\delta[k+2] - \delta[k-2])$ and both are based on the same T_F and $\mathrm{z}(t) = \mathrm{x}(t)\mathrm{y}(t)$, $Z[k]$ can be written in the form

$$\mathrm{Z}[k] = A(\delta[k-a] - \delta[k-b] + \delta[k-c] - \delta[k-d])$$

Find the numerical values of A , a, b, c and d.

$$\mathrm{Z}[k] = \mathrm{X}[k] * \mathrm{Y}[k] = 7(\delta[k-3] + \delta[k+3]) * j3(\delta[k+2] - \delta[k-2])$$

$$\mathrm{Z}[k] = j21(\delta[k-1] - \delta[k-5] + \delta[k+5] - \delta[k+1])$$