Solution of EECS 315 Test 9 F13

1. If $\mathrm{x}(t) \xleftrightarrow[200\,\mathrm{ms}]{\mathcal{FS}} 4(\delta[k-1]+3\delta[k]+\delta[k+1])$,

   (a) What is numerical average value of $\mathrm{x}(t)$? Average value = 12

   (b) Is $\mathrm{x}(t)$ an even or an odd function or neither? Even

   (c) If $\mathrm{y}(t)=\frac{d}{dt}(\mathrm{x}(t))$, what is the numerical value of $\mathrm{c}_y[1]$?

$$\mathrm{c}_y[k]=(j2\pi k/T)\mathrm{c}_x[k]=j10\pi k\times 4(\delta[k-1]+3\delta[k]+\delta[k+1])$$

$$\mathrm{c}_y[k]=j40\pi k(\delta[k-1]+3\delta[k]+\delta[k+1])\Rightarrow \mathrm{c}_y[1]=j40\pi$$

2. An LTI continuous-time system is described by the differential equation $2\mathrm{y}'(t)+5\mathrm{y}(t)=\mathrm{x}'(t)$. Choose the correct description of this systems frequency response.

   (a) The system attenuates low frequencies more than high frequencies.

   (b) The system attenuates high frequencies more than low frequencies.

   (c) The system has the same effect on all frequencies.

   (a) Harmonic response is $\dfrac{j2\pi k/T_0}{j4\pi k/T_0+5}$.

3. If $\mathrm{x}(t) \xleftrightarrow[2s]{\mathcal{FS}} j(\delta[k+1]-\delta[k-1])$ and $\mathrm{y}(t) \xleftrightarrow[2s]{\mathcal{FS}} j(\delta[k+5]-\delta[k-5])$

   (a) Find the CTFS harmonic function of $\mathrm{z}(t)=\mathrm{x}(t)\mathrm{y}(t)$.

   $\mathrm{z}(t) \xleftrightarrow[2s]{\mathcal{FS}} \mathrm{c}_x[k]*\mathrm{c}_y[k]=-\delta[k+6]+\delta[k+4]+\delta[k-4]-\delta[k-6]$

   (b) What are the numerical fundamental periods of $\mathrm{x}(t)$, $\mathrm{y}(t)$ and $\mathrm{z}(t)$?

   x is at the fundamental frequency of 1/2 Hz. y is at the 5th harmonic of 1/2 Hz which is 5/2 Hz.

   z has both the 6th and 4th harmonics of 1/2 Hz which are 3 and 2 Hz whose GCD is 1 Hz.

   T_{0x} = 2 seconds T_{0y} = 2/5 seconds T_{0z} = 1 seconds

Solution of EECS 315 Test 9 F13

1. If $\mathrm{x}(t) \xleftrightarrow[400\,\mathrm{ms}]{\mathcal{FS}} 7(\delta[k-1]+3\delta[k]+\delta[k+1])$,

   (a) What is numerical average value of $\mathrm{x}(t)$? Average value = 21

   (b) Is $\mathrm{x}(t)$ an even or an odd function or neither? Even

   (c) If $\mathrm{y}(t)=\dfrac{d}{dt}(\mathrm{x}(t))$, what is the numerical value of $\mathrm{c}_y[1]$?

$$\mathrm{c}_y[k]=(j2\pi k/T)\mathrm{c}_x[k]=j5\pi k\times 7(\delta[k-1]+3\delta[k]+\delta[k+1])$$

$$\mathrm{c}_y[k]=j35\pi k(\delta[k-1]+3\delta[k]+\delta[k+1])\Rightarrow \mathrm{c}_y[1]=j35\pi$$

2. An LTI continuous-time system is described by the differential equation $2\mathrm{y}'(t)+5\mathrm{y}(t)=\mathrm{x}(t)$. Choose the correct description of this systems frequency response.

   (a) The system attenuates low frequencies more than high frequencies.

   (b) The system attenuates high frequencies more than low frequencies.

   (c) The system has the same effect on all frequencies.

   (b) Harmonic response is $\dfrac{1}{j4\pi k/T_0+5}$.

3. If $\mathrm{x}(t) \xleftrightarrow[4s]{\mathcal{FS}} j(\delta[k+1]-\delta[k-1])$ and $\mathrm{y}(t) \xleftrightarrow[4s]{\mathcal{FS}} j(\delta[k+5]-\delta[k-5])$

   (a) Find the CTFS harmonic function of $\mathrm{z}(t)=\mathrm{x}(t)\mathrm{y}(t)$.

   $\mathrm{z}(t) \xleftrightarrow[4s]{\mathcal{FS}} \mathrm{c}_x[k]*\mathrm{c}_y[k]=-\delta[k+6]+\delta[k+4]+\delta[k-4]-\delta[k-6]$

   (b) What are the numerical fundamental periods of $\mathrm{x}(t)$, $\mathrm{y}(t)$ and $\mathrm{z}(t)$?

   x is at the fundamental frequency of 1/4 Hz. y is at the 5th harmonic of 1/4 Hz which is 5/4 Hz.

   z has both the 6th and 4th harmonics of 1/4 Hz which are 3/2 and 1 Hz whose GCD is 1/2 Hz.

   T_{0x} = 4 seconds T_{0y} = 4/5 seconds T_{0z} = 2 seconds

Solution of EECS 315 Test 9 F13

1. If $x(t) \xleftrightarrow[100\,\text{ms}]{\mathcal{FS}} 9(\delta[k-1]+3\delta[k]+\delta[k+1])$,

   (a) What is numerical average value of $x(t)$? Average value = 27

   (b) Is $x(t)$ an even or an odd function or neither? Even

   (c) If $y(t)=\frac{d}{dt}(x(t))$, what is the numerical value of $c_y[1]$?

$$c_y[k]=(j2\pi k/T)c_x[k]=j20\pi k\times 9(\delta[k-1]+3\delta[k]+\delta[k+1])$$

$$c_y[k]=j180\pi k(\delta[k-1]+3\delta[k]+\delta[k+1]) \Rightarrow c_y[1]=j180\pi$$

2. An LTI continuous-time system is described by the differential equation $2y'(t)+5y(t)=x'(t)$. Choose the correct description of this systems frequency response.

   (a) The system attenuates low frequencies more than high frequencies.

   (b) The system attenuates high frequencies more than low frequencies.

   (c) The system has the same effect on all frequencies.

   (a) Harmonic response is $\frac{j2\pi k/T_0}{j4\pi k/T_0+5}$.

3. If $x(t) \xleftrightarrow[5s]{\mathcal{FS}} j(\delta[k+1]-\delta[k-1])$ and $y(t) \xleftrightarrow[5s]{\mathcal{FS}} j(\delta[k+5]-\delta[k-5])$

   (a) Find the CTFS harmonic function of $z(t)=x(t)y(t)$.

   $z(t) \xleftrightarrow[5s]{\mathcal{FS}} c_x[k]*c_y[k]=-\delta[k+6]+\delta[k+4]+\delta[k-4]-\delta[k-6]$

   (b) What are the numerical fundamental periods of $x(t)$, $y(t)$ and $z(t)$?

   x is at the fundamental frequency of 1/5 Hz. y is at the 5th harmonic of 1/5 Hz which is 1 Hz.

   z has both the 6th and 4th harmonics of 1/5 Hz which are 6/5 and 4/5 Hz whose GCD is 2/5 Hz.

   T_{0x} = 5 seconds T_{0y} = 1 seconds T_{0z} = 5/2 seconds