Solution to ECE 315 Test #6 F03

1. The complex CTFS harmonic function for a real-valued even periodic time function is purely real.

   The complex CTFS harmonic function for a real-valued odd periodic time function is purely imaginary.

2. The CTFS harmonic function, $\mathrm{X}[k]$, for the signal, $\mathrm{x}(t) = 5\cos(20\pi t)$ using a representation time, T_F, that is twice the fundamental period of $\mathrm{x}(t)$ is of the form, $\mathrm{X}[k] = A(\delta[k-a] + \delta[k+a])$. Find A and a.

A = 5/2 , a = 2

From the table, $\cos(2\pi f_0 t) \xleftrightarrow{FS} \frac{1}{2}(\delta[k-m] + \delta[k+m]) \quad , \quad T_F = mT_0 \Rightarrow m = 2$

$$\cos(20\pi t) \xleftrightarrow{FS} \frac{1}{2}(\delta[k-2] + \delta[k+2]) \quad , \quad T_F = 2 \times \frac{1}{10} = \frac{1}{5}$$

Multiplying both sides by 5, $5\cos(20\pi t) \xleftrightarrow{FS} \frac{5}{2}(\delta[k-2] + \delta[k+2]) \quad , \quad T_F = \frac{1}{5}$

3. The CTFS harmonic function, $\mathrm{X}[k]$, for the signal,

$$\mathrm{x}(t) = \mathrm{rect}(2(t-1)) * \mathrm{comb}\left(\frac{t}{3}\right) = \mathrm{rect}(2t) * \mathrm{comb}\left(\frac{t-1}{3}\right),$$

is of the form, $\mathrm{X}[k] = A\,\mathrm{sinc}(ak)e^{-jb\pi k}$. Find A, a and b using a representation time, $T_F = T_0$.

A = 1/2 , a = 1/6 , b = 2/3

From the table, $\mathrm{rect}\left(\frac{t}{w}\right) * \frac{1}{T_0}\mathrm{comb}\left(\frac{t}{T_0}\right) \xleftrightarrow{FS} \frac{w}{T_0}\mathrm{sinc}\left(\frac{w}{T_0}k\right)$

$$\mathrm{rect}(2t) * \frac{1}{3}\mathrm{comb}\left(\frac{t}{3}\right) \xleftrightarrow{FS} \frac{1}{6}\mathrm{sinc}\left(\frac{k}{6}\right)$$

Multiplying both sides by 3, $\mathrm{rect}(2t) * \mathrm{comb}\left(\frac{t}{3}\right) \xleftrightarrow{FS} \frac{1}{2}\mathrm{sinc}\left(\frac{k}{6}\right)$

Transforming t into t - 1 in the rectangle function and using the time-shifting property,

$$\mathrm{rect}(2(t-1)) * \mathrm{comb}\left(\frac{t}{3}\right) \xleftrightarrow{FS} \frac{1}{2}\mathrm{sinc}\left(\frac{k}{6}\right)e^{-j2\pi k f_0 (1)}$$

where $f_0 = \frac{1}{T_0} = \frac{1}{3}$. Therefore $\mathrm{rect}(2(t-1)) * \mathrm{comb}\left(\frac{t}{3}\right) \xleftrightarrow{FS} \frac{1}{2}\mathrm{sinc}\left(\frac{k}{6}\right)e^{-j\frac{2\pi k}{3}}$.