

Solution to ECE 315 Test #7 F03

1. Based on a representation time, $N_F = 4$, the DTFS harmonic function, $X[k]$, of a DT signal, $x[n]$, has the following values,

$$X[-1] = 2 - j2, \quad X[0] = 4, \quad X[1] = 2 + j2, \quad X[2] = 3$$

(a) What is the numerical value of $X[3]$?

The DTFS is periodic with a period of 4. Therefore $X[3] = X[-1] = 2 - j2$

(b) What is the numerical value of $X[22]$?

Again, using the periodicity of the DTFS, $X[22] = X[22 - 4 \times 5] = X[2] = 3$

(c) What is the numerical average value of $x[n]$?

Since $x[n]$ is periodic its average power is $\frac{1}{N_F} \sum_{n=\langle N_F \rangle} x[n]$. From the DTFS harmonic function definition, $X[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} x[n] e^{-j2\pi \frac{kn}{N_F}}$ and $X[0] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} x[n]$.

Therefore

$$\text{Average value of } x[n] = X[0] = 4.$$

2. The DTFS harmonic function, $X[k]$, of $x[n] = \text{sinc}\left(\frac{n}{4}\right) * \text{comb}_{12}[n]$ with $N_F = N_0$, can be expressed in the form, $X[k] = A \text{rect}_B[k] * \text{comb}_C[k]$.

(a) Find the numerical values of A , B and C .

From the DTFS table,

$$\text{sinc}\left(\frac{n}{w}\right) * \text{comb}_{N_0}[n] \xleftrightarrow{FS} \frac{w}{N_0} \text{rect}_M[k] * \text{comb}_{N_0}[k]$$

(where M is the greatest integer in $\frac{N_0}{2w}$)

$$w = 4, \quad N_0 = 12$$

Therefore $M = \text{greatest integer in } \frac{12}{2 \times 4} = 1$ and

$$\text{sinc}\left(\frac{n}{4}\right) * \text{comb}_{12}[n] \xleftrightarrow{FS} \frac{4}{12} \text{rect}_1[k] * \text{comb}_{12}[k]$$

and $A = 1/3, B = 1, C = 12$.

(b) What is the numerical average signal power of $x[n]$?

By Parseval's theorem, Average signal power of $x[n] = \text{Average signal power of } X[k]$.

$$\text{Average signal power of } X[k] = \sum_{k=\langle N_0 \rangle} |X[k]|^2 = \sum_{k=-1}^1 \left| \frac{1}{3} \right|^2 = \frac{1}{3}$$