Solution to ECE 315 Test #6 F02

1. Which of the periodic CT functions with fundamental period, 2, have trigonometric CTFS harmonic functions for which $X_c[k]=0$, for all $k \geq 1$.

To meet this requirement, the signal must either be an odd function or the sum of an odd function and a constant.

2. A CT function with a fundamental period, T_0, has a CTFS harmonic function,

$$X[k]=a(\delta[k-k_a]+\delta[k+k_a])+jb(\delta[k+k_b]-\delta[k-k_b]) \ .$$

Using only real-valued functions (no j's) write a mathematical expression for the CT function, $x(t)$, corresponding to this CTFS harmonic function assuming that $T_f=T_0$.
Using

$$\sin(2\pi m f_0 t) \xleftrightarrow{\mathcal{FS}} \frac{j}{2}(\delta[k+m]-\delta[k-m])$$
$$(m \text{ an integer})$$
$$\cos(2\pi m f_0 t) \xleftrightarrow{\mathcal{FS}} \frac{1}{2}(\delta[k-m]+\delta[k+m])$$
$$(m \text{ an integer})$$

we get $x(t)=2a\cos(2\pi k_a f_0 t)+2b\sin(\cos 2\pi k_b f_0 t)$.

3. Find the average value, $X[0]$, of a periodic CT function, $x(t)$, with fundamental period, T_0.

The average value is $\frac{1}{T_0}\int_{T_0} x(t)dt$. The functions graphed were all constant in 4 regions, each of which occupied one-fourth of the period. Therefore the average value of the signal is

$$\frac{1}{T_0}\left[\int_{\frac{T_0}{4}} K_1 dt+\int_{\frac{T_0}{4}} K_2 dt+\int_{\frac{T_0}{4}} K_3 dt+\int_{\frac{T_0}{4}} K_4 dt\right]$$

or

$$\frac{1}{T_0}\left[K_1\frac{T_0}{4}+K_2\frac{T_0}{4}+K_3\frac{T_0}{4}+K_4\frac{T_0}{4}\right]=\frac{K_1+K_2+K_3+K_4}{4}$$

which is simply the average of those four values.