Solution to ECE 315 Test 6 F04

1. A periodic signal, $\mathrm{x}(t)$, with fundamental period, $T_0 = 3$ has a CTFS harmonic function, $\mathrm{X}[k] = 5\mathrm{sinc}\left(\frac{k}{8}\right)$, based on a representation time that is equal to its fundamental period, $T_F = T_0$. Let the representation time be changed to $T_F = 3T_0$.

(a) (4 pts) Find the new harmonic function, $\mathrm{X}_3[k]$. The new harmonic function is

$$\mathrm{X}_3[k] = \left\{ \begin{array}{lll} \mathrm{X}\left[\frac{k}{3}\right] & , & \frac{k}{3} \ an \ integer \\ 0 & , & otherwise \end{array} \right\} = \left\{ \begin{array}{lll} 5\mathrm{sinc}\left(\frac{k}{24}\right) & , & \frac{k}{3} \ an \ integer \\ 0 & , & otherwise \end{array} \right\}$$

(b) (4 pts) What is the numerical value of the new harmonic function, $\mathrm{X}_3[k]$ at $k = 9$?

$$\mathrm{X}_3[9] = 5\mathrm{sinc}\left(\frac{9}{24}\right) = 5\frac{\sin\left(\frac{9\pi}{24}\right)}{\frac{9\pi}{24}} = 3.9211$$

(c) (2 pts) Is $\mathrm{x}(t)$even, odd or neither even nor odd? (Circle the correct answer.) Even
Explain how you know. The harmonic function is purely real.

(d) (3 pts) If $\mathrm{y}(t) = \frac{d}{dt}(\mathrm{x}(t))$, find its CTFS harmonic function, $\mathrm{Y}[k]$, based on the original representation time, $T_F = T_0$.
$\mathrm{Y}[k] = j2\pi k f_0 \times 5\mathrm{sinc}\left(\frac{k}{8}\right) = \frac{j10\pi k}{3}\mathrm{sinc}\left(\frac{k}{8}\right)$

(e) (3 pts) What is the numerical value of the harmonic function, $\mathrm{Y}[k]$ at $k = 2$?
$\mathrm{Y}[2] = \frac{j20\pi}{3}\mathrm{sinc}\left(\frac{2}{8}\right) = j18.856$

(f) (2 pts) Is $\mathrm{y}(t)$ even, odd or neither even nor odd? (Circle the correct answer.) Odd
Explain how you know. Its harmonic function is purely imaginary.

(g) (3 pts) If $\mathrm{z}(t) = \int_{-\infty}^{t} \mathrm{x}(\lambda)d\lambda$ is it possible to find a CTFS harmonic function for $\mathrm{z}(t)$ using the original representation time, $T_F = T_0$? (Circle the correct answer.)
No Explain how you know. $\mathrm{X}[0] \neq 0$ therefore the average value of $\mathrm{x}(t)$ is not zero and its integral is not periodic.

Solution to ECE 315 Test 6 F04

1. A periodic signal, $\mathrm{x}(t)$, with fundamental period, $T_0 = 3$ has a CTFS harmonic function, $\mathrm{X}[k] = 4\mathrm{sinc}\left(\frac{k}{5}\right)$, based on a representation time that is equal to its fundamental period, $T_F = T_0$. Let the representation time be changed to $T_F = 3T_0$.

(a) (4 pts) Find the new harmonic function, $\mathrm{X}_3[k]$. The new harmonic function is

$$\mathrm{X}_3[k] = \left\{ \begin{array}{lll} \mathrm{X}\left[\frac{k}{3}\right] & , & \frac{k}{3} \ an \ integer \\ 0 & , & otherwise \end{array} \right\} = \left\{ \begin{array}{lll} 4\mathrm{sinc}\left(\frac{k}{15}\right) & , & \frac{k}{3} \ an \ integer \\ 0 & , & otherwise \end{array} \right\}$$

(b) (4 pts) What is the numerical value of the new harmonic function, $\mathrm{X}_3[k]$ at $k = 9$?

$$\mathrm{X}_3[9] = 4\mathrm{sinc}\left(\frac{9}{15}\right) = 4\frac{\sin\left(\frac{9\pi}{15}\right)}{\frac{9\pi}{15}} = 2.0182$$

(c) (2 pts) Is $\mathrm{x}(t)$even, odd or neither even nor odd? (Circle the correct answer.) Even
Explain how you know. The harmonic function is purely real.

(d) (3 pts) If $\mathrm{y}(t) = \frac{d}{dt}(\mathrm{x}(t))$, find its CTFS harmonic function, $\mathrm{Y}[k]$, based on the original representation time, $T_F = T_0$.
$\mathrm{Y}[k] = j2\pi k f_0 \times 4\mathrm{sinc}\left(\frac{k}{5}\right) = \frac{j8\pi k}{3}\mathrm{sinc}\left(\frac{k}{5}\right)$

(e) (3 pts) What is the numerical value of the harmonic function, $\mathrm{Y}[k]$ at $k = 2$?
$\mathrm{Y}[2] = \frac{j16\pi}{3}\mathrm{sinc}\left(\frac{2}{5}\right) = j12.6808$

(f) (2 pts) Is $\mathrm{y}(t)$ even, odd or neither even nor odd? (Circle the correct answer.) Odd
Explain how you know. Its harmonic function is purely imaginary.

(g) (3 pts) If $\mathrm{z}(t) = \int_{-\infty}^{t} \mathrm{x}(\lambda)d\lambda$ is it possible to find a CTFS harmonic function for $\mathrm{z}(t)$ using the original representation time, $T_F = T_0$? (Circle the correct answer.)
No Explain how you know. $\mathrm{X}[0] \neq 0$ therefore the average value of $\mathrm{x}(t)$ is not zero and its integral is not periodic.