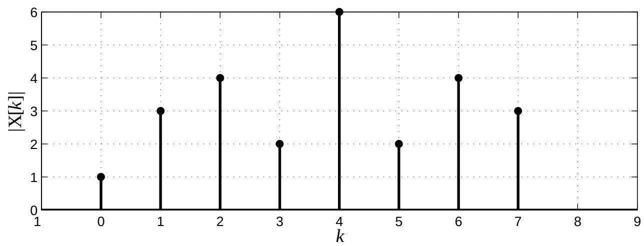
Solution to ECE 315 Test 7 F04

1. A DT signal, $\mathrm{x}[n]$, with a fundamental period of 8 has a DTFS harmonic function, $\mathrm{X}[k]$, using one fundamental period, N_0, as the representation time, N_F, whose magnitude is displayed below over one fundamental period. A new DT signal, $\mathrm{z}[n]$, is formed by expanding $\mathrm{x}[n]$ in time by putting 3 zeros between adjacent values of $\mathrm{x}[n]$,

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}\left[\frac{n}{4}\right] & , \ \frac{n}{4} \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$$

Its harmonic function, $\mathrm{Z}[k]$, is formed using the fundamental period of $\mathrm{z}[n]$ as the representation time, N_F.



(a) (1 pt) What is the fundamental period of $\mathrm{z}[n]$? z is expanded in time by a factor of 4. Therefore its fundamental period is 32.

The time expansion factor is 4. Therefore$\mathrm{Z}[k] = \frac{1}{4}\mathrm{X}[k]$.

(b) (2 pts) What is the numerical value of $|\mathrm{Z}[3]|$? $|\mathrm{Z}[3]| = \frac{1}{4}|\mathrm{X}[3]| = \frac{1}{2}$

(c) (4 pts) What is the numerical value of $|\mathrm{Z}[-1]|$? The DTFS harmonic function, $\mathrm{X}[k]$, is periodic with period, 8. Therefore $|\mathrm{X}[-1]| = |\mathrm{X}[7]| = 3$ and $|\mathrm{Z}[-1]| = |\mathrm{Z}[7]| = \frac{1}{4}|\mathrm{X}[7]| = \frac{3}{4}$.

2. For each DT signal, $\mathrm{x}[n]$, find the requested numerical values of its DTFS harmonic function, $\mathrm{X}[k]$, using the representation time, N_F, specified.

(a) (6 pts) $\mathrm{x}[n] = 4\sin\left(\frac{2\pi n}{12}\right)$, $N_F = 12$, $\mathrm{X}[0] = 0$, $\mathrm{X}[1] = -j2$

$\mathrm{X}[k] = j2\left(\mathrm{comb}_{12}[k+1] - \mathrm{comb}_{12}[k-1]\right)$

(b) (10 pts) $\mathrm{x}[n] = 8\cos\left(\frac{\pi n}{2}\right)\sin\left(\frac{\pi n}{5}\right)$, $N_F = 20$

$\mathrm{X}[2] = 0$, $\mathrm{X}[5] = 0$, $\mathrm{X}[3] = j2$, $\mathrm{X}[-7] = j2$

The two individual periods are 4 and 10. Therefore their harmonic numbers based on a representation time of 20 would be 5 and 2 respectively.

$\mathrm{X}[k] = 4\left(\mathrm{comb}_{20}[k-5] + \mathrm{comb}_{20}[k+5]\right) \circledast \frac{j}{2}\left(\mathrm{comb}_{20}[k+2] - \mathrm{comb}_{20}[k-2]\right)$

$\mathrm{X}[k] = j2\left(\mathrm{comb}_{20}[k-5] + \mathrm{comb}_{20}[k+5]\right) * \left(\delta[k+2] - \delta[k-2]\right)$

$\mathrm{X}[k] = j2\left(\mathrm{comb}_{20}[k-3] - \mathrm{comb}_{20}[k-7] + \mathrm{comb}_{20}[k+7] - \mathrm{comb}_{20}[k+3]\right)$

(c) (6 pts) The values of $\mathrm{x}[n]$ over one fundamental period are $\{3, 6, -1, -5\}$.

$$\mathrm{X}[k] = \frac{1}{4}\sum_{n=\langle 4\rangle} \mathrm{x}[n] e^{-j2\pi kn/4}$$

$$\mathrm{X}[0] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] = \frac{1}{4}(3+6-1-5) = \frac{3}{4}$$

$\mathrm{X}[0] = \frac{3}{4}$

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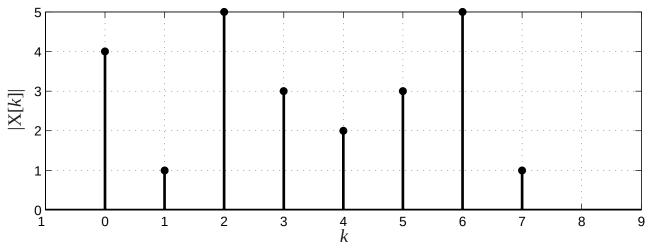
1. A DT signal, $\mathrm{x}[n]$, with a fundamental period of 8 has a DTFS harmonic function, $\mathrm{X}[k]$, using one fundamental period, N_0, as the representation time, N_F, whose magnitude is displayed below over one fundamental period. A new DT signal, $\mathrm{z}[n]$, is formed by expanding $\mathrm{x}[n]$ in time by putting 3 zeros between adjacent values of $\mathrm{x}[n]$,

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}\left[\frac{n}{4}\right] & , \quad \frac{n}{4} \text{ an integer} \\ 0 & , \quad \text{otherwise} \end{cases}$$

Its harmonic function, $\mathrm{Z}[k]$, is formed using the fundamental period of $\mathrm{z}[n]$ as the representation time, N_F.



(a) (1 pt) What is the fundamental period of $\mathrm{z}[n]$? z is expanded in time by a factor of 4. Therefore its fundamental period is 32.

The time expansion factor is 4. Therefore$\mathrm{Z}[k] = \frac{1}{4}\mathrm{X}[k]$.

(b) (2 pts) What is the numerical value of $|\mathrm{Z}[3]|$? $|\mathrm{Z}[3]| = \frac{1}{4}|\mathrm{X}[3]| = \frac{3}{4}$

(c) (4 pts) What is the numerical value of $|\mathrm{Z}[-1]|$? The DTFS harmonic function, $\mathrm{X}[k]$, is periodic with period, 8. Therefore $|\mathrm{X}[-1]| = |\mathrm{X}[7]| = 3$ and $|\mathrm{Z}[-1]| = |\mathrm{Z}[7]| = \frac{1}{4}|\mathrm{X}[7]| = \frac{1}{4}$.

2. For each DT signal, $\mathrm{x}[n]$, find the requested numerical values of its DTFS harmonic function, $\mathrm{X}[k]$, using the representation time, N_F, specified.

(a) (6 pts) $\mathrm{x}[n] = 7\sin\left(\frac{2\pi n}{15}\right)$, $N_F = 15$, $\mathrm{X}[0] = 0$, $\mathrm{X}[1] = -j\frac{7}{2} = -j3.5$

$\mathrm{X}[k] = j2\frac{7}{2}\left(\mathrm{comb}_{15}[k+1] - \mathrm{comb}_{15}[k-1]\right)$

(b) (10 pts) $\mathrm{x}[n] = 3\cos\left(\frac{\pi n}{3}\right)\sin\left(\frac{\pi n}{5}\right)$, $N_F = 30$

$\mathrm{X}[3] = 0$, $\mathrm{X}[5] = 0$, $\mathrm{X}[4] = 0$, $\mathrm{X}[-16] = 0$

The two individual periods are 6 and 10. Therefore their harmonic numbers based on a representation time of 30 would be 5 and 3 respectively.

$\mathrm{X}[k] = \frac{3}{2}\left(\mathrm{comb}_{30}[k-5] + \mathrm{comb}_{30}[k-5]\right) \circledast \frac{j}{2}\left(\mathrm{comb}_{30}[k+3] - \mathrm{comb}_{30}[k-3]\right)$

$\mathrm{X}[k] = j\frac{3}{4}\left(\mathrm{comb}_{30}[k-5] + \mathrm{comb}_{30}[k+5]\right) * \left(\delta[k+3] - \delta[k-3]\right)$

$\mathrm{X}[k] = j\frac{3}{4}\left(\mathrm{comb}_{30}[k-2] - \mathrm{comb}_{30}[k-8] + \mathrm{comb}_{30}[k+8] - \mathrm{comb}_{30}[k+2]\right)$

(c) (6 pts) The values of $\mathrm{x}[n]$ over one fundamental period are $\{5, 9, -3, -2\}$.

$$\mathrm{X}[k] = \frac{1}{4}\sum_{n=\langle 4\rangle} \mathrm{x}[n] e^{-j2\pi kn/4}$$

$$\mathrm{X}[0] = \frac{1}{4}\sum_{n=0}^{3} \mathrm{x}[n] = \frac{1}{4}(5+9-3-2) = \frac{9}{4}$$

$\mathrm{X}[0] = \frac{9}{4}$