Solution of ECE 315 Test 8 F05

1. A periodic DT signal is described over one fundamental period by $\mathrm{x}[n] = \begin{cases} 2 & , \ 0 \le n < 2 \\ 0 & , \ 2 \le n < 5 \end{cases}$. Its harmonic function, with $N_F = N_0$, is $\mathrm{X}[k]$. Find the numerical values of

   (a) $\mathrm{X}[0]$ $\qquad \mathrm{X}[0] = \underline{4/5}$

   (b) $\mathrm{X}[1]$ $\qquad \mathrm{X}[1] = \underline{\mathrm{X}[1] = 0.524 - j0.38 = 0.6472 e^{-j0.628}}$

$$\mathrm{X}[k] = \frac{1}{N_F} \sum_{n=\langle N_F \rangle} \mathrm{x}[n] e^{-j2\pi nk/N_F} = (1/5) \sum_{0}^{1} 2 e^{-j2\pi nk/5} = (2/5)\left(1 + e^{-j2\pi k/5}\right)$$

$$\mathrm{X}[0] = (2/5)(1+1) = 4/5$$
$$\mathrm{X}[1] = (2/5)\left(1 + e^{-j2\pi/5}\right) = 0.524 - j0.38 = 0.6472 e^{-j0.628}$$

2. A periodic DT signal $\mathrm{x}[n]$ with fundamental period 12 has a harmonic function $\mathrm{X}[k]$ with $N_F = N_0$. Let $\mathrm{y}[n] = \begin{cases} \mathrm{x}[n/3] & , \ n/3 \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases}$. Based on a representation time $N_F = 3N_0$ its harmonic function is $\mathrm{Y}[k]$. If $\mathrm{X}[3] = 2 - j5$, what are the numerical values of the harmonic function $\mathrm{Y}[k]$ at the following points. (If it is impossible to determine a numerical value with the information given just write "unknown".)

   Using the time-scaling property of the DTFS

$$\mathrm{z}[n] = \begin{cases} \mathrm{x}[n/m] & , \ n/m \text{ an integer} \\ 0 & , \ \text{otherwise} \end{cases} , N_F \to mN_F \Rightarrow \mathrm{Z}[k] = (1/m)\mathrm{X}[k]$$

   and $\mathrm{X}[k] = \mathrm{X}[k + qN_F]$ for any integer q and $\mathrm{X}[k] = \mathrm{X}^*[-k]$,

   Since $\mathrm{X}[k]$ is periodic with period 12 and $\mathrm{Y}[k] = (1/3)\mathrm{X}[k]$, $\mathrm{Y}[k]$ also has a period of 12, even though its representation time is $N_F = 3 \times 12 = 36$.

   (a) $\mathrm{Y}[3] = \frac{1}{3}\mathrm{X}[3] = \frac{2 - j5}{3} = \frac{2}{3} - j\frac{5}{3} = 1.795 e^{-j1.19}$

   (b) $\mathrm{Y}[39] = \mathrm{Y}[39 + (-3) \times 12] = \mathrm{Y}[3] = \frac{2}{3} - j\frac{5}{3} = 1.795 e^{-j1.19}$

   (c) $\mathrm{Y}[33] = \mathrm{Y}[33 - 3 \times 12] = \mathrm{Y}[-3] = \mathrm{Y}^*[3] = \frac{2}{3} + j\frac{5}{3} = 1.795 e^{+j1.19}$

(d) $Y[-15]$

$$Y[-15] = Y[-15+12] = Y[-3] = Y^*[3] = \frac{2}{3} + j\frac{5}{3} = 1.795e^{+j1.19}$$

(e) $Y[-6]$ $Y[-6]$ = <u>unknown</u>

Solution of ECE 315 Test 8 F05

1. A periodic DT signal is described over one fundamental period by $\mathrm{x}[n]=\begin{cases}5 & , \ 0\le n<2 \\ 0 & , \ 2\le n<5\end{cases}$. Its harmonic function, with $N_F = N_0$, is $\mathrm{X}[k]$. Find the numerical values of

   (a) $\mathrm{X}[0]$ $\qquad\qquad \mathrm{X}[0] = \underline{2}$

   (b) $\mathrm{X}[1]$ $\qquad\qquad \mathrm{X}[1] = \underline{\mathrm{X}[1] = 1.309 - j0.9511 = 1.618e^{-j0.628}}$

$$\mathrm{X}[k]=\frac{1}{N_F}\sum_{n=\langle N_F\rangle}\mathrm{x}[n]e^{-j2\pi nk/N_F}=(1/5)\sum_{0}^{1}5e^{-j2\pi nk/5}=1+e^{-j2\pi k/5}$$

$$\mathrm{X}[0]=1+1=2 \quad \mathrm{X}[1]=1+e^{-j2\pi/5}=1.309-j0.9511=1.618e^{-j0.628}$$

2. A periodic DT signal $\mathrm{x}[n]$ with fundamental period 12 has a harmonic function $\mathrm{X}[k]$ with $N_F = N_0$. Let $\mathrm{y}[n]=\begin{cases}\mathrm{x}[n/3] & , \ n/3 \text{ an integer} \\ 0 & , \ \text{otherwise}\end{cases}$. Based on a representation time $N_F = 3N_0$ its harmonic function is $\mathrm{Y}[k]$. If $\mathrm{X}[3]=1+j3$, what are the numerical values of the harmonic function $\mathrm{Y}[k]$ at the following points. (If it is impossible to determine a numerical value with the information given just write "unknown".)

Using the time-scaling property of the DTFS

$$\mathrm{z}[n]=\begin{cases}\mathrm{x}[n/m] & , \ n/m \text{ an integer} \\ 0 & , \ \text{otherwise}\end{cases}, N_F \to mN_F \Rightarrow \mathrm{Z}[k]=(1/m)\mathrm{X}[k]$$

and $\mathrm{X}[k]=\mathrm{X}[k+qN_F]$ for any integer q and $\mathrm{X}[k]=\mathrm{X}^*[-k]$,

Since $\mathrm{X}[k]$ is periodic with period 12 and $\mathrm{Y}[k]=(1/3)\mathrm{X}[k]$, $\mathrm{Y}[k]$ also has a period of 12, even though its representation time is $N_F = 3\times 12 = 36$.

(a) $\mathrm{Y}[3]=\frac{1}{3}\mathrm{X}[3]=\frac{1+j3}{3}=\frac{1}{3}+j=1.0541e^{j1.249}$

(b) $\mathrm{Y}[39]=\mathrm{Y}[39+(-3)\times 12]=\mathrm{Y}[3]=\frac{1}{3}+j=1.0541e^{j1.249}$

(c) $\mathrm{Y}[33]=\mathrm{Y}[33-3\times 12]=\mathrm{Y}[-3]=\mathrm{Y}^*[3]=\frac{1}{3}-j=1.0541e^{-j1.249}$

(d) $\mathrm{Y}[-15]$

$$\mathrm{Y}[-15]=\mathrm{Y}[-15+12]=\mathrm{Y}[-3]=\mathrm{Y}^*[3]=\frac{1}{3}-j=1.0541e^{-j1.249}$$

(e) $Y[-6]$ $Y[-6]$ = <u>unknown</u>