# Solution of ECE 315 Test 3 F06

1. Below is graphed exactly one period of a periodic function $\mathrm{x}(t)$. Its harmonic function $\mathrm{X}[k]$ (with $T_F = T_0$) can be written as

$$\mathrm{X}[k] = A\,\mathrm{g}(bk)e^{jck} .$$

What is the name of the function $\mathrm{g}(\cdot)$ ? g is <u>$\mathrm{sinc}^2$</u>

What are the numerical values of $A$, $b$ and $c$?
$A$ = <u>9/5</u> , $b$ = <u>3/10</u> , $c$ = <u>$-\pi/5$</u>

$$\mathrm{x}(t) = 6\,\mathrm{tri}((t-2)/6) * \delta_{20}(t)$$

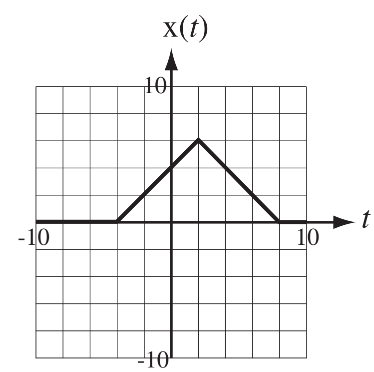
$$(1/w)\,\mathrm{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\mathrm{sinc}^2(wkf_0)$$

$$(1/6)\,\mathrm{tri}(t/6) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (1/20)\,\mathrm{sinc}^2(6k/20)$$

$$6\,\mathrm{tri}(t/6) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (9/5)\,\mathrm{sinc}^2(3k/10)$$

$$6\,\mathrm{tri}((t-2)/6) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (9/5)\,\mathrm{sinc}^2(3k/10)e^{-j2\pi k(2)/20}$$

$$6\,\mathrm{tri}((t-2)/6) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (9/5)\,\mathrm{sinc}^2(3k/10)e^{-j\pi k/5}$$



2. Below is graphed exactly one period of a periodic function $\mathrm{x}[n]$. Its harmonic function $\mathrm{X}[k]$ (with $N_F = N_0$) can be written in the form

$$\mathrm{X}[k] = A\left(Be^{jbk} - e^{jck} + De^{jdk}\right)$$

Find the numerical values of the constants.

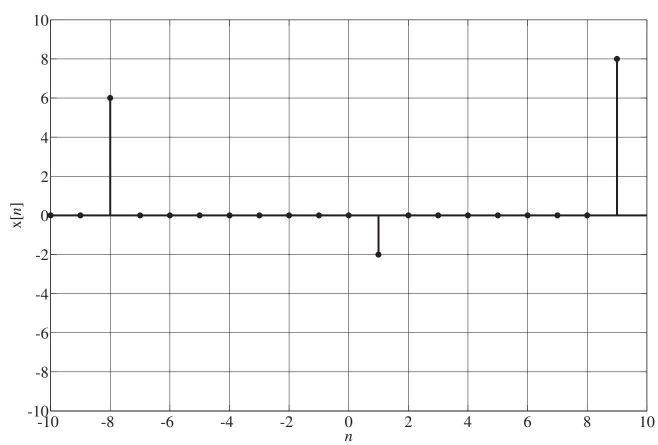
$A =$ <u>1/10</u> , $B =$ <u>3</u> , $b =$ <u>$4\pi/5$</u>, $c =$ <u>$-\pi/10$</u>
$D =$ <u>4</u> , $d =$ <u>$-9\pi/10$</u>

$$\mathrm{x}[n] = 6\delta_{20}[n+8] - 2\delta_{20}[n-1] + 8\delta_{20}[n-9]$$

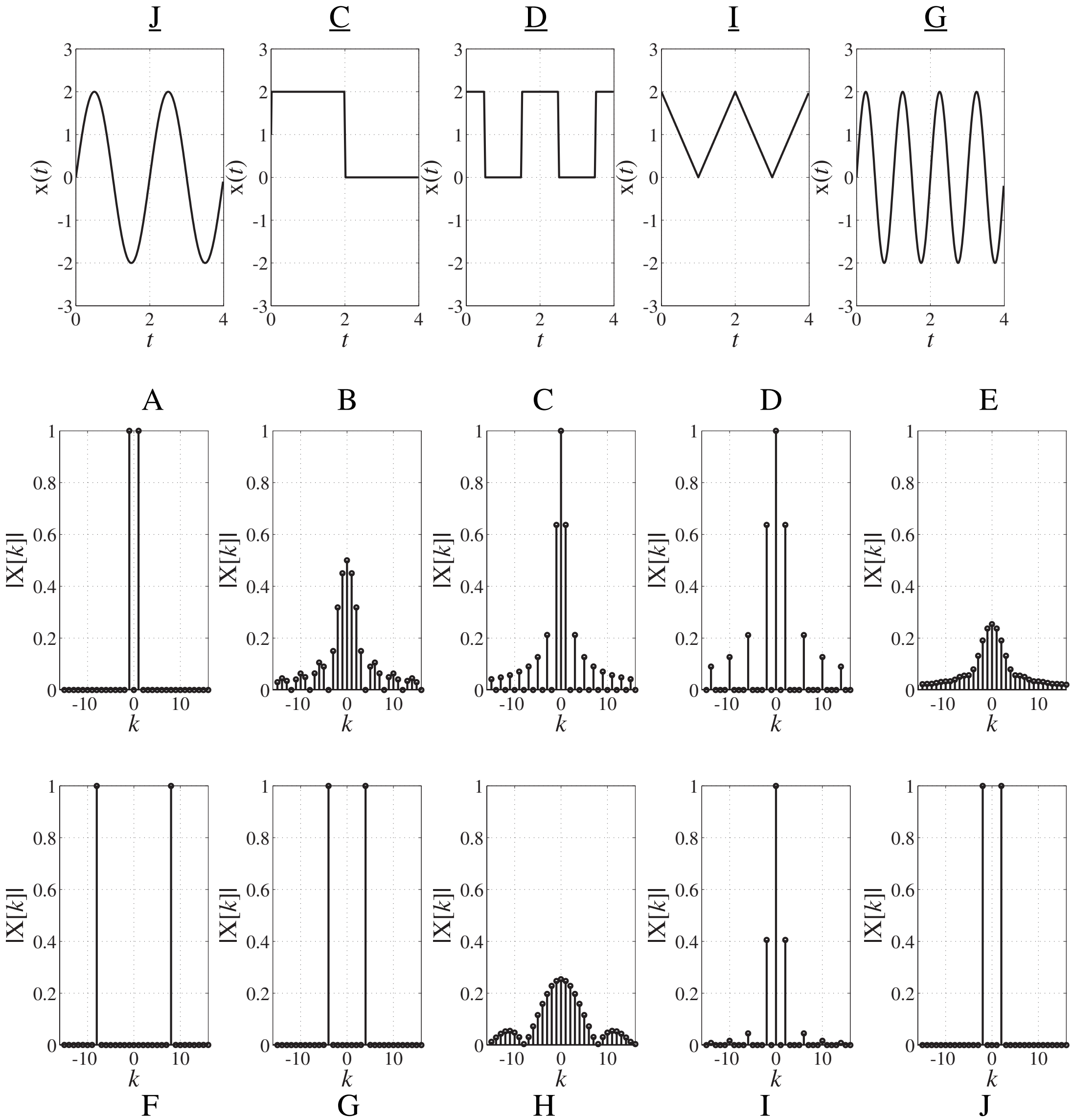
Using $\delta_{N_F}[n] \xleftrightarrow{\mathcal{FS}} 1/N_F$ and $\mathrm{x}[n-n_0] \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k]e^{-j2\pi n_0 k/N_F}$

$$\mathrm{X}[k] = 6(1/20)e^{j16\pi k/20} - 2(1/20)e^{-j2\pi k/20} + 8(1/20)e^{-j18\pi k/20}$$

$$\mathrm{X}[k] = (1/10)\left(3e^{j4\pi k/5} - e^{-j\pi k/10} + 4e^{-j9\pi k/10}\right)$$



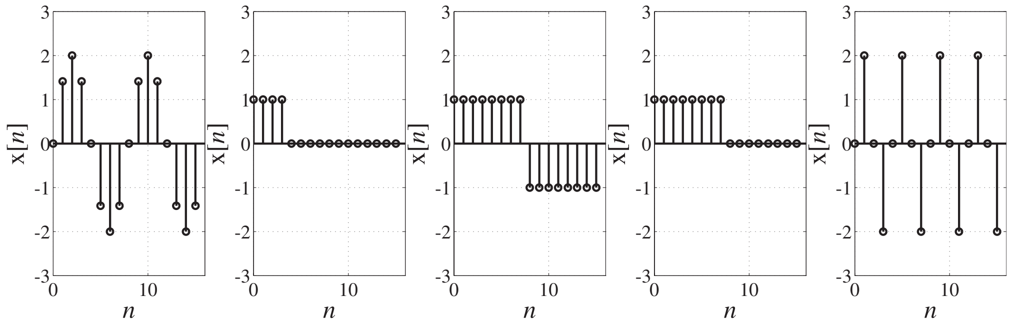
3. Match the CTFS magnitude graphs to the time functions. (In all cases $T_F = 4$.)

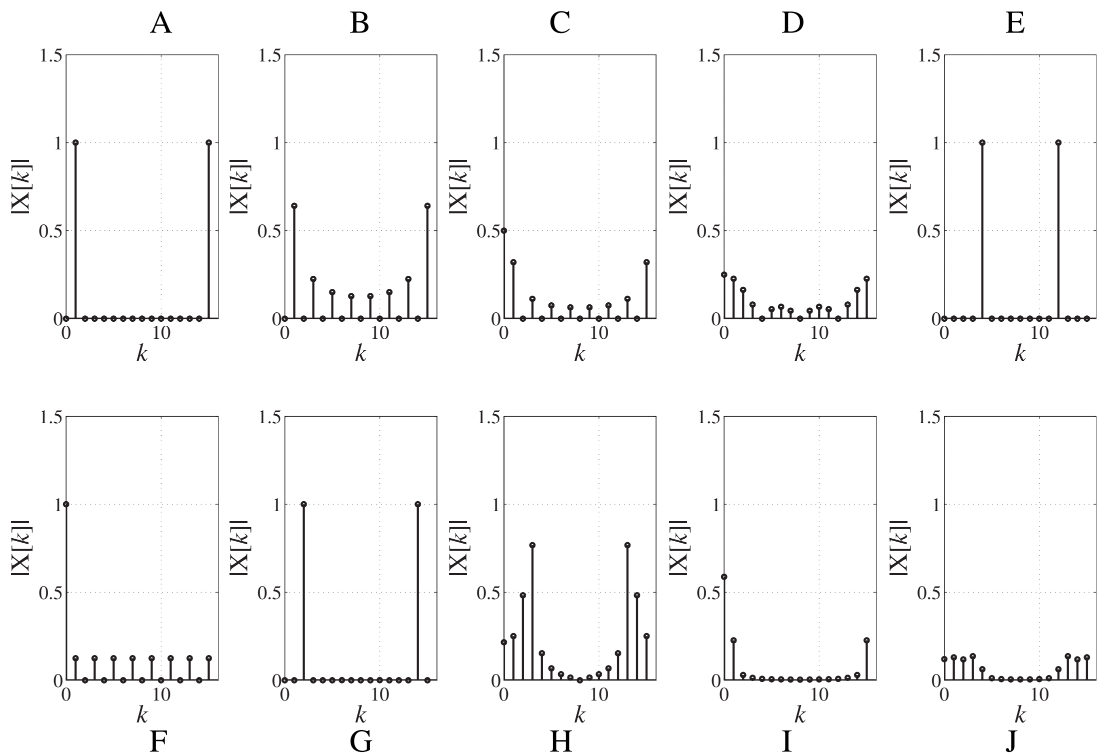



1. Second-harmonic sine function. Non-zero impulses only at $k = \pm 2$.
2. One fundamental period of a square wave with an average value of 1. Harmonic function is a sinc function with a non-zero impulse at the fundamental $k = 1$.
3. Two fundamental periods of a square wave with an average value of 1. Harmonic function is a sinc function with a zero impulse at the fundamental $k = 1$ and a non-zero impulse at $k = 2$.
4. Two fundamental periods of a triangle wave with an average value of 1. Harmonic function is a sinc squared function with a zero impulse at the fundamental $k = 1$ and a non-zero impulse at $k = 2$. The impulses are smaller than for a square wave because of the faster rate of decay of sinc squared compared with sinc.
5. Fourth-harmonic sine function. Non-zero impulses only at $k = \pm 4$.

4. Match the DTFS magnitude graphs to the time functions. (In all cases $N_F = 16$.)

G D B C E




1. Second-harmonic sine function. Non-zero impulses only at $k = \pm 2$ repeated periocidally with period 16.
2. Pulse train with average value of 1/4. Harmonic function has a Dirichlet shape (periodically-repeated sinc functions).
3. Square wave with an average value of zero and fundamental period equal to representation time. Zero at $k = 0$ and non-zero at $k = \pm 1$ repeated periodically with period 16.
4. Pulse train with average value of 1/2. Harmonic function has a Dirichlet shape (periodically-repeated sinc functions) narrower than #2 because the time-domain pulse is twice as wide.
5. Fourth-harmonic sine function. Non-zero impulses only at $k = \pm 4$ repeated periodically with period 16.

# Solution of ECE 315 Test 3 F06

1. Below is graphed exactly one period of a periodic function $x(t)$. Its harmonic function $X[k]$ (with $T_F = T_0$) can be written as

$$X[k] = A\,g(bk)e^{jck}.$$

What is the name of the function $g(\cdot)$ ? g is $\underline{\text{sinc}^2}$

What are the numerical values of *A*, *b* and *c*?
$A = \underline{8/5}$ , $b = \underline{1/5}$ , $c = \underline{-\pi/5}$

$$x(t) = 8\,\text{tri}((t-2)/4) * \delta_{20}(t)$$

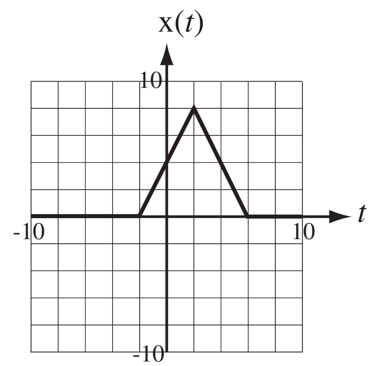
$$(1/w)\,\text{tri}(t/w) * \delta_{T_0}(t) \xleftrightarrow{\mathcal{FS}} f_0\,\text{sinc}^2(wkf_0)$$

$$(1/4)\,\text{tri}(t/4) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (1/20)\,\text{sinc}^2(4k/20)$$

$$8\,\text{tri}(t/4) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (8/5)\,\text{sinc}^2(k/5)$$

$$8\,\text{tri}((t-2)/4) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (8/5)\,\text{sinc}^2(k/5)e^{-j2\pi k(2)/20}$$

$$8\,\text{tri}((t-2)/4) * \delta_{20}(t) \xleftrightarrow{\mathcal{FS}} (8/5)\,\text{sinc}^2(k/5)e^{-j\pi k/5}$$



2. Below is graphed exactly one period of a periodic function $\mathrm{x}[n]$. Its harmonic function $\mathrm{X}[k]$ (with $N_F = N_0$) can be written in the form

$$\mathrm{X}[k] = A\left(e^{jbk} + Ce^{jck} + De^{jdk}\right)$$

Find the numerical values of the constants.

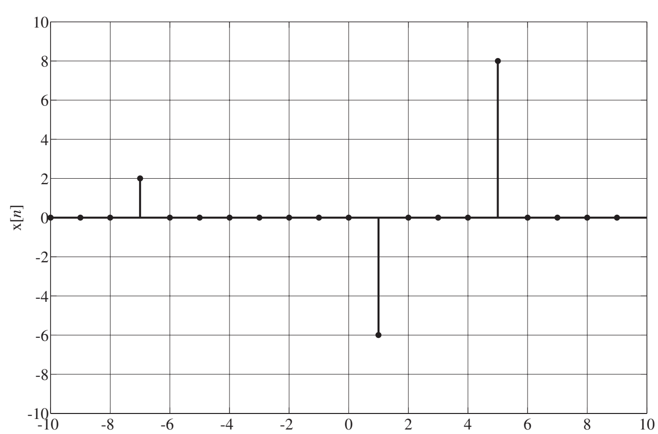
$A =$ <u>1/10</u> , $b =$ <u>$7\pi/10$</u> , $C =$ <u>-3</u>, $c =$ <u>$-\pi/10$</u>
$D =$ <u>4</u> , $d =$ <u>$-\pi/2$</u>

$$\mathrm{x}[n] = 2\delta_{20}[n+7] - 6\delta_{20}[n-1] + 8\delta_{20}[n-5]$$

Using $\delta_{N_F}[n] \xleftrightarrow{\mathcal{FS}} 1/N_F$ and $\mathrm{x}[n-n_0] \xleftrightarrow{\mathcal{FS}} \mathrm{X}[k]e^{-j2\pi n_0 k/N_F}$

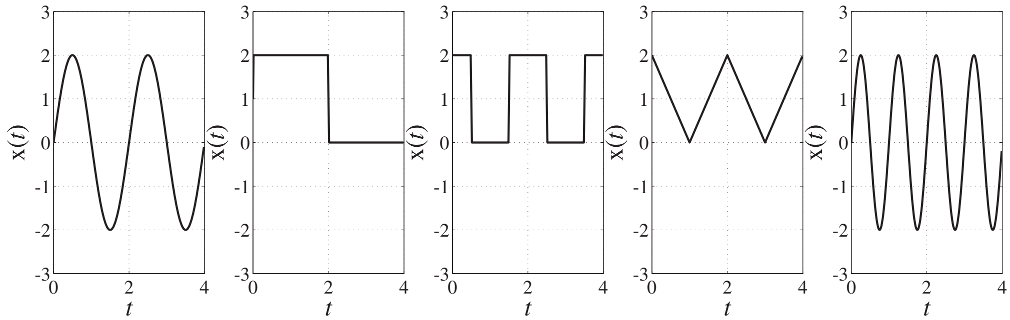
$$\mathrm{X}[k] = 2(1/20)e^{j14\pi k/20} - 6(1/20)e^{-j2\pi k/20} + 8(1/20)e^{-j10\pi k/20}$$

$$\mathrm{X}[k] = (1/10)\left(e^{j7\pi k/10} - 3e^{-j\pi k/10} + 4e^{-j\pi k/2}\right)$$

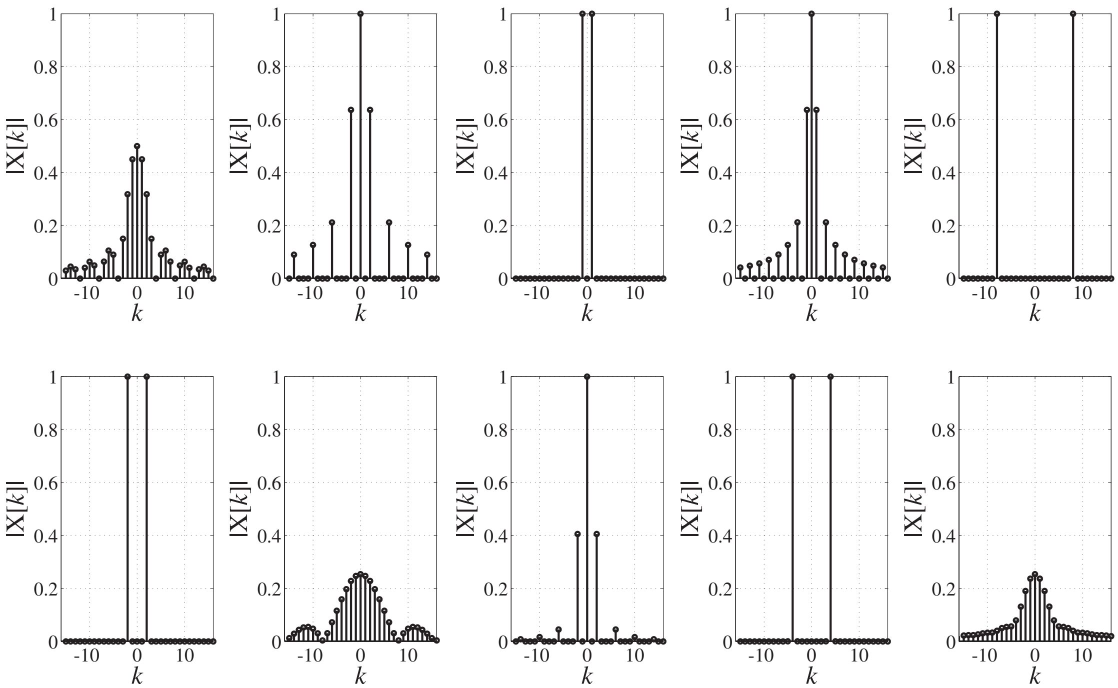


3. Match the CTFS magnitude graphs to the time functions. (In all cases $T_F = 4$.)

<u>F</u> <u>D</u> <u>B</u> <u>H</u> <u>I</u>




A B C D E

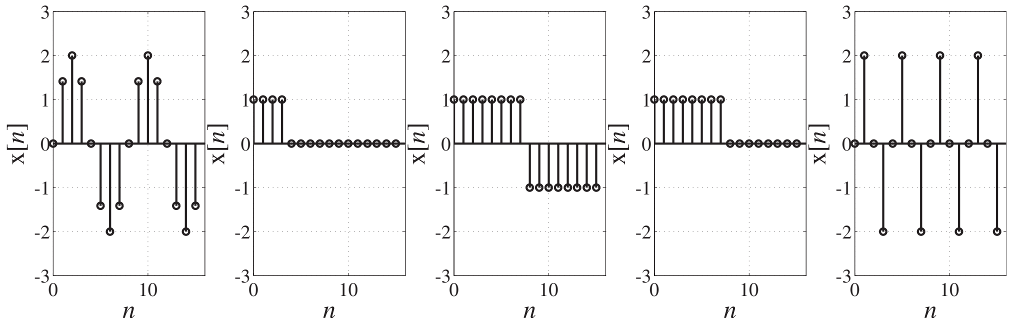



F G H I J

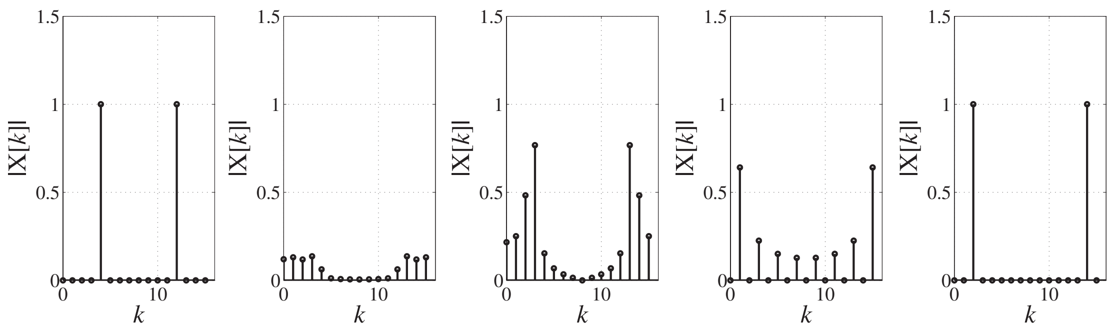
1. Second-harmonic sine function. Non-zero impulses only at $k = \pm 2$.
2. One fundamental period of a square wave with an average value of 1. Harmonic function is a sinc function with a non-zero impulse at the fundamental $k = 1$.
3. Two fundamental periods of a square wave with an average value of 1. Harmonic function is a sinc function with a zero impulse at the fundamental $k = 1$ and a non-zero impulse at $k = 2$.
4. Two fundamental periods of a triangle wave with an average value of 1. Harmonic function is a sinc squared function with a zero impulse at the fundamental $k = 1$ and a non-zero impulse at $k = 2$. The impulses are smaller than for a square wave because of the faster rate of decay of sinc squared compared with sinc.
5. Fourth-harmonic sine function. Non-zero impulses only at $k = \pm 4$.

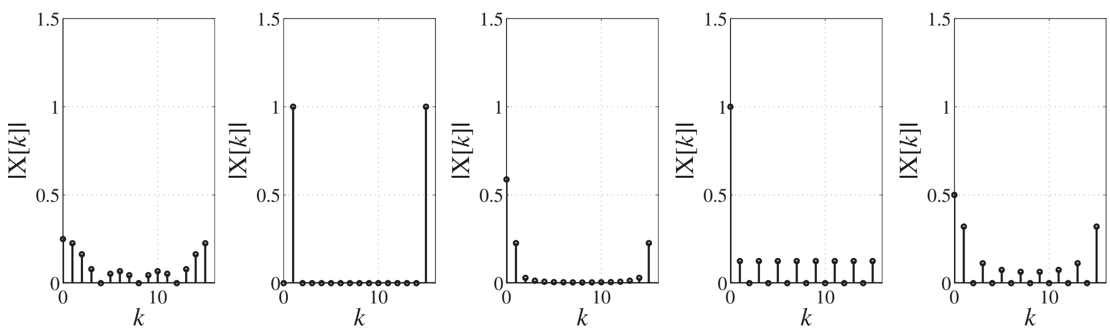
4. Match the DTFS magnitude graphs to the time functions. (In all cases $N_F = 16$.)

<u>E</u>  <u>F</u>  <u>D</u>  <u>J</u>  <u>A</u>




A  B  C  D  E




F  G  H  I  J

1. Second-harmonic sine function. Non-zero impulses only at $k = \pm 2$ repeated periocidally with period 16.
2. Pulse train with average value of 1/4. Harmonic function has a Dirichlet shape (periodically-repeated sinc functions).
3. Square wave with an average value of zero and fundamental period equal to representation time. Zero at $k = 0$ and non-zero at $k = \pm 1$ repeated periodically with period 16.
4. Pulse train with average value of 1/2. Harmonic function has a Dirichlet shape (periodically-repeated sinc functions) narrower than #2 because the time-domain pulse is twice as wide.
5. Fourth-harmonic sine function. Non-zero impulses only at $k = \pm 4$ repeated periodically with period 16.