

Solution to ECE 315 Test #8 F03

- Saying a signal has an infinite period and saying that it is aperiodic are really saying the same thing. (I also accepted “non-periodic”, “not periodic”, “unperiodic” and “non-repeating” as correct.)
- The magnitude of the CTFT of any real-valued time-domain signal is always an even function of frequency and the phase of the CTFT is always an odd function of frequency.
- Parseval’s theorem for the CTFT states that the signal energy of a signal is preserved in the transformation process.
- Below are two lists, one of time-domain functions and one of frequency-domain functions. By writing the number of a time-domain function, match the frequency-domain functions to their inverse CTFT’s in the list of time-domain functions. The match must be exact. If no exact match exists, just write “none”. Some of the functions may be written in an unusual or unfamiliar form but still be correct, so check carefully.

	<u>Time Domain</u>	<u>Frequency Domain</u>
1	$-4 \text{comb}(8t)$	<u>14</u> $5[\delta(f - 200) + \delta(f + 200)]$
2	$5 \text{sinc}(2(t+2))$	<u>5</u> $\frac{5}{2} \text{rect}\left(\frac{f}{2}\right) e^{-j4\pi f}$
3	$3\delta(3t-9)$	<u>11</u> $180 \text{sinc}(20f) e^{-j8\pi f}$
4	$-7 \text{sinc}^2\left(\frac{t}{12}\right)$	<u>4</u> $-84 \text{tri}(12f)$
5	$5 \text{sinc}(2(t-2))$	<u>13</u> $-96 \text{sinc}(4f) e^{j2\pi f}$
6	$5 \cos(200\pi t)$	<u>1</u> $-\frac{1}{2} \text{comb}\left(-\frac{f}{8}\right)$
7	$2 \text{tri}\left(\frac{t+5}{10}\right)$	<u>3</u> $e^{-j6\pi f}$
8	$3\delta(t-3)$	<u>none</u> $10 \text{sinc}^2(5f) e^{j10\pi f}$
9	$-24[u(t+1) - u(t-3)]$	
10	$-8 \text{comb}(-4t)$	
11	$9 \text{rect}\left(\frac{t-4}{20}\right)$	
12	$2 \text{tri}\left(\frac{t+10}{5}\right)$	
13	$-24[u(t+3) - u(t-1)]$	
14	$10 \cos(400\pi t)$	

The CTFT’s of these time-domain signals are

$$\begin{aligned}
 -4 \text{comb}(8t) &\xrightarrow{\mathcal{F}} -\frac{1}{2} \text{comb}\left(\frac{f}{8}\right) = -\frac{1}{2} \text{comb}\left(-\frac{f}{8}\right), \text{ (an unshifted comb is an even function)} \\
 5 \text{sinc}(2(t+2)) &\xrightarrow{\mathcal{F}} \frac{5}{2} \text{rect}\left(\frac{f}{2}\right) e^{j4\pi f}, \quad 3\delta(3t-9) = \delta(t-3) \xrightarrow{\mathcal{F}} e^{-j6\pi f} \\
 -7 \text{sinc}^2\left(\frac{t}{12}\right) &\xrightarrow{\mathcal{F}} -84 \text{tri}(12f), \quad 5 \text{sinc}(2(t-2)) \xrightarrow{\mathcal{F}} \frac{5}{2} \text{rect}\left(\frac{f}{2}\right) e^{-j4\pi f} \\
 5 \cos(200\pi t) &\xrightarrow{\mathcal{F}} \frac{5}{2} [\delta(f-100) + \delta(f+100)], \quad 2 \text{tri}\left(\frac{t+5}{10}\right) \xrightarrow{\mathcal{F}} 20 \text{sinc}^2(10f) e^{j10\pi f} \\
 3\delta(t-3) &\xrightarrow{\mathcal{F}} 3e^{-j6\pi f}, \quad -24[u(t+1) - u(t-3)] = -24 \text{rect}\left(\frac{t-1}{4}\right) \xrightarrow{\mathcal{F}} -96 \text{sinc}(4f) e^{-j2\pi f} \\
 -8 \text{comb}(-4t) &\xrightarrow{\mathcal{F}} -2 \text{comb}\left(-\frac{f}{4}\right) = -2 \text{comb}\left(\frac{f}{4}\right), \quad 9 \text{rect}\left(\frac{t-4}{20}\right) \xrightarrow{\mathcal{F}} 180 \text{sinc}(20f) e^{-j8\pi f} \\
 2 \text{tri}\left(\frac{t+10}{5}\right) &\xrightarrow{\mathcal{F}} 10 \text{sinc}^2(5f) e^{j20\pi f}, \quad -24[u(t+3) - u(t-1)] = -24 \text{rect}\left(\frac{t+1}{4}\right) \xrightarrow{\mathcal{F}} -96 \text{sinc}(4f) e^{j2\pi f} \\
 10 \cos(400\pi t) &\xrightarrow{\mathcal{F}} 5[\delta(f-200) + \delta(f+200)]
 \end{aligned}$$