# Solution of ECE 315 Test 10 F06

1. Let $\mathrm{x}[n]$ be a periodic signal with period $N_0 = 5$, one period of which is described by $\mathrm{x}[n] = \begin{Bmatrix} 3 \ , \ 0 \le n < 2 \\ 0 \ , \ 2 \le n < 5 \end{Bmatrix} \ , \ 0 \le n < 5$. Its DTFT can be written in the form

$$\mathrm{X}(F) = Ae^{jBF}\cos(CF)\mathrm{comb}(DF) \ .$$

Find the numerical values of $A$, $B$, $C$ and $D$.

$A$ = 6 , $B$ = -π , $C$ = π , $D$ = 5

$$\mathrm{x}[n] = 3\left(\mathrm{comb}_5[n] + \mathrm{comb}_5[n-1]\right)$$

$$\mathrm{X}(F) = 3\left[\mathrm{comb}(5F) + \mathrm{comb}(5F)e^{-j2\pi F}\right]$$

$$\mathrm{X}(F) = 3\mathrm{comb}(5F)\left(1 + e^{-j2\pi F}\right) = 3\mathrm{comb}(5F)e^{-j\pi F}\left(e^{j\pi F} + e^{-j\pi F}\right)$$

$$\mathrm{X}(F) = 6e^{-j\pi F}\cos(\pi F)\mathrm{comb}(5F)$$

2. Let $\mathrm{X}(F) = 2 - j3\sin(4\pi F)$. Its inverse DTFT is $\mathrm{x}[n]$. Fill in the table below with numbers.

| $n$ | –3 | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $\mathrm{x}[n]$ | 0 | –3 / 2 | 0 | 2 | 0 | 3 / 2 |

$$\mathrm{X}(F) = 2 - j3\frac{e^{j4\pi F} - e^{-j4\pi F}}{j2} = 2 - 3\frac{e^{j4\pi F} - e^{-j4\pi F}}{2}$$

$$\mathrm{x}[n] = 2\delta[n] - (3/2)\left(\delta[n+2] - \delta[n-2]\right)$$

| $n$ | –3 | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $\mathrm{x}[n]$ | 0 | –3 / 2 | 0 | 2 | 0 | 3 / 2 |

# Solution of ECE 315 Test 10 F06

1. Let $\mathrm{x}[n]$ be a periodic signal with period $N_0 = 6$, one period of which is described by $\mathrm{x}[n] = \begin{cases} 0 \ , \ 0 \le n < 4 \\ 5 \ , \ 4 \le n < 6 \end{cases} \ , \ 0 \le n < 6$. Its DTFT can be written in the form

$$\mathrm{X}(F) = Ae^{jBF}\cos(CF)\mathrm{comb}(DF) \ .$$

Find the numerical values of *A* , *B*, *C* and *D*.

*A* = 10 , *B* = -9π , *C* = π , *D* = 6

$$\mathrm{x}[n] = 5\left(\mathrm{comb}_6[n-4] + \mathrm{comb}_6[n-5]\right)$$

$$\mathrm{X}(F) = 5\left[\mathrm{comb}(6F)e^{-j8\pi F} + \mathrm{comb}(6F)e^{-j10\pi F}\right]$$

$$\mathrm{X}(F) = 5\,\mathrm{comb}(6F)\left(e^{-j8\pi F} + e^{-j10\pi F}\right) = 5\,\mathrm{comb}(6F)e^{-j9\pi F}\left(e^{j\pi F} + e^{-j\pi F}\right)$$

$$\mathrm{X}(F) = 10e^{-j9\pi F}\cos(\pi F)\mathrm{comb}(6F)$$

2. Let $\mathrm{X}(F) = -1 - j4\sin(6\pi F)$. Its inverse DTFT is $\mathrm{x}[n]$. Fill in the table below with numbers.

| $n$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $\mathrm{x}[n]$ | −2 | 0 | 0 | −1 | 0 | 0 | 2 |

$$\mathrm{X}(F) = -1 - j4\frac{e^{j6\pi F} - e^{-j6\pi F}}{j2} = -1 - 4\frac{e^{j6\pi F} - e^{-j6\pi F}}{2}$$

$$\mathrm{x}[n] = -\delta[n] - 2\left(\delta[n+3] - \delta[n-3]\right)$$

| $n$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $\mathrm{x}[n]$ | −2 | 0 | 0 | −1 | 0 | 0 | 2 |